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Financial Structure, Cycle and Instability

Kenshiro Ninomiya

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Center for Risk Research
Faculty of Economics
SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,
SHIGA 522-8522, JAPAN

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Kenshiro Ninomiya†
Shiga University

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Abstract

The subprime loan mortgage crisis has revived scholarly interest in Minsky's financial instability hypothesis. The related mathematical models present two types of Minskian financial structures. We construct macrodynamic models that consider both structures and discuss financial instability and cycles. We also demonstrate that one of the financial cycles occurs when a real factor stabilizes the economy. The burden of interest-bearing debt is an important determinant of the cycle. We posit that the escalating financial fragility in this cycle is a more appropriate interpretation of the Minskian financial structure that refers to hedging, speculative and Ponzi behaviors. We further demonstrate that another financial structure destabilizes the economy. If the instability occurs at the point of fragility, then the economy may deteriorate into financial crisis. Fragility then becomes instability.

Keywords: Minskian financial structure, financial fragility, business cycle, financial instability.
JEL classifications: E12, E32, E33, E43

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†Professor of Economics, Faculty of Economics, Shiga University. 1-1-1 Banba, Hikone, Shiga, JAPAN, 522-8522. E-mail: k-nino@biwako.shiga-u.ac.jp, Tel: +81-749-1158, Fax: +81-749-1032.

1 Introduction

The financial instability hypothesis proposed by Hyman P. Minsky (1975, 1982, 1986) has attracted renewed attention since the subprime loan mortgage crisis. Many authors, mainly post-Keynesian economists, employ two types of financial structures in their mathematical models.

Taylor and O'Connell (1985) formulated that lenders' liquidity preferences intensify with a decrease in the expected profit rate (ρ). They hypothesized that an increase in the expected profit rate (ρ) reduces the interest rate (i). They also asserted that a true Minsky crisis occurs when the value of derivatives (i_ρ) turns significantly negative. Kregel (1997) emphasized that the margins of safety proposed by Minsky are significant for financial instability. When an economic boom reduces lenders' risks, banks, including commercial ones, promote lending despite erosion in the margin of safety.

Ninomiya (2007) considered these factors in a Kaldorian business cycle model and discusses financial instability as a cycle. Ninomiya and Tokuda (2017) demonstrated that Japan's financial structure has been fragile since the mid-1990s by expanding upon the work of Taylor and O'Connell (1985) and applying VAR analysis. Ninomiya and Tokuda (2012) demonstrated that Korea's financial structure stabilized after the Asian monetary crisis[1]. We identify financial structures similar to aforementioned structures (Japan and Korea) as the lenders' risk type (LR).

Minsky emphasizes increasing financial fragility, which refers to hedging, speculation, and Ponzi finance. His financial instability hypothesis is an endogenous financial business cycle theory. Therefore, related mathematical models interpret enlargement

[1] Kregel (2000) regards the Asian monetary crisis as traditional Minskian financial instability. Ninomiya (2007) and Ninomiya and Tokuda (2012) emphasize risks faced by international lenders in an open economy.

in firms' debt burdens as the source of increasing financial fragility and introduce a dynamic equation for debt burden into a Kaldorian business cycle and Goodwin models[2]. Ninomiya (2015) constructed a macrodynamic model that considers the burden of interest-bearing debt as a source of financial instability and cycles.

Some studies explicitly consider the latter type of Minskian financial structure[3], which we identify as the hedge, speculative and Ponzi type (HSP). Nishi (2012) proposed a revised Minskian financial structure and introduces the burden of interest-bearing debt into a Kaleckian model. Although his definition of hedge finance differs from Foley's (2003), it promises to be widely accepted. Nonetheless, he focused on the long run without discussing financial cycles and assumes a constant interest rate. He did not consider an LR financial structure[4].

This paper constructs simple macrodynamic models, introduces two types of Minskian financial structures and discusses financial instability and cycles. It focuses on the business cycle because the financial instability hypothesis is an endogenous theory of the business cycle. We present a numerical simulation of financial cycles and describe an HSP financial structure. We emphasize the importance of considering both LR and HSP structures in dynamic systems.

Section 2 introduces the two types of Minskian financial structures, presents a basic macrodynamic model in which the interest rate is constant and, finally, discusses finan-

[2] See, for example, Keen (1995), Asada (2006), and Ninomiya (2006).

[3] See Foley (2003), Lima and Meirelles (2007), Charles (2008b), and Sasaki and Fujita (2014)

[4] Charles (2008b) constructs a macroeconomic model linking accumulation of capital and the state of the financial structure. The interest rate in his model is an endogenous variable that depends on the state of the financial structure. However, he did not examine financial cycles.

Sasaki and Fujita(2014) consider dividends in a Kalckain model and suggest that cyclical fluctuations can occur such that the financial structure of firms changes periodically between speculative finance and Ponzi finance. Since we do not consider dividends, we adopts Nishi's definition. Note also that Sasaki and Fujita (2014) also assume a constant interest rate and do not consider the LR financial structure.

cial instability and cycles. Section 3 presents extended models featuring an endogenous interest rate. Section 4 concludes.

2 Financial Structures and Basic Dynamics

2.1 Minskian Financial Structures

We first clarify LR and HSP Minskian financial structure. Real gross profit Π is defined as follows:

$$\Pi = Y^d - \frac{W}{p}N, \tag{1}$$

where, Y^d is the demand side of goods, W is the nominal wage, p is the price level and N represents the level of employment. Following Asada (1995), we assume that disequilibrium in the goods market is compensated by inventory fluctuation and the demand side is always realized ($Y^d = Y$). We also assume that the economy is oligopolistic and the price level p is decided by the mark-up principle as follows:

$$p = (1+\tau)\frac{WN}{Y}, \tag{2}$$

where, τ is the mark-up rate. Therefore, real gross profit Π is:

$$\Pi = Y - \frac{W}{p}N = \frac{\tau}{1+\tau}Y = \theta Y, \tag{3}$$

where, θ is the rate of profit sharing.

We assume that an interest payment iD is distributed to rentiers. Firms retain their remaining profit as internal reserves V, obtained by

$$V = \Pi - iD = \theta Y - iD, \tag{4}$$

where i is the interest rate and D denotes firms' debt burdens.

Following Nishi (2012), who formulated the HSP-type Minskian financial structure, we formalize the financial regimes as follows:

$$\Pi \geqq \dot{D} + iD, \qquad \text{(hedge finance)} \tag{5}$$

$$\Pi \geqq iD, \qquad \text{(speculative finance)} \tag{6}$$

$$\Pi < iD, \qquad \text{(Ponzi finance)} \tag{7}$$

where $\dot{D}$ denotes the change in debt burden D. For example, hedge finance means that internal reserves $V(= \Pi - iD)$ exceed the increase in debt burden D. Ponzi finance means that a firm's gross profit (net operating revenue Π) cannot cover its interest payment iD.

Suppose that investment demand must be financed by adding debt if it is not financed via internal reserves. The dynamic equation expressing debt burden D becomes

$$\dot{D} = I - V = I - (\theta Y - iD). \tag{8}$$

The investment function I is defined as

$$I = g_1 Y - g_2 iD - g_0, \quad g_i > 0, \tag{9}$$

where g_1 represents animal spirits or appropriate investment opportunities. For example, a paucity of appropriate opportunities reduces g_1 even though income Y rises. g_2 implies that a firm curtails investment demand because its debt burden rises. $-g_0$ is a depreciation that indicates that I falls when Y is sufficiently small.

We, first, begin to examine a basic dynamic system and assume that the interest rate is constant in the system as follows:

$$i = i_0. \tag{10}$$

By ordering (3)-(10), we obtain the following financial regimes:

$$D \leqq \frac{(g_1 - 2\theta)}{(g_2 - 2)i_0}Y - \frac{g_0}{(g_2 - 2)i_0} \qquad \text{(hedge finance)} \tag{11}$$

$$D \leqq \frac{\theta}{i_0}Y, \qquad \text{(speculative finance)} \tag{12}$$

$$D > \frac{\theta}{i_0}Y, \qquad \text{(Ponzi finance)} \tag{13}$$

Figure 1 here.

Figure 1 presents one of the regions in (D, Y) space to the different regimes. The boundary of hedge finance (11) depends on the signs of $g_1 - 2\theta$ and $g_2 - 2$. For example, the coefficient of Y is positive and the intercept on the Y axis is negative when $g_2 - 2 > 0$ and $g_1 - 2\theta > 0$. We suppose that an economy is expanding. $g_1 - 2\theta > 0$ indicates that growth in investment demand I exceeds growth in internal reserves. Therefore, debt burden D increases. In contrast, $g_2 - 2 > 0$ indicates that the decline in investment demand I exceeds the increasing burden of interest-bearing debt iD. These effects lead to the decrease in debt burden. Therefore, the region of hedge finance expands with the rise in income (Figure1-1).

Conversely, the coefficient of Y is negative and the intercept on the Y axis is positive when $g_2-2<0$ and $g_1-2\theta>0$. We again suppose the economy is expanding. $g_2-2<0$ indicates that the increase in burden of interest-bearing debt iD exceeds the decline in investment demand I. This effect leads to the increase in debt burden. Therefore, the region of hedge finance shrinks in an expanding economy in this instance (Figure1-2).

The boundary between speculative and Ponzi finance depends on the parameter θ and the interest rate i_0. The region of Ponzi finance expands when θ falls or i_0 rises. The fall in θ reduces internal reserves, and the rise in i_0 enlarges the burden of interest-bearing debt. Therefore, firms' financial conditions deteriorate.

Note that the region of Ponzi(2) in Figure 1-2 satisfies the condition of hedge finance through the reduction in debt burden D. Accordingly, the decline in investment demand I is highly significant in covering payment obligations iD. Therefore, the economy in the region of Ponzi(2) is as grave as the economy in the region of Ponzi(1).

We, next, formulate the basic dynamic system assuming that interest rate is constant. This means that we cannot consider the LR structure in the basic dynamic system.

Real wage income H_w is obtained from Equation (3) as follows:

$$H_w = \frac{W}{p}N = \frac{1}{1+\tau}Y = (1-\theta)Y, \quad 0<\theta<1. \tag{14}$$

The consumption function C is assumed to be a linear function of H_w:

$$C = cH_w + C_0 = c(1-\theta)Y + C_0, \quad 0<c<1, \quad C_0>0, \tag{15}$$

where c is the marginal propensity to consume and C_0 is basic consumption. We assume all interest payments are saved.

The dynamic equation for income Y is formulated as

$$\dot{Y} = \alpha(C + I + G - Y), \quad \alpha > 0. \tag{16}$$

Equation (16) describes the quantity adjustment in the goods market, and α is the speed of adjustment.

Ordering (8), (9), (10), (15) and (16) obtains the following dynamic system (S_a.1):

$$\dot{Y} = \alpha[c(1-\theta)Y + C_0 + g_1 Y - g_2 i_0 D - g_0 + G - Y] \tag{S_a.1.1}$$

$$\dot{D} = g_1 Y - g_2 i_0 D - g_0 - \theta Y + i_0 D \tag{S_a.1.2}$$

We adopt the following assumption:

$$g_1 - s > 0 \tag{A.1}$$

Assumption A.1 indicates that the real factor destabilizes the economy. Kaldorian business cycle models employ a similar assumption[5].

The loci of $\dot{Y} = 0$ and $\dot{D} = 0$ are as follows:

$$D_{(\dot{Y}=0)} = \frac{g_1 - s}{g_2 i_0} Y + \frac{C_0 + G - g_0}{g_2 i_0}, \tag{17}$$

$$D_{(\dot{D}=0)} = \frac{g_1 - \theta}{g_2 - 1} Y - \frac{g_0}{(g_2 - 1) i_0}. \tag{18}$$

The locus of $\dot{Y} = 0$ is positive by assumption A.1, but the locus of $\dot{D} = 0$ depends on the sign of $g_2 - 1$. The slope is negative when $g_2 - 1 < 0$ (Figure 2-1) and positive when

[5] See, for example, Asada (1995) and, Ninomiya (2007).

$g_2 - 1 > 0$ (Figure 2-2)[6].

The Jacobian matrix of the dynamic system (S_a.1) at equilibrium can be expressed as

$$J_a = \begin{pmatrix} \alpha(g_1 - s) & -\alpha g_2 i_0 \\ g_1 - \theta & (1 - g_2)i_0 \end{pmatrix}, \tag{19}$$

where, $s = 1 - c(1 - \theta)$. Therefore, we obtain

$$\mathrm{tr} J_a = \alpha(g_1 - s) + (1 - g_2)i_0, \tag{20}$$

$$\det J_a = \alpha i_0[g_1 - s + (s - \theta)g_2] > 0. \tag{21}$$

We obtain $\det J_a > 0$ by adopting assumption A.1. Therefore, the stability of the system (S_a.1) depends on only the $\mathrm{tr} J_a$.

Figure 2 here.

The dynamic system (S_a.1) becomes unstable when $1 - g_2 > 0$ in Figure 2-1 through the following mechanism. Suppose income Y descends below equilibrium during an economic downturn. The decrease leads to a decline in profit Π and an expansion in debt burden D. Expansion in D restrains investment demand I. However, debt burdens rise because the upsurge in interest payments iD exceeds the decline in investment demand I. Therefore, D rises with the decline in Y. This mechanism indicates that financial factors destabilize the economy alongside real factors. Note that the burden

[6] Equilibrium income Y^* is

$$Y^* = \frac{g_2(C_0 + G) - (C_0 - g_0 + G)}{g_2(1 - \theta)(1 - c) + (g_1 - s)}.$$

of interest-bearing debt iD is a crucial contributor to financial instability.

$$Y \downarrow \Rightarrow \Pi \downarrow \Rightarrow D \uparrow \Rightarrow I \downarrow < iD \uparrow \Rightarrow D \uparrow$$

In addition, there is one parameter value α_a at which Hopf bifurcation occurs when $1 - g_2 < 0$. Figure 2-2 shows at least one closed orbit around the equilibrium in the system (S_a.1) in this case, when α is close to α_a (Appendix 1). This is a financial cycle with income Y and debt burden D. This cycle occurs via the following mechanism. Suppose the economy occupies Point A in Figure 2-2. Income Y and debt burden D increase at Point A. Rising D restrains investment demand I, and the economy enters recession. In this instance, however, erosion in investment demand I exceeds the greater burden of interest-bearing debt iD. Therefore, debt burden D shrinks. The financial factor stabilizes the economy[7].

$$D \uparrow \Rightarrow I \downarrow (Y \downarrow) > iD \uparrow \Rightarrow D \downarrow$$

Figure 3 presents the relation between an HSP structure (Figure 1) and the dynamic in Figure 2. That is, Figure 3 shows escalating financial fragility during a business cycle. Suppose the economy operates under a hedge finance regime at Point A in Figure 3-1. Income Y increases, and the financial regime shifts from hedge to speculative (Point B). The debt burden D also expands, and the financial regime shifts to Ponzi finance (Point C)[8]. Consequently, the economy enters depression (Point D).

Figure 3-2 shows the other process of escalating financial fragility during the business cycle. We emphasize that the financial factor has a stabilizing role in the economy,

[7] Only this cycle occurs when the interest rate is constant.

[8] If g_2 becomes small at Point C, the economy might fall into a financial crisis and the debt burdens D would continue to increase.

although the financial regime becomes more fragile from the hedge finance to the speculative finance and the Ponzi finance.

Figure 3 here.

Next, we simulate the financial cycle in the basic dynamic system numerically. By enumerating parameters as $c = 0.6$, $\theta = 0.7$, $C_0 = 25$, $g_1 = 2$, $g_2 = 3$, $i = 1(\%)$, $g_0 = 35$ and $G = 20$, we rewrite dynamic system (S_a.2) as follows (See Appendix 2):

$$\dot{Y} = \alpha[1.18Y - 3D + 10] \tag{S_a.2.1}$$

$$\dot{D} = 1.3Y - 2D - 35 \tag{S_a.2.2}$$

By considering (11), (12) and (13), the financial regimes are

$$D \leqq 0.6Y - 35, \qquad \text{(hedge finance)} \tag{22}$$

$$D \leqq 0.7Y, \qquad \text{(speculative finance)} \tag{23}$$

$$D > 0.7Y, \qquad \text{(Ponzi finance)} \tag{24}$$

Figure 4 illustrates there is a closed orbit in the dynamic system (S_a.2) when $\alpha = 1.695$. The equilibrium value of Y is $Y^* \fallingdotseq 82.32$. Figure 4 also illustrates the relation between the financial cycle and the financial structure. The hedge finance regime of (22) satisfies $g_1 - 2\theta > 0$ and $g_2 - 2 > 0$. Figure 4 also illustrates the escalating fragility of an HSP financial structure in the business cycle[9].

Figure 4 here.

[9]The cycle in Figure 4 contains the period $D < 0$. This indicates that firms lend or firms' bank balances exceed their outstanding loans because investment demand is extremely low.

3 Lenders' Risks and Instability

Section 2 assumed a constant interest rate, leaving us unable to examine an LR financial structure in the dynamic system (S_a.1). We now consider the dynamic system in which the interest rate is an endogenous variable. That is, we consider an LR financial structure in addition to the HSP structure.

Following Rose (1969) and Ninomiya (2007, 2015), we define the money supply function M^s as

$$M^s = \mu(Y, i)H, \quad \mu_Y \equiv \frac{\partial \mu}{\partial Y} > 0, \quad \mu_i \equiv \frac{\partial \mu}{\partial i} > 0, \tag{25}$$

where, μ is a monetary multiplier. $\mu_Y > 0$ implies that money supply increases when a bank lends to an expanding economy, This effect expresses LR. The monetary multiplier μ includes the behavior of commercial banks[10]. We assume that high-powered money H is constant ($H = \bar{H}$).

The money demand function M^d is

$$M^d = L(Y, i), \quad L_Y \equiv \frac{\partial L}{\partial Y} \gtreqless 0, \quad L_i \equiv \frac{\partial L}{\partial i} < 0, \tag{26}$$

where, $L_Y < 0$ implies that lenders' liquidity preference intensify with the decrease in income Y. This effect also expresses LR.

Ordering (25) and (26), the interest rate i is determined by equilibrium in the money market as follows:

$$L(Y, i) = \mu(Y, i)\bar{H}. \tag{27}$$

[10] Lima and Meirelles (2007) and Ryoo (2013) introduce the effect of bank profitability on credit supply.

Solving Equation (27) with respect to interest rate i, we obtain

$$i = i(Y), \quad i_Y \left(\equiv \frac{di}{dY} \right) = -\frac{L_Y - \mu_Y \bar{H}}{L_i - \mu_i \bar{H}} \gtrless 0. \tag{28}$$

Equation (28) also shows that interest rate i is reflected by LRs. This is a financial structure of the LR type.

As mentioned, LRs are expressed by L_Y and μ_Y. The sign of i_Y depends on the sign of $L_Y - \mu_Y \bar{H}$. We obtain $i_Y < 0$ when $L_Y - \mu_Y \bar{H} < 0$. For example, we obtain $i_Y < 0$ when μ_Y is significant. The monetary multiplier μ includes the behavior of commercial banks. Kregel (1997) emphasized that the margins of safety proposed by Minsky are significant for financial instability. When an economic boom reduces LRs, lenders, including commercial banks, promote lending despite erosion in margins of safety.

We also obtain $i_Y < 0$ when $L_Y < 0$. This is similar to Taylor and O'Connell's (1985) study. They presented that an economy would fall into a financial crisis when a decline in expected profit rates aggravated the financial condition of firms and increased household preference for liquidity.

Ninomiya (2007) introduced the factors $L_Y < 0$ and $\mu_Y > 0$, and discusses financial instability when $i_Y < 0$. He indicated that the economy becomes unstable even when the real factor $(g_1 - s)$ stabilizes the economy when $i_Y < 0$. We call this instability the "Taylor-O'Connell type financial instability (T-O type)." The mechanism of this instability is as follows. We suppose that an economy is in recession. A decline in income Y raises the interest rate i. An increase in interest rate i restrains investment demand I, and a financial crisis ensues.

$$Y \downarrow \Rightarrow i \uparrow \Rightarrow I \downarrow \Rightarrow Y \downarrow$$

By ordering (8), (9), (15), (16), and (28), we obtain the following dynamic system $(S_b.1)$:

$$\dot{Y} = \alpha[c(1-\theta)Y + C_0 + g_1 Y - g_2 i(Y)D - g_0 + G - Y] \qquad (S_b.1.1)$$

$$\dot{D} = g_1 Y - g_2(iY)D - g_0 - \theta Y + (iY)D \qquad (S_b.1.2)$$

The Jacobian matrix of the dynamic system $(S_b.1)$ at equilibrium can be expressed as

$$J_b = \begin{pmatrix} \alpha[(g_1 - s) - g_2 i_Y D & -\alpha g_2 i \\ g_1 - \theta + (\delta - g_2) i_Y D & (\delta - g_2) i \end{pmatrix} \qquad (29)$$

Therefore we obtain

$$\mathrm{tr} J_b = \alpha\{(g_1 - s) - g_2 i_Y D\} + (\delta - g_2) i, \qquad (30)$$

$$\det J_b = \alpha i[(g_1 - s)\delta + (s - \theta) g_2] > 0, \qquad (31)$$

We obtain $\det J_b > 0$. Therefore, the stability depends on the sign of $\mathrm{tr} J_b$ as indicated:

$$
\begin{array}{ll}
1) & g_1 - s - g_2 i_Y D > 0, \quad 1 - g_2 < 0 \Rightarrow \mathrm{tr} J_b \gtreqless 0 : \text{Cycle 1} \\
2) & g_1 - s - g_2 i_Y D > 0, \quad 1 - g_2 > 0 \Rightarrow \mathrm{tr} J_b > 0 : \text{Unstable} \\
3) & g_1 - s - g_2 i_Y D < 0, \quad 1 - g_2 < 0 \Rightarrow \mathrm{tr} J_b < 0 : \text{Stable} \\
4) & g_1 - s - g_2 i_Y D < 0, \quad 1 - g_2 > 0 \Rightarrow \mathrm{tr} J_b \gtreqless 0 : \text{Cycle 2}
\end{array}
$$

Stability depends on the signs of $1-g_2$ and $g_1-s-g_2 i_Y D$. The inequity $g_1-s-g_2 i_Y D > 0$ indicates that the goods market destabilizes the economy. This is usually assumed in closed Kaldorian models. We should note that the condition is satisfied evenwhen $i_Y < 0$ and the absolute value is significant. This means that the LR financial structure is unstable and the financial factor may stabilize the economy when $g_1 - s - g_2 i_Y D > 0$.

There is one parameter value α_b at which Hopf bifurcation occurs when $1 - g_2 < 0$. There is at least one closed orbit around equilibrium in the system (S_b.1), when α is close to α_b (Appendix 3). Cycle 1 is similar to the cycles in the basic dynamic system (S_a). The system (S_b.1) is unstable when $g_1 - s - g_2 i_Y D > 0$ and $1 - g_2 > 0$. We emphasize that the fragile HSP financial structure also destabilizes the economy when $1 - g_2 > 0$.

In contrast, $g_1 - s - g_2 i_Y D < 0$ indicates that marginal propensity to invest ($g_1 - g_2 i_Y D$) is smaller than marginal propensity to save (s). In other words, the indirect effect ($g_2 i_Y D$) is significant. Therefore, the goods market stabilizes the economy despite the destabilizing real factor ($g_1 - s > 0$). This means that the LR financial structure makes the economy stable. Therefore, the dynamic system (S_b) is stable when $g_1 - s - g_2 i_Y D < 0$ and $1 - g_2 < 0$. The HSP financial structure also stabilizes the economy when $1 - g_2 < 0$.

There is one parameter value α_b at which Hopf bifurcation occurs when $g_1 - s - g_2 i_Y D < 0$ and $1 - g_2 > 0$, which means that the HSP financial structure is fragile. There is at least one closed orbit around equilibrium in System (S_b.1), when α is close to α_b (Appendix 3). In other words, Cycle 2 is quite different from the other cycles.

Although economic boom reduces LRs and margins of safety, lenders continue to lend. On the other hand, recession exacerbates LRs, and they may curtail lending rapidly and drastically. If T-O type financial instability occurs as fragility progresses, the economy may boom or a financial crisis might ensue. Fragility then becomes instability. As mentioned, the system (S_b.1) is unstable when $g_1 - s - g_2 i_Y D > 0$ and $1 - g_2 > 0$. Again, we emphasize it is essential to consider both types of financial structures in dynamic systems.

It is worthwhile to describe monetary policy interventions for coping with financial instability. The dynamic system (S_a) shows stability conditions under a constant interest rate. That is, the economy mirrors the system (S_a) if central bank policy targets the interest rate. We regard interest rate targeting useful in avoiding T-O type financial instability.

Next, we present a numerical simulation in the case of $i_Y > 0$ by giving an example and specify Equation (28) as

$$i = i_1 Y, \quad i_1 > 0. \tag{32}$$

By considering Equation (32), we obtain the following dynamic system (S_b.2):

$$\dot{Y} = \alpha[c(1-\theta)Y + C_0 + g_1 Y - g_2 i_1 Y D - g_0 + G - Y] \qquad (S_b.2.1)$$

$$\dot{D} = g_1 Y - g_2 i_1 Y D - g_0 - \theta Y + i_1 Y D \qquad (S_b.2.2)$$

By ordering (3)-(9), and (32), we obtain these financial regimes in the dynamic system (S_b.2):

$$Y \geqq -\frac{g_0}{(2\theta - g_1) + (g_2 - 2)i_1 D}, \qquad \text{(hedge finance)} \tag{33}$$

$$D \leqq \frac{\theta}{i_2}, \qquad \text{(speculative finance)} \tag{34}$$

$$D > \frac{\theta}{i_2}, \qquad \text{(Ponzi finance)} \tag{35}$$

The boundary of hedge finance (33) depends on the signs of $2\theta - g_1$, $2 - g_2$ and i_1. We offer the following numerical simulation as an example because there are many patterns. The boundary between speculative and Ponzi finance depends on the parameter θ and the parameter of interest rate i_1. The region of Ponzi finance expands when θ decreases or i_1 rises. The decrease in θ reduces internal reserves. The rise in i_1 enhances the burden of interest-bearing debt via the increase in LRs.

We present a numerical simulation of the financial cycle. We enumerate parameters as $c = 0.8$, $\theta = 0.8$, $C_0 = 15$, $g_1 = 2$, $g_2 = 1.1$, $i_1 = 0.1$, $g_0 = 35$, and $G = 10$. Therefore, we rewrite the dynamic system (S_b.2) as follows (see Appendix 4):

$$\dot{Y} = \alpha[1.16Y - 0.11YD - 10] \tag{S_b.3.1}$$

$$\dot{D} = 1.2Y - 0.01YD - 35 \tag{S_b.3.2}$$

By considering (33), (34) and (35), the financial regimes are

$$Y \geqq -\frac{35}{-0.4 - 0.09D}, \qquad \text{(hedge finance)} \tag{36}$$

$$D \leqq 8, \qquad \text{(speculative finance)} \tag{37}$$

$$D > 8. \qquad \text{(Ponzi finance)} \tag{38}$$

Figure 5 shows that there is a closed orbit in dynamic system $(S_b.3)$ when $\alpha = 0.9696$ and the financial structure is HSP. The equilibrium value of Y is $Y^* \risingdotseq 31.15$. This simulation is an example of Cycle 2. Figure 5 also shows the escalation of financial fragility in the business cycle. In addition, the financial factor destabilizes the economy in Cycle 2. Therefore, escalating financial fragility depicted in Cycle 2 is a more appropriate interpretation of an HSP Minskian structure. As previously mentioned, the financial factor may stabilize the economy in Cycle 1.

Figure 5 here.

In the dynamic system (S_b), we suppose that the interest rate i depends on the income$Y$. However, some studies supposed that the interest rate i depends on the debt burden D. We also construct the following dynamic system (S_c) in which the interest rate i depends on the debt burden D.

We define the money supply function M^s as

$$M^s = \mu(i, D)H, \quad \mu_i \equiv \frac{\partial \mu}{\partial i} > 0, \quad \mu_D \equiv \frac{\partial \mu}{\partial D} < 0, \tag{39}$$

$\mu_D < 0$ implies that the money supply shrinks as banks, concerned about firms' increased debt burden D, curtail lending. This effect also expresses LRs.

The money demand function M^d is:

$$M^d = L(i, D), \quad L_i \equiv \frac{\partial L}{\partial i} < 0, \quad L_D \equiv \frac{\partial L}{\partial D} > 0, \tag{40}$$

where, $L_D > 0$ also implies that lenders' liquidity preferences intensify with the decrease in income Y and the increase in firms' debt burden D. This effect also expresses LRs.

Ordering (39) and (40), the interest rate i is determined by equilibrium in the money market as follows:

$$L(i, D) = \mu(i, D)\bar{H}. \tag{41}$$

Solving Equation (41) with respect to interest rate i, we obtain

$$i = i(D), \quad i_D\left(\equiv \frac{\partial i}{\partial D}\right) = \frac{L_D + \mu_D \bar{H}}{L_i - \mu_i \bar{H}} \gtreqless 0, \tag{42}$$

The sign of i_D depends on the sign of $L_D + \mu_D \bar{H}$. We assume $i_D > 0$[11] and specifies as follows:

$$i = i_2 D, \quad i_2 > 0, \tag{43}$$

By ordering (8), (9), (15), (16), and (43), we obtain dynamic system (S_c):

$$\dot{Y} = \alpha[c(1-\theta)Y + C_0 + g_1 Y - g_2(i_2 D)D - g_0 + G - Y] \tag{S_c.1}$$

$$\dot{D} = g_1 Y - g_2(i_1 D)D - g_0 - \theta Y + (i_2 D)D \tag{S_c.2}$$

The Jacobian matrix of the system (S_c) at equilibrium can be expressed as

$$J_c = \begin{pmatrix} \alpha(g_1 - s) & -2\alpha g_2 i_2 D \\ g_1 - \theta & 2(1-g_2)i_2 D \end{pmatrix}. \tag{44}$$

Therefore, we obtain

$$\mathrm{tr} J_c = \alpha(g_1 - s) + (1 - g_2)2 i_2 D, \tag{45}$$

$$\det J_c = \alpha 2 i_2 D[g_1 - s + (s - \theta)g_2] > 0. \tag{46}$$

[11] Many studies assume $i_D > 0$. See Keen (1995), Asada (2006) and Charles (2008a). Ninomiya (2006, 2015) examine the instance when $i_D < 0$ and discuss financial instability.

We also obtain det $J_c > 0$ by adopting assumption A.1 in this case. Therefore, stability of the system depends solely on the sign of trJ_c. The dynamic system (S_c) is unstable when $1 - g_2 > 0$ (Figure 6-1). However, there is one parameter value α_c at which Hopf bifurcation occurs when $1 - g_2 < 0$. In this case, at least one closed orbit around equilibrium in the system (S_c) occurs when α is close to α_c (Figure 6-2) (Appendix 5). These properties are identical to those in the dynamic system (S_a) because $i_D(= i_2) > 0$ also stabilizes the dynamic system (S_c).

Figure 6 here.

By ordering (3)-(9), (43), we obtain the following financial regimes in the dynamic system (S_c):

$$Y \geqq \frac{(2 - g_2)i_2}{(2\theta - g_1)}D^2 - \frac{g_0}{(2\theta - g_1)}, \qquad \text{(hedge finance)} \tag{47}$$

$$Y \geqq \frac{i_2}{\theta}D^2, \qquad \text{(speculative finance)} \tag{48}$$

$$Y < \frac{i_2}{\theta}D^2. \qquad \text{(Ponzi finance)} \tag{49}$$

The boundary of hedge finance also depends on the sign of $2\theta - g_1$ and $2 - g_2$. For example, the coefficient of D^2 and the intercept are positive when $2 - g_2 < 0$ and $2\theta - g_1 < 0$. The former indicates that the decline in investment demand I via the increase in debt burden D exceeds the rising burden of interest-bearing debt iD. This effect leads to the decrease in debt burden. In this case, therefore, the region of hedge finance also expands with the economy.

The boundary between speculative and Ponzi finance depends on the parameter θ and the parameter of interest rate i_2. The region of Ponzi finance expands when θ falls or i_2 rises. A decrease in θ reduces internal reserves. Parameter i_2 captures LRs. For example, an increase in i_2 enhances the burden of interest-bearing debt via the increase in LRs. Therefore, the increase in i_2 reduces the region of hedge finance and enlarges the region of Ponzi finance.

Figure 7 presents one relationship between an HSP structure and the cycle in Figure 6-2. Figure 7 also shows escalating financial fragility in the business cycle. The progression of fragility in the dynamic system (S_c) resembles that in the dynamic system (S_a), although parameter i_2 contains LRs.

Figure 7 here.

4 Conclusion

This study considered two types of Minskian financial structures—LR finance and HSP finance—and discussed financial instability and cycles. Kregel (1997) emphasized the significance of margins of safety for financial instability. LRs affect margins of safety and interest rates. We also simulated financial cycles numerically.

We examined three instances of dynamic systems: 1) when the interest rate is constant (system (S_a)) 2) when it depends on income (system (S_b)) and 3) when it depends on debt burdens (system (S_c)). The system (S_a) can display only the process of escalating financial fragility, which refers to HSP finance during a business cycle. The systems (S_b) and (S_c) can examine LR type financial structures. However, the progression of financial fragility in the system (S_c) resembles that in the system (S_a), although the system (S_c) considers the effects of lenders' risks. We noted that the financial factor has a sabilizing effect in the busuness cycles of the system (S_a) and (S_c).

In contrast, one cycle in the system (S_b) occurs when the financial factor causes economic instability. Therefore, we posit that one of the process of increasing financial fragility in the system (S_b) is a more appropriate interpretation of an HSP Minskian financial structure. Furthermore, we presented Taylor-O'Connell type (T-O type) financial instability occurring in the system (S_b). If instability occurs during the progression of increasing financial fragility of the HSP type, then the economy may deteriorate into financial crisis. Fragility becomes instability. Targeting the interest rate helps to avoid the T-O type financial instability. We emphasized the significance of considering both financial structures in dynamic systems.

However, the models in this paper are only two dimensional systems in debt burden D and income Y. We need to consider the dynamics of price and income share, and examine monetary policy to avoid this instability[12]. We also need to extend our research to an open economy and construct a model in difference equations[13]. Furthermore, our study is a theoretical analysis following Ninomiya and Tokuda (2012), who examine T-O type financial instability using VAR analysis. In a future study we will examine HSP-type instability empirically.

(Appendix 1)[14]

Suppose $1 - g_2 < 0$. The characteristic equation of system (S_a) is

$$\lambda^2 + (-\mathrm{tr}J)\lambda + (\det J) = 0.$$

[12] Ninomiya (2016) examines Taylor-O'Connell financial instability and the effect of inflation-targeting in a mixed competitive-oligopolistic system.

We assume that the rate of profit sharing θ is constant. Sasaki and Fujita (2014) show that the range of fluctuations in business cycles depends on the retention ratio.

[13] See Fazzari, Ferri and Greenberg (2008). Godley and Lavoie (2007) and Dos Santos and Zezza (2008) develop a stock-flow-consistent model. Okishio (1986) examines stock-flow relations among the central bank, commercial banks, firms and households, and, presents it as an IS-BB analysis..

[14] The method of the proof in Appendices 1, 2 and 3 is based on Gandolfo (1997).

A necessary condition of the Hopf bifurcation for complex roots is $\det J_a > 0$, which is satisfied from (21). Regarding $\mathrm{tr} J_a$, we find that

$$\mathrm{tr} J_a \lesseqgtr 0 \Leftrightarrow \alpha_a \lesseqgtr 0, \qquad \alpha_a = \frac{-(1-g_2)i_0}{g_1 - s}.$$

Roots of the characteristic equation are

$$\lambda_{1,2} = -\frac{1}{2}(-\mathrm{tr} J) \pm \sqrt{(-\mathrm{tr} J)^2 - 4(\det J)}.$$

Because $\mathrm{tr} J_a = 0$ for critical value α_a of the parameter, the characteristic equation has a pair of pure imaginary roots, $\lambda_{1,2} = \pm i\sqrt{(\det J_a)}$ (where, $i = \sqrt{-1}$). Roots of the above equation remain a complex conjugate for $(-\mathrm{tr} J_a)$ sufficiently small, namely for α sufficiently near α_a.

We obtain

$$\frac{d(\mathrm{tr} J_a/2)}{d\alpha}_{\alpha=\alpha_a} = \frac{g_1 - s}{2} \neq 0.$$

From the preceding discussion, all conditions for Hopf bifurcation are satisfied at Point $\alpha = \alpha_a$.

(Appendix 2)

We specify the consumption function (15) and investment function (9) as follows:

$$C = c(1-\theta)Y + C_0 = 0.6(1-0.7)Y + 15, \tag{15'}$$

$$I = g_1 Y - g_2 i D - g_0 = 2Y - 3D - 35, \tag{9'}$$

where, $c = 0.6$, $\theta = 0.7$, $C_0 = 25$, $g_1 = 2$, $g_2 = 3$, $i = 1(\%)$, and $g_0 = 35$.

Ordering (8), (16), (9'), (15') and $G = 20$, we obtain

$$\dot{Y} = \alpha[0.6(1 - 0.7)Y + 25 + 2Y - 3D - 35 + 20 - Y],$$
$$\dot{D} = 2Y - 3D - 35 - 0.7Y + 1D.$$

Therefore, we obtain the dynamic system $(S_a.2)$.

(Appendix 3)

$\det J_b > 0$ is satisfied from (31) when $i_1 > 0$. Regarding $\mathrm{tr} J_b$, we find that

$$\mathrm{tr} J_b \lesseqqgtr 0 \Leftrightarrow \alpha_b \lesseqqgtr 0, \qquad \text{when } (g_1 - s) - g_2 i_2 D > 0 \text{ and } 1 - g_2 < 0,$$
$$\mathrm{tr} J_b \lesseqqgtr 0 \Leftrightarrow \alpha_b \gtreqqless 0, \qquad \text{when } (g_1 - s) - g_2 i_2 D < 0 \text{ and } 1 - g_2 > 0,$$
$$\alpha_b = \frac{-(1 - g_2) i_2 Y}{(g_1 - s) - g_2 i_2 D} > 0.$$

Because $\mathrm{tr} J_b = 0$ for the critical value α_b of the parameter. We obtain

$$\left. \frac{d(\mathrm{tr} J_b/2)}{d\alpha} \right|_{\alpha = \alpha_b} = \frac{(g_1 - s) - g_2 i_1 D}{2} \neq 0.$$

From the proceeding discussion, all conditions in which Hopf bifurcation occurs are satisfied at the point $\alpha = \alpha_b$.

(Appendix 4)

We specify the consumption function (15) and investment function (9) as follows:

$$C = c(1 - \theta)Y + C_0 = 0.8(1 - 0.8)Y + 15, \qquad (15")$$
$$I = g_1 Y - g_2 i D - g_0 = 2Y - 1.1 i D - 35, \qquad (9")$$

where, $c = 0.8$, $\theta = 0.8$, $C_0 = 15$, $g_1 = 2$, $g_2 = 1.1$, $i_1 = 0.1$, and $g_0 = 35$. We also specify Equation (32) as follows:

$$i = i_1 Y = 0.1Y. \tag{32'}$$

Ordering (8), (16), (9"), (15"), (32') and $G = 10$, we obtain

$$\dot{Y} = \alpha[0.8(1-0.8)Y + 15 + 2Y - 1.1 * 0.1YD - 35 + 10 - Y],$$
$$\dot{D} = 2Y - 1.1 * 0.1YD - 35 - 0.8Y + 0.1YD,$$

Therefore, we obtain the dynamic system (S_b.3).

(Appendix 5)

Suppose $1 - g_2 < 0$. $\det J_c > 0$ is satisfied from (46). Regarding $\mathrm{tr}J_c$, we find that

$$\mathrm{tr}J_c \lesseqqgtr 0 \Leftrightarrow \alpha_c \lesseqqgtr 0, \qquad \alpha_c = \frac{-(1-g_2)2i_1D}{g_1 - s} > 0.$$

Because $\mathrm{tr}J_c = 0$ for the critical value α_c of the parameter. We obtain

$$\left.\frac{d(\mathrm{tr}J_c/2)}{d\alpha}\right|_{\alpha=\alpha_c} = \frac{g_1 - s}{2} \neq 0.$$

From the preceding discussion, all conditions in which Hopf bifurcation occurs are satisfied at Point $\alpha = \alpha_c$.

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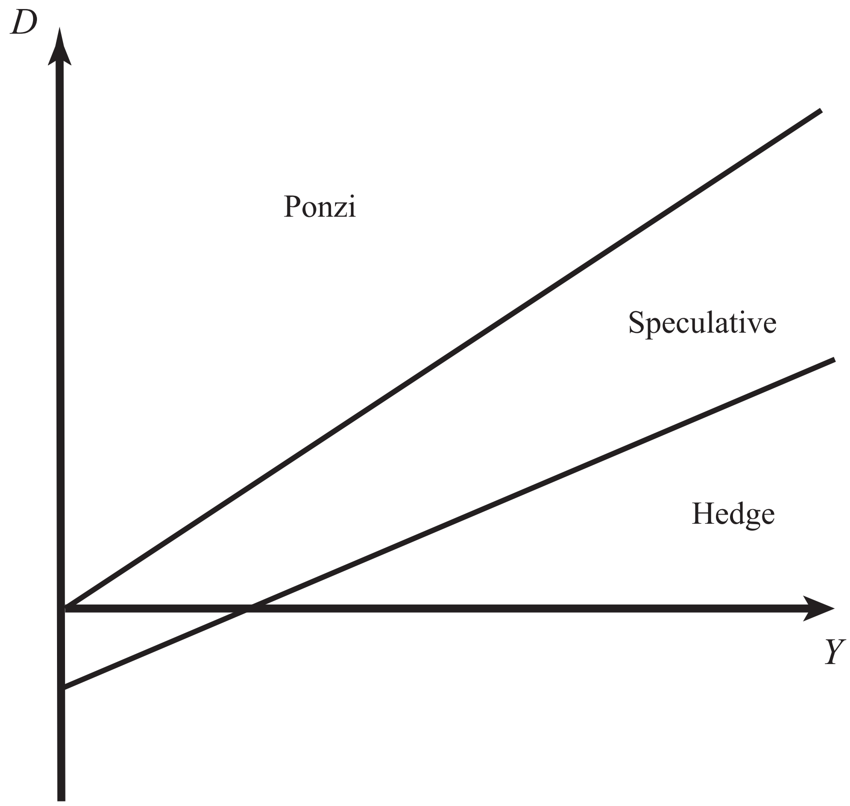

Figure 1-1

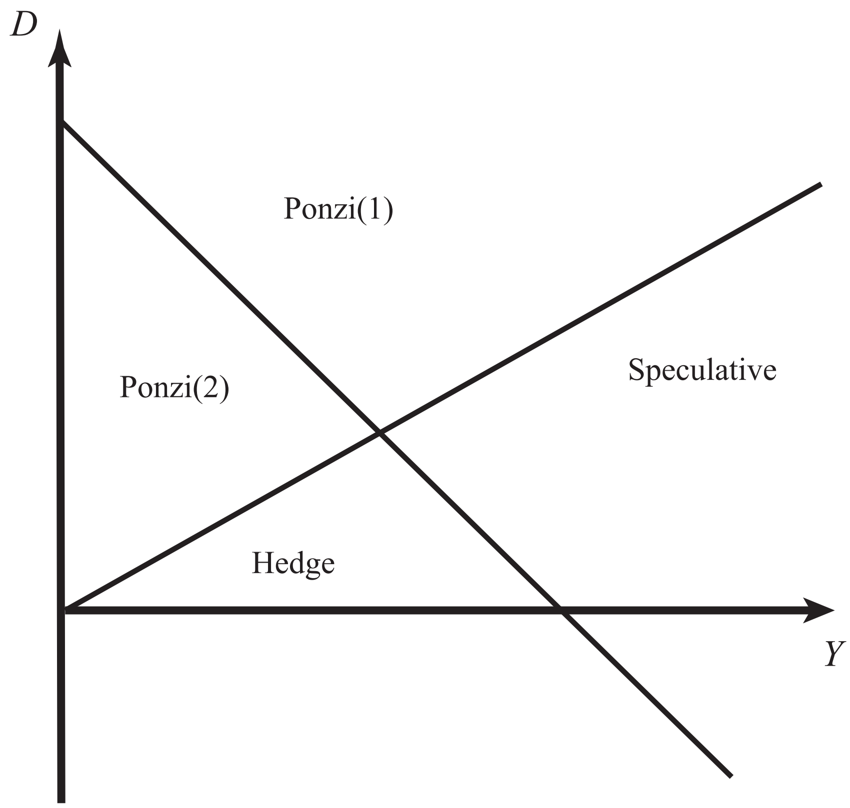

Figure 1-2

Figure 1　Financial Regimes

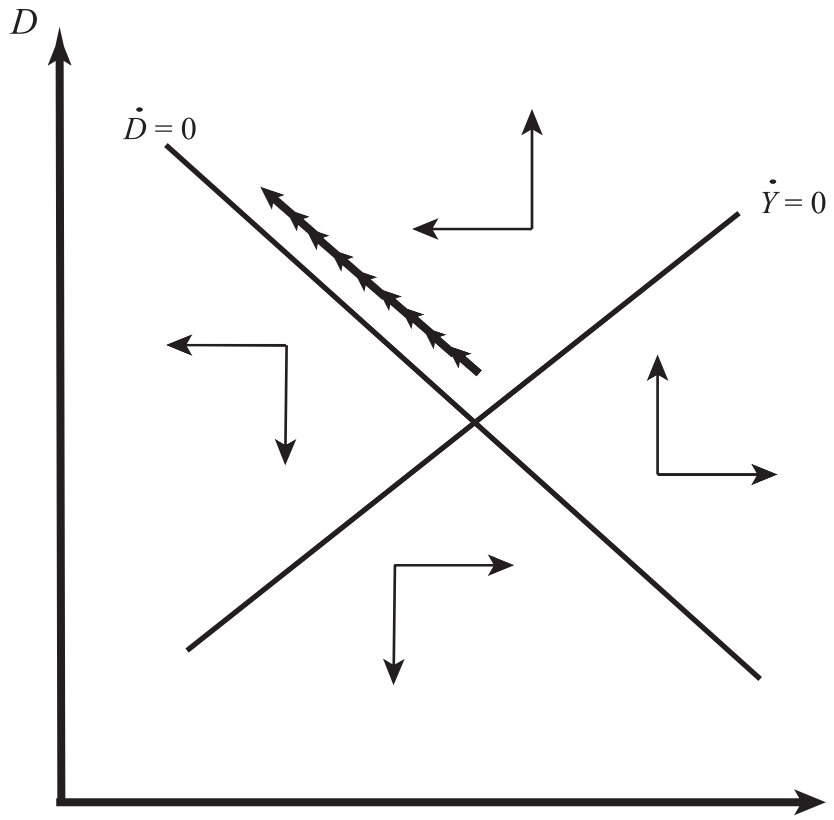

Figure 2-1

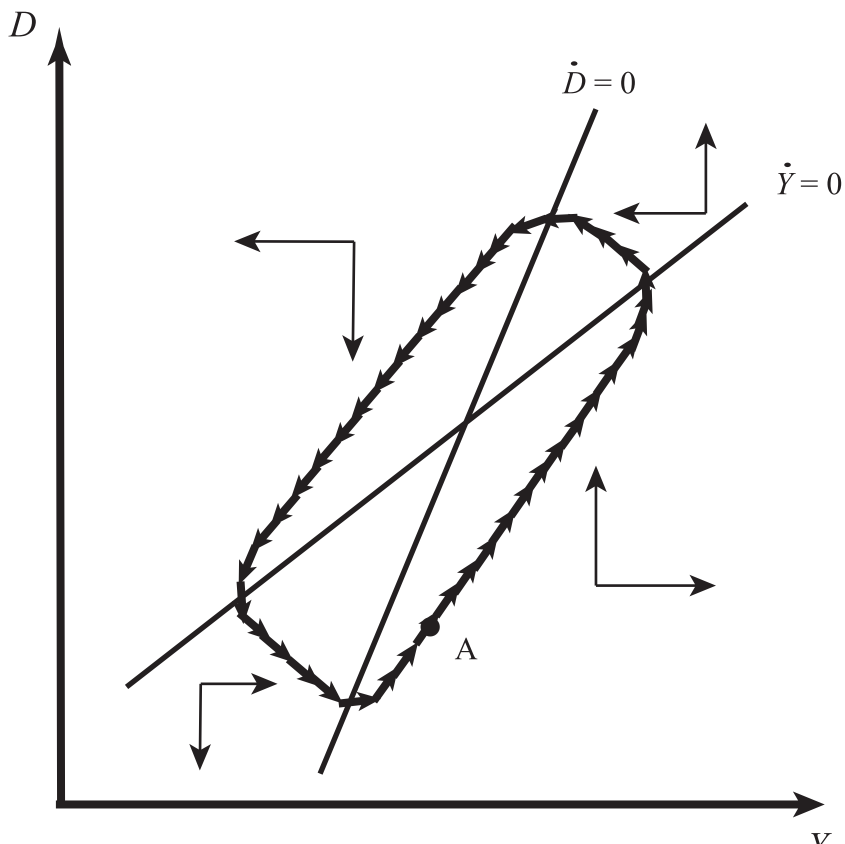

Figure 2-2

Figure 2 Dynamic System S_a

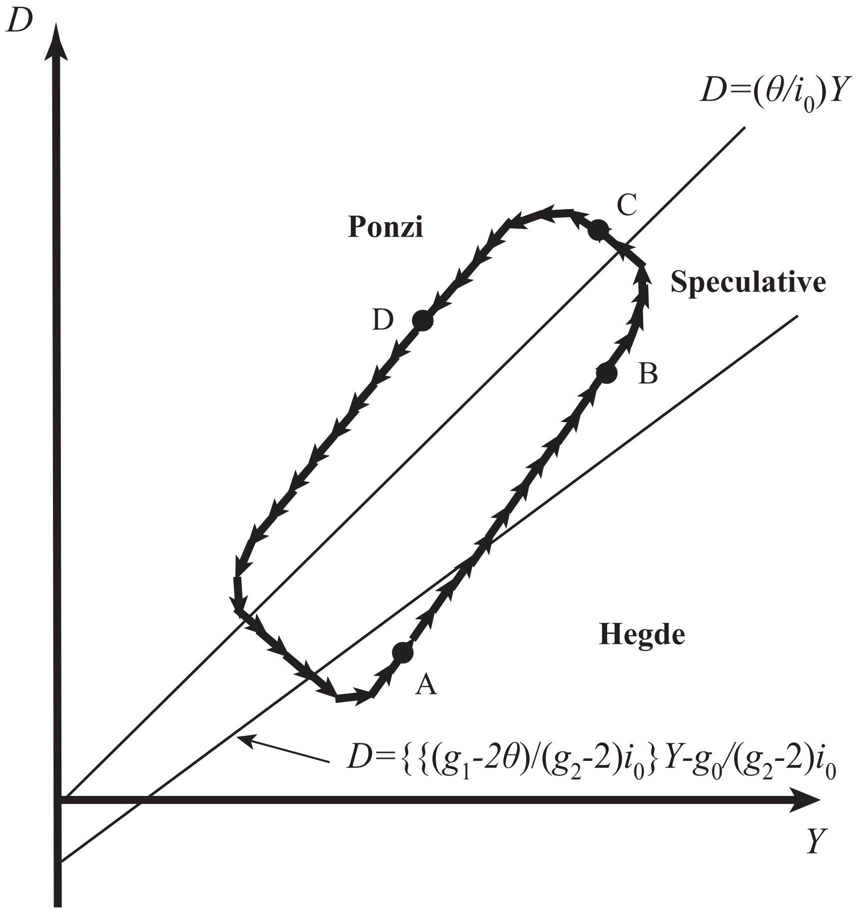

Figure 3-1

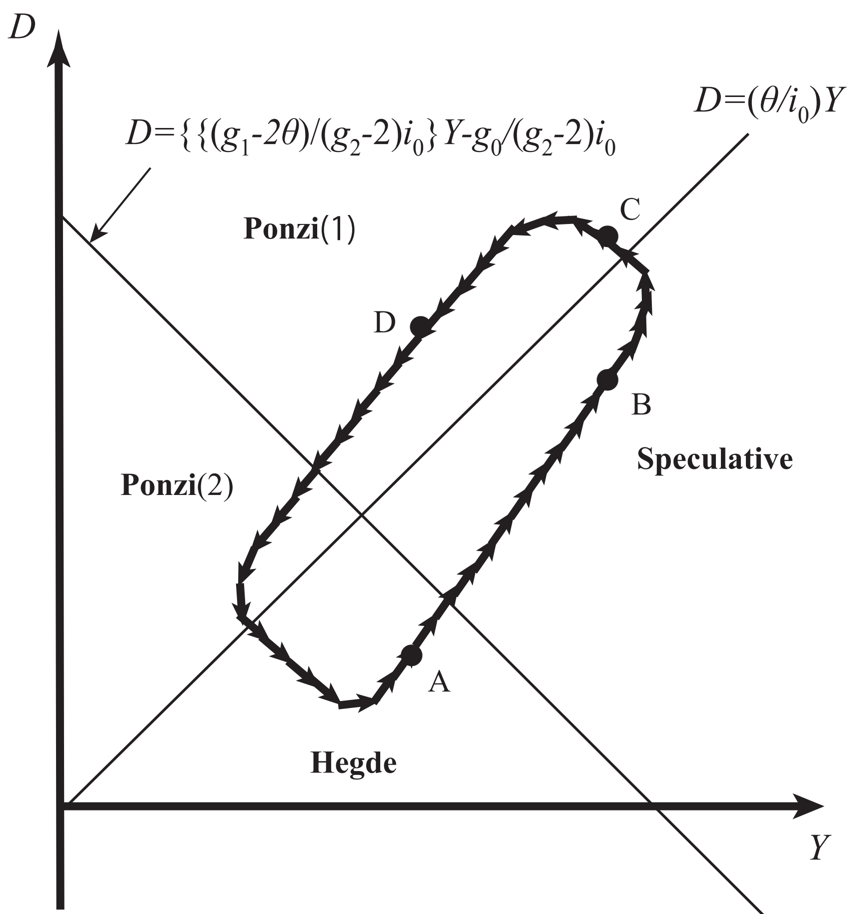

Figure 3-2

Figure 3 Financial Regimes and Cycles

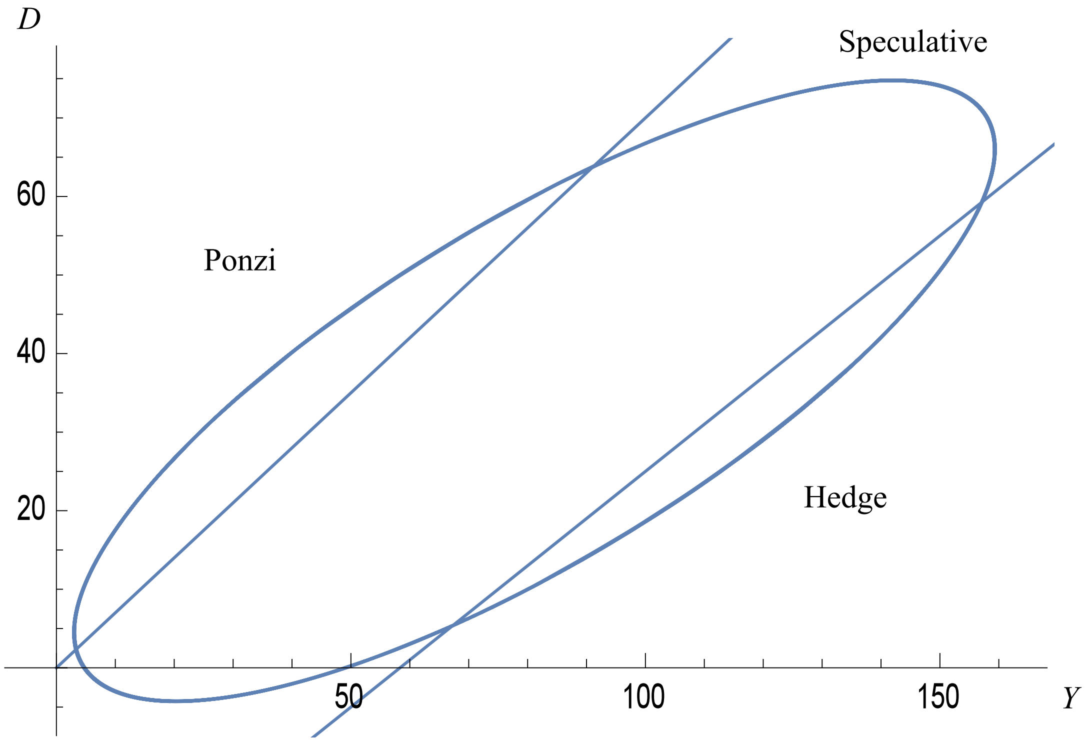

Figure 4 Numerical Simulation: SystemS_a

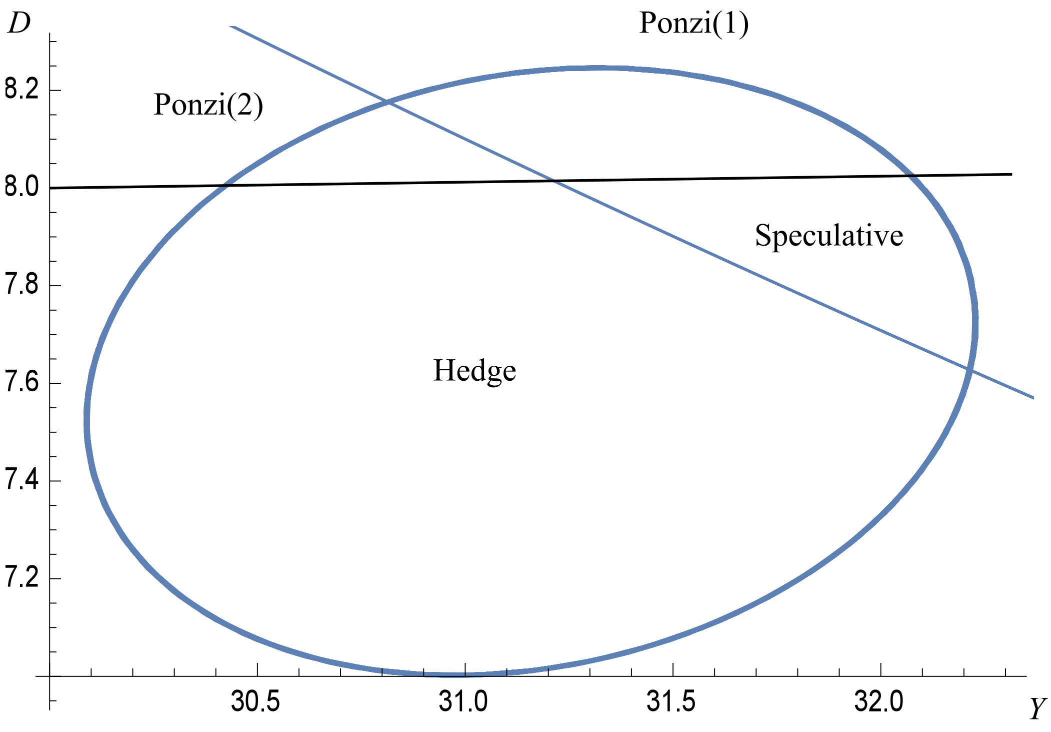

Figure 5 Numerical Simulation: System S_b

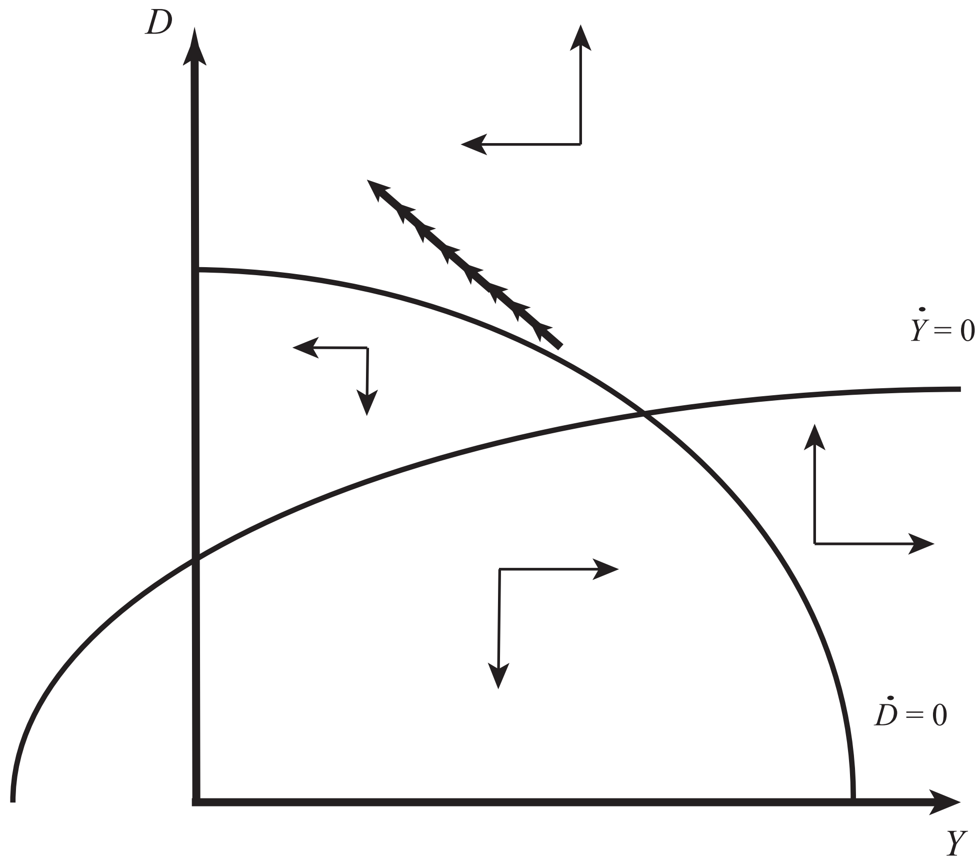

Figure 6-1

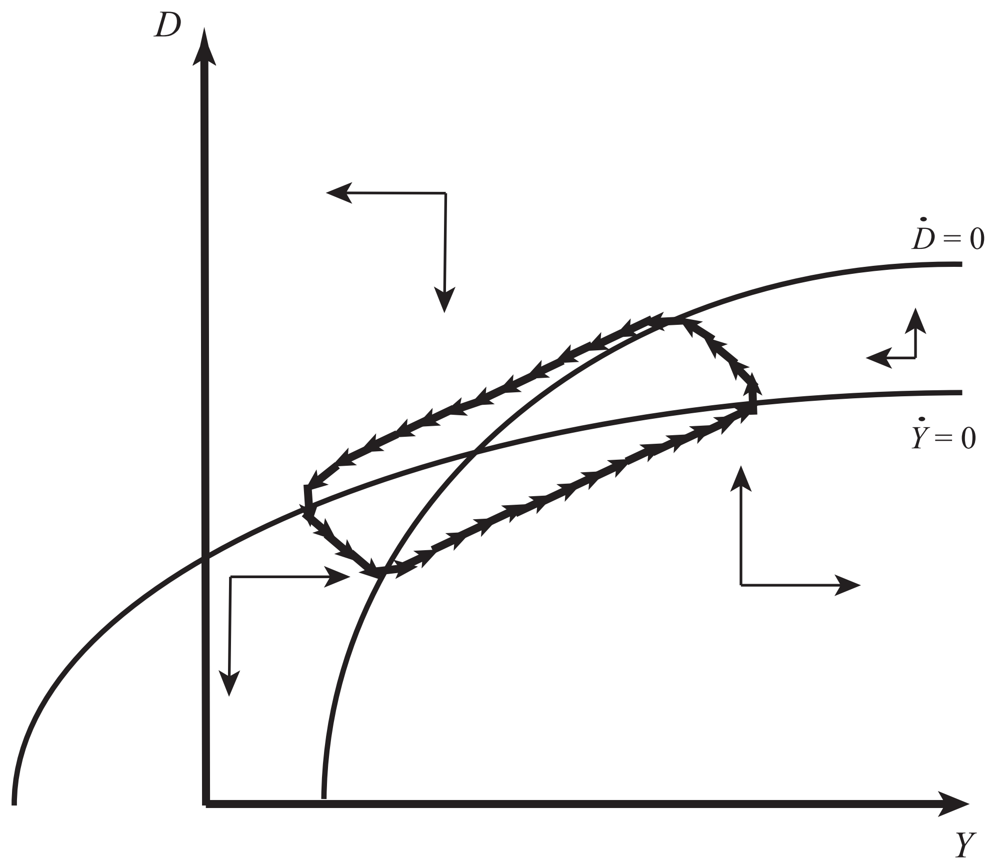

Figure 6-2

Figure 5 Dynamic System S_c

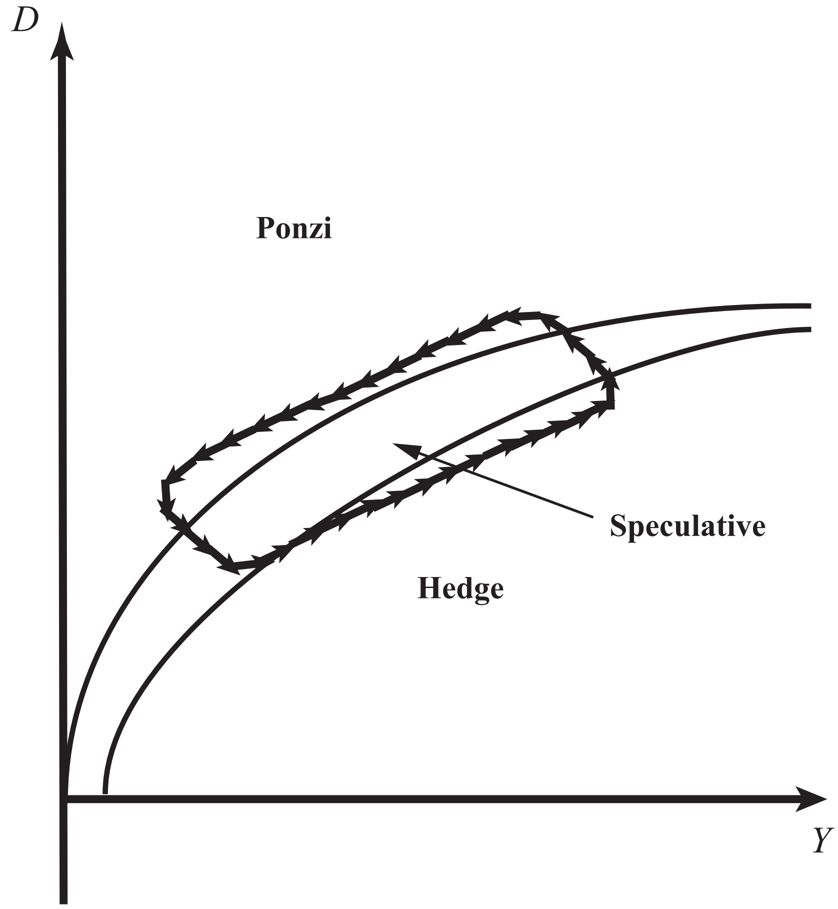

Figure 7 Financial Regimes and Cycle: System S_c