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Financial Structure and Instability in an Open Economy

Kenshiro Ninomiya

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Center for Risk Research
Faculty of Economics
SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,
SHIGA 522-8522, JAPAN

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Kenshiro Ninomiya†
Shiga University

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Abstract

The subprime loan mortgage crisis has revived scholarly interest in Minsky's financial instability hypothesis. The related mathematical models present two types of Minskian financial structures, which we identify as the lenders' risk type (LR) and the hedge, speculative and Ponzi type (HSP)

We construct macrodynamic models in a fixed and floating exchange rate system which considers both the LR and HSP financial structures. We examine the effects of international capital mobility and international lenders' risks and demonstrate the significance of the LR and HSP financial structures in the fixed and floating exchange rate system. We emphasize the significance of stable financial structures in order to stabilize dynamic systems in an open economy.

Keywords: Minskian financial structure, financial fragility, financial instability, international capital mobility
JEL classifications: E12, E32, E33, E43

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†Professor of Economics, Faculty of Economics, Shiga University. 1-1-1 Banba, Hikone, Shiga, JAPAN, 522-8522. E-mail: k-nino@biwako.shiga-u.ac.jp, Tel: +81-749-1158, Fax: +81-749-1032.

1 Introduction

The financial instability hypothesis proposed by Hyman P. Minsky (1975, 1982, 1986) has attracted renewed attention since the subprime loan mortgage crisis. Many authors, mainly post-Keynesian economists, employ two types of financial structures in their mathematical models.

Taylor and O'Connell (1985) formulated that lenders' liquidity preferences intensify with a decrease in the expected profit rate ρ. They hypothesized that an increase in the expected profit rate ρ reduces the interest rate i. They also asserted that a true Minsky crisis occurs when the value of derivatives i_ρ turns significantly negative. Semmler (1987) developed Taylor and O'Connell (1985) and explored financial cycles by applying the Hopf bifurcation theorem. Kregel (1997) emphasized that the margins of safety proposed by Minsky are significant for financial instability. When an economic boom reduces lenders' risks, banks - including commercial varieties thereof - promote lending despite erosion in the margin of safety. We identify one of the financial structures as the lenders' risks type (the LR financial structure).

Ninomiya and Tokuda (2017) demonstrated through VAR analysis that Japan's LR financial structure has been fragile since the mid-1990s by expanding upon the work of Taylor and O'Connell (1985). Ninomiya (2016) discussed the financial instability, of the Taylor and O'Connell type (T-O type), and the effect of inflation targeting in a mixed competitive-oligopolistic system that considers the LR financial structure..

Minsky also emphasized increasing financial fragility, which refers to hedging, speculation and Ponzi finance. His financial instability hypothesis is an endogenous financial business cycle theory. Therefore, related mathematical models interpreted enlargement in firms' debt burdens as the source of increasing financial fragility and introduced a dynamic equation for debt burden into models such as the Kaldorian business cycle, Goodwin and Kaleckian models[1].

Some studies explicitly considered the latter type of Minskian financial structure, which we identify as the hedge, speculative and Ponzi type (the HSP financial structure). For example, Nishi (2012a) proposed a Minskian financial structure and introduced the burden of interest-bearing debt into a Kaleckian model[2]. However, he focused on the long run and assumed a constant interest rate: thus, he did not consider an LR financial structure[3].

[1] See, for example, Franke and Semmler (1989), Jarsulic (1990), Adachi (1994), Keen (1995), Asada (2006a, 2006b), Ninomiya (2006), Hein (2007), Charles (2008a, 2008b), Ryoo (2010, 2013), Sasaki and Fujita (2012), Nishi (2012b), and Ninomiya and Tokuda (2017)

Stock-Flow Consistent models also considered the burden of interest-bearing debt and examined debt-led and burdend economic growth. See, for example, Godley and Lavoie(2007), and Dos Santos,and Zezza (2008). Ryoo(2013) also considered the stock-flow relation.

[2] Foley (2003), Lima and Meirelles (2007), Charles (2008c), and Sasaki and Fujita (2014) also considered Minskian financial structures

[3] Charles (2008c) constructed a macroeconomic model that linked capital accumulation and the

Ninomiya (2015) constructed a macrodynamic model that considers the burden of interest-bearing debt as a source of financial instability and cycles. Ninomiya(2016a) constructed simple macrodynamic models, introduced both the LR and HSP financial structures and discussed financial instability and cycles[4].

On the other hand, some studies have discussed financial instability and cycles in an open economy contexts, Sethi (1992), for example, considered financial instability in a fixed exchange rate system and suggested that an increase in the domestic money supply resulting from a current account surplus would destabilize an economy. Asada(1995) demonstrated that when international capital mobility is sufficiently high, the fixed exchange rate system destabilizes an economy. By contrast, the floating exchange rate system imposes stability[5]. Ninomiya (2007) considered the LR financial structure in an open economy and discusses financial instability. Using VAR analysis, Ninomiya and Tokuda (2012) demonstrated that Korea's LR financial structure stabilized after the Asian monetary crisis[6]. However, these studies did not consider the HSP financial structure.

This paper constructs macrodynamic models in a fixed and floating exchange rate system which considers both the LR and HSP financial structures. The paper examines the effects of international capital mobility and international lenders' risks and demonstrates the significance of the LR and HSP financial structures in the fixed and floating exchange rate system. We emphasize the significance of stable financial structures in order to stabilize dynamic systems in an open economy.

The remainder of the paper is organised as follows. Section 2 presents a basic macrodynamic model which considers LR and HSP Minskian financial structure. Section 3 extends the model into fixed and floating rate systems and discusses financial instability. Section 4 concludes.

2 Basic Model

Following Ninomiya (2017), we first clarify LR and HSP Minskian financial structures and present a basic dynamic model.

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state of the financial structure. Sasaki and Fujita (2014) considered dividends in a Kalckain model and assert that cyclical fluctuations can occur such that the financial structure of firms changes periodically between speculative finance and Ponzi finance. Charles (2008c) and Sasaki and Fujita (2014) assumed a constant interest rate and did not consider the LR financial structure.

[4] Ninomiya and Tokuda (2017) also considered the burden of interest-bearing debt.

[5] However, he did not discuss financial instability.

[6] Kregel (2000) regarded the Asian monetary crisis as an example of traditional Minskian financial instability. Ninomiya (2007) and Ninomiya and Tokuda (2012) emphasized risks faced by international lenders in an open economy.

We define the money demand function M^d and the money supply function M^s as

$$M^d = L(Y, i), \quad L_Y \equiv \frac{\partial L}{\partial Y} \gtreqless 0, \quad L_i \equiv \frac{\partial L}{\partial i} < 0, \tag{1}$$

$$M^s = \mu(Y, i)H, \quad \mu_Y \equiv \frac{\partial \mu}{\partial Y} > 0, \quad \mu_i \equiv \frac{\partial \mu}{\partial i} > 0, \tag{2}$$

where, μ is a monetary multiplier. $L_Y < 0$ imply that lenders' liquidity preferences intensify with the decrease in income Y. $\mu_Y > 0$ implies that money supply increases when a bank lends to an expanding economy. The monetary multiplier μ includes the behavior of commercial banks[7]. These effects express lenders' risks (LRs).

Ordering (1) and (2), the interest rate i is determined by equilibrium in the money market as follows:

$$L(Y, i) = \mu(Y, i)H. \tag{3}$$

Solving Equation (3) with respect to the interest rate i, we obtain

$$i = i(Y, H) = i_0 + i_1 Y - i_2 H, \tag{4}$$

$$i_1 \left(\equiv \frac{\partial i}{\partial Y} \right) = -\frac{L_Y - \mu_Y \bar{H}}{L_i - \mu_i \bar{H}} \gtreqless 0.$$

$$i_2 \left(\equiv \frac{\partial i}{\partial H} \right) = -\frac{\mu}{L_i - \mu_i H} > 0,$$

Equation (4) also shows that the interest rate i is reflected by LRs. This is the financial structure of the lenders' risks type (the LR financial structure). As mentioned, LRs are expressed by L_Y and μ_Y. The sign of i_1 depends on the sign of $L_Y - \mu_Y \bar{H}$. We obtain $i_1 < 0$ when $L_Y - \mu_Y \bar{H} < 0$. For example, we obtain $i_1 < 0$ when μ_Y is significant. The monetary multiplier μ includes the behaviour of commercial banks[8]. We assume that high-powered money H is constant ($H = \bar{H}$) in this section.

Real gross profit Π and real wage income H_w are defined as follows:

$$\Pi = \theta Y, \quad 0 < \theta < 1. \tag{5}$$

$$H_w = (1 - \theta)Y, \tag{6}$$

where, θ is the rate of profit sharing.

We assume that an interest payment iD is distributed to rentiers. Firms retain their remaining profit as internal reserves V, obtained by

$$V = \Pi - \delta i D = \theta Y - \delta i D, \quad \delta > 0, \tag{7}$$

[7] Lima and Meirelles (2007) and Ryoo (2013) introduced the effect of bank profitability on credit supply.

[8] Kregel (1997) emphasized that the margins of safety proposed by Minsky are significant for financial instability. When an economic boom reduces LRs, lenders, including commercial banks, promote lending despite erosion in margins of safety.

where i is the interest rate and D denotes firms' debt burdens. δ is an important parameter that expresses one of the effect of interest-bearing debt burden and reflects a risk premium. As δ affects financial conditions of firms, we call this effect the "financial condition effect."

Suppose that investment demand must be financed by adding debt if it is not financed via internal reserves. The dynamic equation expressing debt burden D becomes

$$\dot{D} = I - V = I - (\theta Y - \delta i D). \tag{8}$$

The investment function I is defined as

$$I = g_1 Y - g_2 i D - g_0, \quad g_i > 0, \tag{9}$$

where g_1 represents animal spirits or appropriate investment opportunities. For example, a paucity of appropriate opportunities reduces g_1 even though income Y rises. g_2 implies that a firm curtails investment demand because its debt burden rises. g_2 is another effect of interest-bearing debt burden and expresses the debt effect. $-g_0$ is a depreciation, indicating that investment I falls when income Y is sufficiently small.

Following Nishi (2012), who formulated the HSP-type Minskian financial structure, Ninomiya (2017) formalized the financial regimes as follows:

$$\begin{aligned} &\Pi \geqq \dot{D} + iD, && \text{(hedge finance)} \\ &\Pi \geqq iD, && \text{(speculative finance)} \\ &\Pi < iD, && \text{(Ponzi finance)} \end{aligned} \tag{10}$$

For example, hedge finance means that internal reserves $V (= \Pi - iD)$ exceed the increase in debt burden D. Ponzi finance means that a firm's gross profit (net operating revenue Π) cannot cover its interest payment iD. It is quite important that the interest payment iD is introduced into a dynamic system in order to examine effects of the HSP financial structure.

The consumption function C is assumed to be a linear function of H_w as follows:

$$C = cH_w + C_0 = c(1-\theta)Y + C_0, \quad 0 < c < 1, \quad C_0 > 0, \tag{11}$$

where c is the marginal propensity to consume and C_0 is basic consumption. We assume all interest payments are saved.

The dynamic equation for income Y is formulated as

$$\dot{Y} = \alpha(C + I + G - Y), \quad \alpha > 0. \tag{12}$$

Equation (12) describes the quantity adjustment in the goods market, and α is the speed of adjustment.

Ordering (4), (8), (9), (11) and (12), we obtain the following dynamic system (S_a):

$$\dot{Y} = \alpha[c(1-\theta)Y + C_0 + g_1 Y - g_2 i(Y)D - g_0 + G - Y] \tag{$S_a.1$}$$

$$\dot{D} = g_1 Y - g_2 i(Y)D - g_0 - \theta Y + \delta i(Y)D \tag{$S_a.2$}$$

We adopt the following assumption:

$$g_1 - s > 0 \tag{$A.1$}$$

Kaldorian business cycle models employ a similar assumption[9]. Assumption A.1 indicates that the real factor destabilizes the economy.

The Jacobian matrix of the system (S_a) at equilibrium can be expressed as

$$J_a = \begin{pmatrix} \alpha[(g_1 - s) - g_2 i_1 D & -\alpha g_2 i \\ g_1 - \theta + (\delta - g_2) i_1 D & (\delta - g_2) i \end{pmatrix} \tag{13}$$

Therefore we obtain

$$\mathrm{tr} J_a = \alpha\{(g_1 - s) - g_2 i_1 D\} + (\delta - g_2) i, \tag{14}$$

$$\det J_a == \alpha i[(g_1 - s)\delta + (s - \theta) g_2] > 0, \tag{15}$$

Since we obtain $\det J_a > 0$, stability depends on the sign of $\mathrm{tr} J_a$ as indicated:

1) $g_1 - s - g_2 i_1 D > 0, \quad \delta - g_2 < 0 \Rightarrow \mathrm{tr} J_a \gtreqless 0$: Cycle 1
2) $g_1 - s - g_2 i_1 D > 0, \quad \delta - g_2 > 0 \Rightarrow \mathrm{tr} J_a > 0$: Unstable
3) $g_1 - s - g_2 i_1 D < 0, \quad \delta - g_2 < 0 \Rightarrow \mathrm{tr} J_a < 0$: Stable
4) $g_1 - s - g_2 i_1 D < 0, \quad \delta - g_2 > 0 \Rightarrow \mathrm{tr} J_a \gtreqless 0$: Cycle 2

Stability depends on the signs of $\delta - g_2$ and $g_1 - s - g_2 i_2 D$. The inequality $g_1 - s - g_2 i_2 D > 0$ indicates that the goods market destabilizes the economy. This is usually assumed in closed Kaldorian business cycle models. The dynamic system (S_a) is unstable when $\delta - g_2 > 0$. Both δ and g_2 express the HSP financial structure. The parameter δ (g_2) has a destabilizing (stabilizing) effect on the system[10]. Therefore, the HSP financial structure is stable when $\delta - g_2 < 0$ and fragile (or unstable) when $\delta - g_2 > 0$.

There is one parameter value α_{a1} at which Hopf bifurcation occurs when $\delta - g_2 < 0$. There is at least one closed orbit around equilibrium in the dynamic system (S_a), when α is close to α_{a1}. The LR financial structure destabilizes the economy when $g_1 - s - g_2 i_1 D > 0$. Cycle 1 is similar to Kaldorian business cycle models.

[9] See, for example, Asada (1995) and, Ninomiya (2007).

[10] Asada (2006a, 2006b) also examined the debt effect in terms of it destabilizing dynamic systems. He introduced wege and price Pillips curves into the models. A decrease in price induces an increase in real debt burden that restrains investment demand.

In contrast, $g_1 - s - g_2 i_1 D < 0$ indicates that marginal propensity to invest ($g_1 - g_2 i_1 D$) is smaller than marginal propensity to save (s). The indirect effect ($g_2 i_1 D$) is significant and the LR financial structure stabilizes the economy, Therefore, the goods market stabilizes the economy despite the destabilizing real factor ($g_1 - s > 0$).

The dynamic system (S_a) is stable when $\delta - g_2 < 0$. There is one parameter value α_{a2} at which Hopf bifurcation occurs when $\delta - g_2 > 0$. There is at least one closed orbit around equilibrium in the system (S_a), when α is close to α_{a2}. The HSP financial structure destabilizes the economy when $\delta - g_2 > 0$. Therefore, Cycle 2 is somewhat different from Cycle 1.

The condition, $\delta - g_2 > 0$, means that the HSP financial structure is fragile. The mechanism of instability is as follows. We assume that the level of income Y diverges from the equilibrium point to an upper level as a result of a disturbance. The investment demand I is stimulated by the rise in income and debt burden D also increases. As the financial condition effect ($\delta i D \uparrow$) is significant, debt burden D increases more and more.

$$Y \uparrow \Rightarrow I \uparrow \Rightarrow D \uparrow \Rightarrow \delta i D \uparrow (> I \downarrow) \Rightarrow D \uparrow$$

On the other hand, the condition, $i_1 < 0$, means that the LR financial structure is unstable[11]. The mechanism of instability is as follows. We assume that the level of income Y diverges from the equilibrium point to an upper level as a result of a disturbance. If the lenders' risks decline sharply with the rise in income, the supply of loanable funds increases. As a result, the interest rate i will fall in spite of the rise in income. The decrease in interest rate will stimulate investment demand and income will also increase.

$$Y \uparrow \Rightarrow i \downarrow \Rightarrow I \uparrow \Rightarrow Y \uparrow$$

3 Open Economy

Following Asada(1995), Ninomiya(2007), and Ninomiya and Tokuda(2012), we now extend the basic model into an open economy as follows:

[11] Ninomoiya (2007) examined finacial instability for the case where $i_1 < 0$.

$$\dot{Y} = \alpha(C + I + J - Y), \quad \alpha > 0, \tag{16}$$

$$Q = \beta\left(i - \gamma\zeta(Y) - i_f - \frac{\pi^e - \pi}{\pi}\right), \quad \beta > 0, \quad \gamma > 0, \quad \zeta_Y < 0, \tag{17}$$

$$A = J + Q, \tag{18}$$

$$J = J(Y, \pi), \quad J_Y < 0, \quad J_\pi > 0, \tag{19}$$

$$i = i(Y, H) = i_0 + i_1 Y - i_2 H, \quad i_1 \gtreqless 0, \quad i_2 > 0, \tag{4}$$

$$\dot{D} = I - \theta Y + \delta i D, \quad \delta > 0, \tag{8}$$

$$I = g_1 Y - g_2 i D - g_0, \quad g_i > 0, \tag{9}$$

$$C = c(1 - \theta)Y + C_0, \tag{11}$$

where, J is the balance of the current account (net export), Q is the balance of the capital account, A is the total balance of payments, π is the value of a unit of foreign currency in terms of domestic currency, π^e is expected exchange rate in the near future, and i_f is the expected rate of return for holding foreign bonds except for exchange risk[12]. β is the parameter which quantifies the degree of international capital mobility. $\zeta(Y)$ represents "international lenders' risks", the degree of which is quantified by parameter γ.

Equation (17) reflects that the balance of the capital account is determined by the difference between the domestic interest rate and the expected rate of return of foreign bonds. Equation (18) defines the total balance of payments.

3.1 The case of the fixed exchange rate system

In the case of the fixed exchange rate system, we add three equations as follows:

$$\pi = \bar{\pi} \tag{20}$$

$$\pi^e = \pi \tag{21}$$

$$\dot{H} = A \tag{22}$$

Equations (20) and (21) denote that the exchange rate is given. Equation (22) explains that unless the central bank adopts a so-called sterilization policy, the money supply becomes an endogenous variable under the fixed exchange rate system.

In the above Eqs. (4), (8), (9), (11) and (16)-(22), the dynamic system of fixed exchange rates is complete. We now have the dynamic system of fixed exchange rate

[12] We assume i_f is constant.

(S_b) as follows:

$$\dot{Y} = \alpha \left[c(1-\theta)Y + C_0 + g_1 Y - g_2(i_0 + i_1 Y - i_2 H)D - g_0 + J(Y, \bar{\pi}) - Y \right] \qquad (S_b.1)$$

$$\dot{D} = [g_1 Y - g_2(i_0 + i_1 Y - i_2 H)D - g_0] - \theta Y + \delta(i_0 + i_1 Y - i_2 H)D \qquad (S_b.2)$$

$$\dot{H} = J(Y, \bar{\pi}) + \beta[(i_0 + i_1 Y - i_2 H) - \gamma\zeta(Y) - i_f] \qquad (S_b.3)$$

The Jacobian matrix of the system (S_b) at the equilibrium point can be expressed as

$$J_b = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix}, \qquad (23)$$

where,

$$f_{11} = \alpha[g_1 - s + J_Y - g_2 i_1 D], \quad f_{12} = -\alpha g_2 i < 0, \quad f_{13} = \alpha g_2 i_2 D > 0,$$
$$f_{21} = g_1 + (\delta - g_2) i_1 D - \theta, \quad f_{22} = (\delta - g_2) i, \quad f_{23} = -(\delta - g_2) i_2 D,$$
$$f_{31} = J_Y + \beta(i_1 - \gamma\zeta_Y), \quad f_{32} = 0, \quad f_{33} = -\beta i_2, \quad s = 1 - c(1-\theta),$$

The characteristic equation of the dynamic system (S_b) is

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \qquad (24)$$

where,

$$\begin{aligned} a_1 &= -f_{11} - f_{22} - f_{33} \\ &= -\alpha[g_1 - s + J_Y - g_2 i_1 D] - (\delta - g_2) i + \beta i_2, \end{aligned} \qquad (25)$$

$$\begin{aligned} a_2 &= f_{22}f_{33} + f_{11}f_{33} - f_{13}f_{31} + f_{11}f_{22} - f_{12}f_{21} \\ &= [-(\delta - g_2)i - \alpha(g_1 - s + J_Y) + \alpha g_2 D\gamma\zeta_Y] i_2 \beta \\ &\quad - \alpha g_2 i_2 D J_Y + \alpha(g_1 i - s + J_Y)(\delta - g_2) + \alpha g_2 i (g_1 - \theta), \end{aligned} \qquad (26)$$

$$\begin{aligned} a_3 &= -(f_{11}f_{22} - f_{21}f_{12})f_{33} - (f_{12}f_{23} - f_{13}f_{22})f_{31} \\ &= [\alpha(g_1 - s)\delta + \alpha(\delta - g_2)J_Y - \alpha(-s + \theta)g_2] i\beta i_2. \end{aligned} \qquad (27)$$

The above discussion proves the propositions below. In the case of low international capital mobility ($\beta \to 0$), we obtain Proposition 1 as follows:

Proposition 1: Suppose that *the degree of international capital mobility is sufficiently low* ($\beta \to 0$). *The dynamic system of fixed exchange rate* (S_b) *is locally stable when the LR financial structure is stable* ($i_1 > 0$), *the HSP financial structure is stable* ($\delta - g_2 < 0$) and $g_1 i - s + J_Y < 0$. By contrast, *the dynamic system* (S_b) *is locally unstable when* *the LR financial structure is unstable* ($i_1 < 0$) *or the HSP financial structure is fragile* ($\delta - g_2 > 0$).

Proof. Suppose that the degree of international capital mobility is sufficiently low ($\beta \to 0$). We have $g_1 - s + J_Y - g_2 i_1 D < 0$ when $i_1 > 0$ and the value is sufficiently large. Therefore, we have $a_1 > 0$ if $g_1 - s + J_Y - g_2 i_1 D < 0$ and $\delta - g_2 < 0$. We have $a_2 > 0$ if $\delta - g_2 < 0$ and $g_1 i - s + J_Y < 0$. We have $a_3 > 0$ if $\delta - g_2 < 0$. When β is sufficiently small, we have

$$\begin{aligned} a_1 a_2 - a_3 = &[-\alpha(g_1 - s + J_Y - g_2 i_1 D) - (\delta - g_2) i] \\ &* [-\alpha g_2 i_2 D J_Y + \alpha(g_1 i - s + J_Y)(\delta - g_2) + \alpha g_2 i (g_1 - \theta)]. \end{aligned}$$

We obtain $a_1 a_2 - a_3 > 0$ if $g_1 - s + J_Y - g_2 i_1 D < 0$, $\delta - g_2 < 0$ and $g_1 i - s + J_Y < 0$. Therefore, the Routh-Hurwitz conditions are satisfied in this case.

When $i_1 < 0$ and the absolute value is sufficiently large, we have $g_1 - s + J_Y - g_2 i_1 D > 0$. We may obtain $a_1 < 0$ if $g_1 - s + J_Y - g_2 i_1 D > 0$ or $\delta - g_2 > 0$. The Routh-Hurwitz conditions are not satisfied if $a_1 < 0$. Q.E.D.

As we have examined in the previous section, i_1 expresses as the LR financial structure. Proposition 1 shows that the stability of the dynamic system (S_b) depends on i_1 in the same mechanism as the previous section when the degree of international capital mobility is sufficiently low ($\beta \to 0$). In addition to the LR financial structure, the HSP financial structure is also significant for stability in the same mechanism as the previous section.

Next, we can prove the following propositions when the degree of international capital mobility is sufficiently high ($\beta \to \infty$) and the degree of international lenders' risks is sufficiently low ($\gamma \to 0$).

Proposition 2: *Suppose that the degree of international capital mobility is sufficiently high* ($\beta \to \infty$) *and the degree of international lenders' risks is sufficiently low* ($\gamma \to 0$). *The dynamic system of fixed exchange rate* (S_b) *is locally stable when the HSP financial structure is stable* ($\delta - g_2 < 0$) *and* $g_1 - s + J_Y < 0$. *By contrast, the dynamic system* (S_b) *is locally unstable when the HSP financial structure is fragile* ($\delta - g_2 > 0$) *or* $g_1 - s + J_Y > 0$ and the value is significant.

Proof. Suppose that the degree of international capital mobility is sufficiently high ($\beta \to \infty$) and the degree of *international lenders' risks is sufficiently low*($\gamma \to 0$).

We obtain $a_1 > 0$ from Eq.(25), $a_2 > 0$ from Eq.(26), and $a_3 > 0$ from Eq,(27), when $\delta - g_2 < 0$ *and* $g_1 - s + J_Y < 0$. About $a_1 a_2 - a_3$, we obtain

$$a_1 a_2 - a_3 = \eta_1 \beta^2 + \eta_2 \beta + \eta_3,$$

where,

$$\eta_1 = [-(\delta - g_2) i - \alpha(g_1 - s + J_Y)] i_2^2.$$

Therefore, we obtain $a_1a_2 - a_3 > 0$ when $\delta - g_2 < 0$ *and* $g_1 - s + J_Y < 0$ when β is sufficiently high. In the above discussion, we have $a_1 > 0$, $a_2 > 0$, $a_3 > 0$ and $a_1a_2 - a_3 > 0$. Therefore, Routh-Hurwitz conditions are satisfied in this case.

By contrast, we obtain $a_2 < 0$ when $\delta - g_2 > 0$. We obtain $a_1a_2 - a_3 < 0$ when $g_1 - s + J_Y > 0$ and the value is significant. Therefore, Routh-Hurwitz conditions are not satisfied when $\delta - g_2 > 0$ or $g_1 - s + J_Y > 0$ and the value is significant. Q.E.D.

Proposition 2 demonstrates that the stability of the dynamic system of fixed exchange rate (S_b) depends on the HSP financial structure and the sign of $g_1 - s + J_Y$. The condition, $g_1 - s + J_Y > 0$, means that the instability in the dynamic system (S_b) arises from only the real factor. High international capital mobility cannot mitigate instability in the system. By contrast, $g_1 - s + J_Y < 0$, means that the instability in the dynamic system (S_b) may arise from the LR financial structure. High international capital mobility can mitigate the instability in the system when the HSP financial structure is stable ($\delta - g_2 < 0$).

For example, the unstable LR financial structure occurs when the interest rate i falls in spite of a rise in income Y ($i_1 < 0$). As a result, the domestic interest rate i declines below the expected rate of return for holding foreign bonds i_f if high international capital mobility induces rapid capital outflow overseas, which means that the domestic money supply is inclined to decrease. Therefore, investment demand I is restrained by the rise in the domestic interest rate i and income Y will begin to decline[13].

$$Y \uparrow \Longrightarrow i \downarrow \Longrightarrow i < r_f \Longrightarrow H \downarrow \Longrightarrow i \uparrow \Longrightarrow I \downarrow \Longrightarrow Y \downarrow$$

However, Proposition 2 demonstrates that the high international capital mobility cannot mitigate the unstable HSP financial structure ($\delta - g_2 > 0$). Asada (1995), who did not consider the HSP financial structure, presented instability in the fixed exchange rate system. Proposition 2 posits further instability in the fixed exchange rate system.

Furthermore, we can prove Proposition 3, which follows, by invokeing the Hopf bifurcation theorem.

Proposition 3: *Suppose that the LR financial structure is unstable* ($i_1 < 0$) *and the degree of international lenders' risk is sufficiently low* ($\gamma \to 0$) *in the dynamic system of fixed exchange rates* (S_b)*. There is one parameter value* β_0 *at which Hopf bifurcation occurs when the HSP financial structure is stable* ($\delta - g_2 < 0$), $g_1 - s + J_Y < 0$, *and* $g_1 i - s + J_Y < 0$. *Then if* β *converges on* β_0*, there exists at least one closed orbit around the equilibrium in the system* (S_b)*.*

Proof. See Appendix.

[13] These results are identical to Ninomiya (2007) who did not consider the HSP financial structure. He asserted that a dynamic system of fixed exchange rates becomes stable with high international capital mobility and low international lenders' risks.

Proposition 3 indicates a cycle in the fixed exchange rate system which occurs when the HSP financial structure is stable. This cycle is similar to Ninomiya (2007) who did not consider the HSP financial structure.

We obtain Proposition 4 when the degree of international lenders' risks is high ($\gamma \to \infty$).

Proposition 4: *The dynamic system of fixed exchange rate (S_b) is locally unstable when the degree of international lenders' risks is sufficiently high ($\gamma \to \infty$).*

Proof. When γ is sufficiently large, we obtain

$$\eta_1 = \alpha g_2 D \zeta_Y i_2^2 \gamma + \cdots < 0.$$

If β is sufficiently large, we obtain $a_1 a_2 - a_3 < 0$. Therefore, Routh-Hurwitz conditions are not satisfied in this case.Q.E.D.

Proposition 4 indicates that the dynamic system of fixed exchange rate (S_b) is always unstable when international lenders' risks are high ($\gamma \to \infty$), The mechanism of instability is as follows. We suppose that the economy is in a recession. There is a strong likelihood of default with the decline in income. This situation induces capital outflows and the economy will be in a depression. The instability does not depend on the LR and the HSP financial structures.

$$Y \downarrow \Longrightarrow \gamma\zeta \uparrow (> i) \Longrightarrow i - \gamma\zeta < i_f \Longrightarrow H \downarrow \Longrightarrow i \uparrow \Longrightarrow I \downarrow \Longrightarrow Y \downarrow$$

However, we believe that the stable LR and HSP financial structures are significant in the fixed exchange rate system. The stable financial structures may induce a decline in international lenders' risks. Proposition 1 and Proposition 2 indicate that the dynamic system of fixed exchange rate (S_b) becomes stable in the case of low international lenders' risks when the LR and the HSP financial structures are stable.

3.2 The case of the floating exchange rate system

Next, let's examine the dynamic system of floating exchange rate. We can formulate the model by adding three equations into Eqs. (4), (8), (9), (11) and (16)-(19);

$$A = 0, \tag{28}$$
$$\dot{\pi^e} = \gamma(\pi - \pi^e), \tag{29}$$
$$H = \bar{H}. \tag{30}$$

Equation (28) represents the equilibrium of the total balance of payment. Equation (29) formalizes the adaptive expectation hypothesis concerning the expected exchange rate. Equation (30) means that the money supply becomes an exogenous variable in the system of floating exchange rates.

We obtain the following dynamic system by ordering Eqs.(4), (8), (9), (11), (16)-(19), and (29)-(30):

$$\dot{Y} = \alpha\left[c(1-\theta)Y + C_0 + g_1Y - g_2(i_0 + i_1Y - i_2\bar{H})D - g_0 + J(Y,\pi) - Y\right], \tag{31}$$

$$\dot{D} = [g_1Y - g_2(i_0 + i_1Y - i_2\bar{H})D - g_0] - \theta Y + \delta(i_0 + i_1Y - i_2\bar{H})D, \tag{32}$$

$$A = J(Y,\pi) + \beta[(i_0 + i_1Y - i_2\bar{H}) - \gamma\zeta(Y) - i_f - (\pi^e/\pi) + 1] = 0, \tag{33}$$

$$\dot{\pi^e} = \rho(\pi - \pi^e), \quad \rho > 0, \tag{34}$$

Solving Eq.(33) with respect to π, we have the following equation:

$$\pi = \pi(Y, \pi^e), \tag{35}$$

$$\pi_Y = -\frac{J_Y\pi + \beta(i_1 - \gamma\zeta_Y)\pi}{J_\pi\pi + \beta} \gtreqless 0, \quad \pi_{\pi^e} = \frac{\beta}{J_\pi\pi + \beta} > 0,$$

Substituting Eq.(35) into Eqs.(31) and (34), we define the dynamic system of floating exchange rate (S_c) as follows:

$$\dot{Y} = \alpha[c(1-\theta)Y + C_0 + g_1Y - g_2(i_0 + i_1Y)D - g_0 + J(Y, \pi(Y,\pi^e)) - Y] \tag{S_c.1}$$

$$\dot{D} = [g_1Y - g_2(i_0 + i_1Y)D - g_0] - \theta Y + \delta(i_0 + i_1Y)D \tag{S_c.2}$$

$$\dot{\pi^e} = \rho[\pi(Y,\pi^e) - \pi^e] \tag{S_c.3}$$

The Jacobian matrix of the system (S_c) is given by

$$J_c = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & 0 \\ g_{31} & 0 & g_{33} \end{pmatrix}, \tag{36}$$

$$g_{11} = \alpha[g_1 - s - g_2i_1D + J_Y + J_\pi\pi_Y], \quad g_{12} = -\alpha g_2 i,$$
$$g_{13} = \alpha J_\pi\pi_{\pi^e} > 0, \quad g_{21} = g_1 + (\delta - g_2)i_1D - \theta,$$
$$g_{22} = (\delta - g_2)i, \quad g_{31} = \rho\pi_Y, \quad g_{33} = \rho(\pi_{\pi^e} - 1).$$

Moreover, the characteristic equation of this system is

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \tag{37}$$

where,

$$b_1 = -g_{11} - g_{22} - g_{33}, \tag{38}$$

$$b_2 = g_{11}g_{22} - g_{12}g_{21} + g_{11}g_{33} - g_{13}g_{31} + g_{22}g_{33}, \tag{39}$$

$$b_3 = -g_{11}g_{22}g_{33} + g_{12}g_{21}g_{33} - g_{13}g_{21}g_{32} + g_{13}g_{22}g_{31}, \tag{40}$$

To begin with, we investigate the case where the degree of international capital mobility is sufficiently low ($\beta \to 0$). When β is sufficiently low, we have $\pi_Y \to -J_Y/J_\pi$, $\pi_{\pi^e} \to 0$ and $g_{33} = -\rho$. Therefore, we obtain

$$\begin{aligned}
b_1 &= -\alpha(g_1 - s - g_2 i_1 D) - (1 - g_2)i + \rho, \\
b_2 &= \alpha g_2 i(s - \theta) + \alpha(g_1 - s - g_2 i_1 D)(\delta i - \rho) + \alpha g_2 i \delta i_1 D, \\
b_3 &= -\rho[-\alpha(g_1 - s)\delta i + \alpha g_2 i(-s + \theta)] > 0.
\end{aligned}$$

We can prove the following proposition when the HSP financial structure is stable ($\delta \to 0$) as follows:

Proposition 5: *Suppose that the degree of international capital mobility is sufficiently low* ($\beta \to 0$) and *the HSP financial structure is stable* ($\delta \to 0$). *The stability of the dynamic system of floating exchange rates* (S_c) *depends on the LR financial structure* (i_1). *The dynamic system* (S_c) *is locally stable if* $i_1 > 0$, the value of i_1 is large, and $g_1 - s - g_2 i_1 D < 0$. By contrast, the dynamic system (S_c) *is locally unstable if* $i_1 < 0$ *and the absolute value of* i_1 *is large.*

Proof. Suppose that the degree of international capital mobility is sufficiently low ($\beta \to 0$) and the HSP financial structure is stable ($\delta \to 0$). We obtain

$$\begin{aligned}
b_1 &= -\alpha(g_1 - s - g_2 i_1 D) + g_2 i + \rho, \\
b_2 &= \alpha g_2 i(s - \theta) - \alpha(g_1 - s - g_2 i_1 D)\rho, \\
b_3 &= \alpha g_2 i(s - \theta)\rho > 0,
\end{aligned}$$

$$\begin{aligned}
b_1 b_2 - b_3 = &-\alpha^2(g_1 - s - g_2 i_1 D) g_2 i(s - \theta) + \alpha^2(g_1 - s - g_2 i_1 D)^2 \rho \\
&+ \alpha(g_2 i)^2(s - \theta) - \alpha(g_1 - s - g_2 i_1 D)\rho g_2 i - \alpha(g_1 - s - g_2 i_1 D)\rho^2.
\end{aligned}$$

Therefore, we obtain $b_1 > 0$, $b_2 > 0$, and $b_1 b_2 - b_3 > 0$ if $i_1 > 0$, the value of i_1 is large, and $g_1 - s - g_2 i_1 D < 0$. Because of $b_3 > 0$, the Routh-Hurwitz conditions are satisfied.

On the other hand, we obtain $b_1 < 0$ if $i_1 < 0$ and the absolute value of i_1 is large. Therefore, the Routh-Hurwitz conditions are satisfied in this case. Q.E.D.

Proposition 6: *Suppose that the degree of international capital mobility is sufficiently low* ($\beta \to 0$). *The dynamic system of floating exchange rates* (S_c) is unstable if *the HSP financial structure is fragile (the financial condition effect is large* ($\delta > 0$) *and the debt effect is small* ($g_2 \to 0$)).

Proof. Suppose that and the degree of international capital mobility is sufficiently low ($\beta \to 0$) and the financial condition effect is large ($\delta > 0$). We have

$$b_1 b_2 - b_3 = -\alpha(g_1 - s) i^2 \delta^2 + \cdots .$$

Therefore, we obtain $b_1 b_2 - b_3 < 0$ since $g_1 - s > 0$. The Routh-Hurwitz conditions are not satisfied in this case. Q.E.D.

Proposition 5 and Proposition 6 indicate that the stability of dynamic system of floating exchange rates (S_c) depends on the financial structures in the case of low international capital mobility ($\beta \to 0$). The stable LR and HSP financial structures are also significant in this case.

We examine the case of high international capital mobility ($\beta \to \infty$)[14]. In this case, we obtain the following propositions. In the case of low international lenders' risks ($\gamma \to 0$), we obtain Proposition 7 as follows:

Proposition 7: *Suppose that the degree of international capital mobility is sufficiently high ($\beta \to \infty$) and the degree of international lenders' risks is sufficiently low ($\gamma \to 0$). The dynamic system of floating exchange rates (S_c) is locally stable when the HSP financial structure is stable ($\delta - g_2 < 0$) and the LR financial structure is stable ($i_1 > 0$). By contrast, the dynamic system (S_c) becomes unstable if the LR financial structure is unstable ($i_1 < 0$) or the HSP financial structure is fragile ($\delta - g_2 > 0$).*

Proof. Suppose that and the degree of international capital mobility is sufficiently high ($\beta \to \infty$) and *the degree of international lenders' risks is sufficiently low ($\gamma \to 0$)*. We obtain

$$\begin{aligned}
b_1 &= \alpha J_\pi \pi i_1 \beta + \cdots, \\
b_2 &= -\frac{J_\pi \pi(\delta - g_2)i}{J_\pi \pi + \beta} i_1 \beta + \frac{\alpha J_\pi \pi}{(J_\pi \pi + \beta)^2} i_1 \beta + \cdots, \\
b_3 &= \frac{1}{(J_\pi \pi + \beta)^2}[-\alpha J_\pi(\delta - g_2) i \rho(i_1) \pi \beta^2 + \cdots].
\end{aligned}$$

We obtain $b_1 > 0$ when $i_1 > 0$. We also obtain $b_2 > 0$ and $b_3 > 0$ when $i_1 > 0$ and $\delta - g_2 < 0$.

Ordering (38), (39) and (40), we have

$$b_1 b_2 - b_3 = -g_{11} b_2 - g_{22}(g_{11} g_{22} - g_{12} g_{21}).$$

Therefore, we obtain $b_1 b_2 - b_3 > 0$ if $g_{11} > 0$, $b_2 > 0$, $g_{22} < 0$, and $g_{11} g_{22} - g_{12} g_{21} > 0$. We have

$$g_{11} g_{22} - g_{12} g_{21} = -\frac{J_\pi \pi(\delta - g_2)i}{J_\pi \pi + \beta}(i_1)\beta + \cdots.$$

Therefore, we obtain $b_1 b_2 - b_3 > 0$ if $i_1 > 0$ and $\delta - g_2 < 0$.

In the above discussion, we obtain $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1 b_2 - b_3 > 0$ when $i_1 > 0$ and $\delta - g_2 < 0$. The Routh-Hurwitz conditions are satisfied in this case.

[14] $g_{33} = \rho(\pi_{\pi^e} - 1) = \rho\left(\frac{\beta}{J_\pi \pi + \beta} - \frac{J_\pi \pi + \beta}{J_\pi \pi + \beta}\right) = \rho\left(\frac{-J_\pi \pi}{J_\pi \pi + \beta}\right)$.Therefore, we obtain $g_{33} = 0$ when β is suffficiently large.

By contrast, we obtain $b_1 < 0$ when $i_1 < 0$. We obtain $b_3 < 0$ when $i_1 > 0$ and $\delta - g_2 > 0$. Therefore, the Routh-Hurwitz conditions are not satisfied when $i_1 < 0$ or $\delta - g_2 > 0$. Q.E.D.

In the case of high international lenders' risks ($\gamma \to 0$), we obtain Proposition 8 as follows:

Proposition 8: *Suppose that the degree of international capital mobility is sufficiently high ($\beta \to \infty$) and the degree of international lenders' risks is sufficiently high ($\gamma \to \infty$). The dynamic system of floating exchange rates (S_c) is locally stable when the effect of the interest-bearing debt burden is stable ($\delta - g_2 < 0$). By contrast, the dynamic system (S_c) becomes unstable if the HSP financial structure is fragile ($\delta - g_2 > 0$).*

Proof. Suppose that and the degree of international capital mobility is sufficiently high ($\beta \to \infty$) and *the degree of international lenders' risks is sufficiently high* ($\gamma \to \infty$). We obtain

$$\begin{aligned} b_1 &= \alpha\gamma J_\pi \zeta_Y \pi\beta + \cdots > 0, \\ b_2 &= \frac{J_\pi \pi(\delta - g_2)i}{J_\pi \pi + \beta}\zeta_Y \gamma\beta - \frac{\alpha J_\pi \pi\rho}{(J_\pi \pi + \beta)^2}\zeta_Y \gamma\beta + \cdots, \\ b_3 &= \frac{1}{(J_\pi \pi + \beta)^2}[-\alpha J_\pi(\delta - g_2)i\rho(-\gamma\zeta_Y)\pi\beta^2 + \cdots], \end{aligned}$$

We obtain $b_1 > 0$. We also obtain $b_2 > 0$ and $b_3 > 0$ when $\delta - g_2 < 0$.

About $b_1 b_2 - b_3$, we have

$$b_1 b_2 - b_3 = -g_{11} b_2 - g_{22}(g_{11} g_{22} - g_{12} g_{21}).$$

Therefore, we obtain $b_1 b_2 - b_3 > 0$ if $g_{11}(\text{or } b_1) > 0$, $b_2 > 0$, $g_{22} < 0$, and $g_{11} g_{22} - g_{12} g_{21} > 0$. We have

$$g_{11} g_{22} - g_{12} g_{21} = -\frac{J_\pi \pi(\delta - g_2)i}{J_\pi \pi + \beta}(-\gamma\zeta_Y)\beta + \cdots.$$

Therefore, we obtain $b_1 b_2 - b_3 > 0$ if $\delta - g_2 < 0$.

In the above discussion, we obtain $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1 b_2 - b_3 > 0$ when $\delta - g_2 < 0$. The Routh-Hurwitz conditions are satisfied in this case.

On the contrary, we obtain $b_3 < 0$ when $\delta - g_2 > 0$. Therefore, the Routh-Hurwitz conditions are not satisfied when $\delta - g_2 > 0$. Q.E.D.

Proposition 7 and Proposition 8 indicate that the stability of dynamic system of floating exchange rates (S_c) depends on the LR and HSP financial structures in the case of high capital mobility. The stable financial structures are also significant in the cases of both high and low international lenders' risks.

Asada (1995) and Ninomiya (2007) posited that the floating exchange rate system is stable in the case of high international capital mobility. Ninomiya (2007) presented that the system is stable in the case of high international capital mobility and high international lenders' risks even if the LR financial structure is unstable. As mentioned, Ninomiya (2007) did not consider the HSP financial structure. Proposition 7 and Proposition 8 indicate that the system of floating exchange rates is unstable in the case of high international capital mobility and high international lenders' risks when the HSP financial structure is fragile.

The fixed exchange rate system was severely criticized in the context of the Asian monetary crisis. The fixed exchange rate system was changed to a floating exchange rate system in many Asian countries. However, Proposition 8 implies that the fragile HSP financial structure might be crucial for the Asian monetary crisis. Monetary crises may occur in both fixed and floating exchange rate systems if the HSP financial structures are fragile.

4 Conclusion

This study considered two types of Minskian financial structures—LR and HSP—and discussed financial instability and cycles in an open economy. Kregel (1997) emphasized the significance of margins of safety for financial instability. The LR financial structure affects margins of safety and interest rates. Minsky also emphasized increasing financial fragility, which refers to hedging, speculation and Ponzi finance (HSP). The related mathematical models interpreted enlargement in firms' debt burdens as the source of increasing financial fragility.

The paper examines the effects of international capital mobility and international lenders' risks and demonstrates the significance of the LR and HSP financial structures in fixed and floating exchange rate systems. The main conclusions of this paper are as follows:

In the dynamic system of fixed exchange rates (S_b),

> 1) The stability of the dynamic system (S_b) depends on the LR and HSP financial structures in the case of low international capital mobility.
>
> 2) The stability of the dynamic system (S_b) depends on the HSP financial structure and the sign of $g_1 - s + J_Y$ in the case of high international capital mobility and low international lenders' risks.
>
> 3) There exists at least one closed orbit around the equilibrium in the dynamic system of fixed exchange rate (S_b) under some conditions.
>
> 4)The dynamic system of (S_b) is always unstable when the degree of international lenders' risks is sufficiently high.

In the dynamic system of floating exchange rates (S_c), 5)The stability of dynamic systems (S_c) depends on the LR and HSP financial structures in the case of low capital mobility.

6) The stability of dynamic systems (S_c) depends on the LR and HSP financial structures in the case of high international capital mobility and low international lenders' risks.

7) The stability of dynamic systems (S_c) depends on the HSP financial structure in the case of high international capital mobility and high international lenders' risks.

These conclusions are reflective of the fact that we considered financial structures in dynamic systems. We would like to emphasize the significance of stable financial structures in order to stabilize an open economy. In other words, these conclusions imply that monetary crises can occur both in systems which exhibit fixed or floating exchange rates.

Many aspects of this paper remain to be developed further. For instance, this study is limited to theoretical exposition. Ninomiya and Tokuda (2012) examined the LR financial structure using VAR analysis. In a future study we will examine the HSP financial structure empirically. It is important to examine monetary policy and institutional frameworks in order to avoid financial instability. Furthermore, we need to introduce a dynamic equation of governments' deficits (governments' debt burden) to examine the sovereign problems such as those experienced in Greece.

Proof of Proposition 3 [15]

We suppose that the LR financial structure is unstable ($i_1 < 0$), the HSP financial structure is stable ($\delta - g_2 < 0$), $g_1 - s + J_Y < 0$, $g_1 i - s + J_Y < 0$. and the degree of international lenders' risk is sufficiently low ($\gamma \to 0$) in the dynamic system of fixed exchange rates (S_b).

If β is sufficiently large, we have $a_1a_2 - a_3 > 0$ from the proof of Proposition 2. By contrast, if β is sufficiently small, we have $a_1a_2 - a_3 < 0$ from the proof of Proposition 1 in spite of $\delta - g_2 < 0$. Since $a_1a_2 - a_3$ is a smooth and continuous function with β, there exists at least one value β_0 at which $a_1a_2 - a_3 = 0$ and $\partial(a_1a_2 - a_3)/\partial\beta|_{\beta=\beta_0} \neq 0$. Furthermore, we have $a_2 > 0$ when $\delta - g_2 < 0$, $g_1 - s + J_Y < 0$, and $g_1 i - s + J_Y < 0$.

Hence, the characteristic equation has a pair of imaginary roots $\lambda_1 = \sqrt{a_2}i$, $\lambda_2 = -\sqrt{a_2}i$ at $\beta = \beta_0$. From the Orlando formation, we obtain

$$\begin{aligned} a_1a_2 - a_3 &= -(\lambda_1 + \lambda_2)(\lambda_2 + \lambda_3)(\lambda_3 + \lambda_1) \\ &= -2h(\lambda_3^3 + 2h_1\lambda_3 + h_1^2 + h_2^2) \end{aligned}$$

[15]The method of this proof is based on Asada (1995) and Yoshida (1999). As regard to the Hopf bifurcation theorem, see Gondolfo(1997).

Differentiating this equation with β, we have

$$\frac{\partial(a_1a_2 - a_3)}{\partial\beta} = -2\left[\frac{\partial h_1}{\partial\beta}(\lambda_3^2 + 2h_1\lambda_3 + h_1^2 + h_2^2) + h_1\frac{\partial(\lambda_3^2 + 2h_1\lambda_3 + h_1^2 + h_2^2)}{\partial\beta}\right]$$

Substituting $h_1 = 0$ and $h_2 = h$ into the above equation, we obtain

$$\frac{\partial(a_1a_2 - a_3)}{\partial\beta}|_{\beta=\beta_0} = -2({\lambda_3}^2 + h^2)\left[\frac{\partial h_1}{\partial\beta}|_{\beta=\beta_0}\right]$$

where, h_1 is the real part of two complex conjugate numbers and h_2 is the absolute value of the imaginary part. Therefore, if

$$\frac{\partial(a_1a_2 - a_3)}{\partial\beta}|_{\beta=\beta_0} \neq 0 \quad \text{then} \quad \frac{\partial h_1}{\partial\beta}|_{\beta=\beta_0} \neq 0$$

From the above discussion, all of the conditions in which Hopf bifurcation occurs are satisfied at the point $\beta = \beta_0$.Q.E.D.

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