CRR DISCUSSION PAPER SERIES A

Discussion Paper No. A-36

# The Hicks-Morishima Approach Reconsidered:

## Another Look at the Interdependence of Several Markets

Yasuhiro Sakai

October 2019

Center for Risk Research

Faculty of Economics

SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,

SHIGA 522-8522, JAPAN

# The Hicks-Morishima Approach Reconsidered:
## Another Look at the Interdependence of Several Markets

**Yasuhiro Sakai**
**Professor Emeritus, Shiga University**

**Abstract** This paper aims to shed some new light on the Hicks-Morishima approach to the interdependence of several markets. In spite of its rather simple and ambitious framework for the interdependence of several markets, it is quite unfortunate that this approach has been rather neglected in the academic circle. I suppose that there are several reasons for this. First, the traditional general equilibrium approach developed by Lionel W. McKenjie, Gerald Debreu and Kenetth W. Arrow exclusively works with the good space rather than the price space. In contrast, the Hicks-Morishima approach based on Hicks' classical book *Value and Capital* exclusively operates on the price space, thus against the current main stream of economic theory. Next, the majority of economics readers are usually familiar with the straightforward notion of demand and supply curves, but not with the twisted concept of excess demand curves. It is one of my main purpose to mend such unfortunate tendency, presumably proceeding toward the establishment of a new grand system of social science. We can learn new lessons from old teachings.

**Keywords** J.R. Hicks・M. Morishima・*Value and Capital*・excess demand curves・general equilibrium analysis・comparative statics

**JEL Classification** B31, C62, D51

## 1. Michio Morishima and John R. Hicks in My Memory

### 1.1 The Two Economics Superstars： An Introduction

There are the two economics superstars who have greatly influenced my academic career. They are Michio Morishima, Osaka University and London School of Economics, and John R. Hicks, London School of Economics and Oxford University. To tell the truth, both of them are no longer with us. I do believe, however, that they are academically still alive, and their outstanding accomplishments will never be forgotten in the academic world . The main aim of this paper is to reconsider and shed new light on a special topic named the Hicks-Morishima approach to the interdependence of several markets. Although such approach has been rather neglected for some time, I am sure that it is worthy of serious reconsideration, presumably giving a new guide toward an integrated grand theory of social science in the 21 century. This is no doubt the direction of future research both Morishima and Hicks in their late years have so strongly desired.

### 1.2 Michio Morishima as My Indirect Mentor

In this connection, I would like to recall a personal episode around 30 years ago. One day, when I was walking around in a big underground shopping street in the Osaka-Umeda District, the very romantic name "Manjiya" or "Ten Thousand Letter Book Shop" suddenly got into my eyes. It was a long-standing secondhand book store supported by so many book lovers. I have been one of those avid readers, thus dropping in the store periodically since my student days. This time, I was greatly surprised to see an old book of archaeology written by Chiujiro Nakaya, who was a younger brother of "Dr. Snow" or well-known scientist Ukichiro Nakaya, and a close friend of Kiyoshi Oka or world-famous mathematician. More exactly, the young Nakaya and the young Oka spent a very happy time when they were in Paris and doing their research in their respective area, namely old Japan archaeology for Nakaya and multi-functional analysis for Oka. In fact, in his well-sold essay (1963), Oka remarked on his close friendship with Nakaya: [1)]

> The best experience I [Nakaya] received from my Paris days, was that Ukichiro and I stayed under the same roof, and exchanged our philosophies and ambitions with each other almost everyday. He was then a young and ambitious archaeologist who without

> any financial help came to Paris via a long and tiresome Siberia route from Japan. Just before he left Japan, he took pains to walk around the North-East Japan district for the collection of unique cord-marked potteries in the ancient times. By carefully studying those ancient potteries, he eventually succeeded in writing a long outstanding paper. No doubt, he was a gifted man of talent and knowledge. Surely then, both he and I had academic ambitions and were never tired by endless discussions between us. We talked with each other day and night as if we had jointly played violins in perfect harmony. (Oka, 1963)

In their younger days in Paris, the two ambitious scientists in roaring spirits, Chiujiro Nakaya and Kiyoshi Oka, had wonderful days of excitement and inspiration. Personally speaking, I had a similar experience perhaps with a lesser degree in my student days in the United States. In fact, during my Rochester days in 1968-1971, I was very fortunate to meet with Mr. Masayoshi Hitota, a "very serious man with highly samurai spirits," who so fondly argued night and day with me about life philosophy, world matters, and of course economic science. Before long, we became true friends. I still remember the very moment when he enthusiastically presented his own view on the comparison between Japan and the United States:

> Mr. Sakai, I [Hirota] can tell you this. I presume that you know well abut Social Economic Research Institute, Osaka University. Believe or not, that institute is now in a position to lead the economics profession over the world. In particular, Professor Michio Morishima is the indisputable leader of the Osaka Group, distinguishing himself as an world-famed economist whose presence may be more striking than Professor Lionel W. McKenzie, the strong leader of the Rochester Group. Being a Japanese myself, I am inclined to think that we the Japanese should not be unduly overpowered by the big names of American economists.

To tell the truth, such strong speech by Mr. Hirota, my newly acquired friend from Osaka, came as a great surprise to me. It really served as a thunderbolt from a clear sky. I was then staying at the University of Rochester, presumably one of most advanced economics centers around the world, studying difficult subjects such as general equilibrium theory, non-liner programming, and economic dynamics. According to Hirota's remark, however, we should not have any fears about the seemingly high levels of American universities. Osaka University seemed to be at least on a par with, and even better than, University of Rochester. Hirota proposed me

to do a joint work on general equilibrium theory with the final aim of publishing academic papers in professional journals of high prestige. We worked very hard by utilizing every space of the Rochester campus, namely the university library, class rooms, student dining rooms, and the graduate student center. After some time of our joint research, there came our final product, Hirota and Sakai (1969), which aimed to open a new horizon in theoretical studies in production theory. Although our joint paper had a humble yet attractive title "Notes on Substitution and Scale Effects in Production Theory," it was quite unfortunate that because of some minor technical reasons, it missed a publication opportunity.

In hindsight, I am sure that my joint research experience with Mr. Hirota, or "Osaka samurai scholar," has constantly given me a great stimulus for the execution of my research project on economic theory. Having been encouraged a great deal by the Hirora-Sakai cooperation, I myself continued to learn many advanced mathematical tools which were applicable not only to production theory, but also to consumption theory as well. The final result of my continuous research effort was the completion of my doctoral dissertation (1972) *Axiomatic Foundations of Consumption and Production Theories* submitted to The University of Rochester, with my main thesis advisor being Professor Lionel W. McKenzie. Besides, after finishing my Ph. D. work and publishing so many theoretical papers in international journals, I could spend a very happy life at Pittsburgh, in which Professor and Mrs. Asatoshi Maeshiro kindly helped me in many ways. All those nice things happened to my rather long American life in 1968-1976.

As the saying goes, all's well that ends well. I have no objection against the utility of this maxim. I would like to add, however, the equal effectiveness of its dual maxim that all's well that begins well. My close friendship with Rochester classmate Mr. Hitora (now, Emeritus Professor Hirota at Tokyo Science University) and my appreciation of late Professor Michio Morishima, a direct teacher of Hirota and my indirect mentor as well, are so great that they will forever be kept in my memory. In short, no praise is enough for Morishima and Hirota.

### 1.3 *Value and Capital*： A Masterpiece Connecting Morishima and Hicks

In my Rochester days, whenever I met Mr. Hirota on the university campus or out of campus, I was almost overwhelmed by his repeated reference to Professor Morishima's powerful presence in the economics profession all over the world. Hirota's favorite phrase sounded like this: "Morishima is really great! He is perhaps more prominent than McKenzie! " Even before I met Hitota, however, I myself

greatly admired Morishima as a leading economist. And I have constantly been influenced by Morishima's great accomplishments. So, in what follows, without reference to Hirota, let me write down my personal indebtedness to Morishima. [2)]

As far as I could recall, sometime in 1967, my first encounter with Morishima took place at the private residence of Professor Akihiro Amano in Higashinada, Kobe. Amano was an outstanding Kobe University graduate as well a respectable Rochester Ph.D. Amano was a very kind person who wrote a strong recommendation letter to Rochester for me. When I was going to leave Amano, I happened to see Morishima at the front door. Morishima kindly talk to me:

> I presume that you are Mr. Sakai, the young man who is planning to do graduate work at Rochester. Believe or not, your name is already familiar to me. I would sincerely hope that you will convey my good wishes to McKenzie, the leader of the Rochester group.

Although this was really a few minute meeting between Morishima and myself, I have never forgotten it since then. Even today, I can still recall it very vividly. At the very end of 1973, my next meeting with Morishima took place in New York City. [3)] More exactly, he was invited to deliver a special lecture at Econometric Society North American Meeting at New York Hilton Hotel, with Professor Lawrence W. Klein of Pennsylvania serving as a respectable chairman. The audience filled the hall to overflowing. Professor Akira Takayama, a noted international economist, sat in the very front seat, taking so many pictures. All the people seemed to have a very happy time by enjoying the presence of the memorable trio, namely Morishima, Klein and Takayama. Then, Morishima stood up and delivered a historical speech:

> Kark Marx is so great! I have no doubt that he is academically still alive after one hundred years of his death!

In an instant, the whole audience stood up and applauded his powerful speech. Since then, until his death, I have met with Morishima in many occasions in Japan and oversees, having lively discussions between us. Besides, I have received a lot of kind advices and encouragement by means of personal correspondences. In what follows, to save the space, I will just pick up Hicks' great book *Value and Capital* (1946), carefully discussing how closely Hicks and Morishima have been connected by this masterpiece.

According to Naoki Komuro (2004), Morishima has been widely regarded as "the Japanese scholar who is the closest to Nobel Economic Prize." Since Morishima was dead, we should now say that Morishima narrowly missed a Nobel prize by a hairsbreadth. Komoro wrote the following impressive sentence:

> During the Second World War, Morishima served as a naval cadet whose duty was to break a secret code of the enemy. When he left home toward a battlefront, he carried an important book with himself. That book was nothing but a newly published text *Value and Capital* by Hicks, then a rising star economist of the United Kingdom, one of Japan's hostile countries. Needless to say, Morishima, who faithfully conducted his secret mission of code breaking, nevertheless dared to exert all his energy to fully understand the Hicks new book, and safely could return home.

During the Second World War, every Japanese citizen had to spend a very difficult time. The young Morishima, then being appointed as a naval officer, should have been no exception for this painful experience. There was one new book, however, which Morishima badly wanted to read, and decided to carry over a remote place as far as a battleground. Astonishingly, that book was *Value and Capital* written by a British young economist J.R, Hicks, presumably a rare book for an enemy officer to have difficulty to acquire. Anyhow, Morishima acquired it! And he devoted all his energy to fully understand and sometimes go beyond it: in fact, he succeeded in doing some important extensions of the original Hicks work in first-rate academic journals.

Later in his life, Morishima (1999) got a sudden surge of nostalgia for his youth:

> When I [Morishima] was a Kyoto University student just after the last war, I was fondly reading *Value and Capital* by J.R. Hicks. He discovered the following law of critical importance: "Let the excess demand for one good increase. Then, not only will its price will rise, but also the prices of other substitutive goods will rise, whereas the prices of complementary goods will decline." I noticed , however, that he carelessly neglected the undeniable truth that any economic activity including market trading had to operate at some distance in a space. My question which would naturally arise in my mind was how Hicks' market laws aforementioned would had to change by the introduction of distance and space. In my student days, I was literally swamped with such a very difficult problem. (Morishima, 1999)

It was quite fair to say that in those days, Morishima stood high and alone in the Japanese academia: indeed, the research level of Morishima went far beyond the one of any ordinary Japanese economist. He was a man of courage and determination. The major aim of this paper is to reconsider Hicks' law of market exchange in a new modern fashion, thus further extending the Hicks-Morishima approach with my unique flavor attached to it. [4)]

I have met with Hicks several times. My most memorable moment took place when I attended the International Conference in Honor of J.R. Hicks, Bologna, Italy, in 1991. Every time I met him, our meeting gave me a great and unforgettable memory.

The outline of this paper is as follows. Section 2 will deal with s simple model of general equilibrium —— the excess demand approach a la Hicks and Morishima. Session 3 will discuss many non-normal cases. The issues of instability, multiple equilibrium and non-existence will successively taken up. In Section 4, we will turn to the problem of comparative statics. The impact of parameter changes on equilibrium values will carefully be investigated. Final remarks will be made in Section 5, with reference to Morishima's life-long dream of a grand synthetic approach to a so-called "symphonic economics."

## 2 A Simple Model of General Equilibrium: The Excess Function Approach a la Hicks and Morishima

Saying is one thing. But doing is another. Now let us discuss the working and performance of a simple model of general equilibrium. To this end, we will adopt the excess demand approach a la Hicks and Morishima. The Hicks-Morishima approach is a very effective one to shed light on interrelation of excess demand curves in the price space for an in investigation of the working and performance of general equilibrium. Although we focus on a simple two good model in this paper, its generalization to any several good model would require no difficult task.

Honestly speaking, in spite of its rather simple and ambitious framework, it is quite unfortunate that the Hicks-Morishima approach has been long neglected in the academic circle. I suppose that there are several reasons for this. First of all, the traditional general equilibrium approach developed by Lionel W. McKenjie, Gerald Debreu and Kenetth W. Arrow exclusively works with the good space rather than the price space. In contrast, the Hicks-Morishima approach exclusively operates on the price space, thus against the current main stream of economic theory. Next, the majority of economics readers are usually familiar with the straightforward notion of

demand and supply curves, but not with the twisted concept of excess demand curves. And of course, many people prefer simplicity to complexity.

I suppose that Morishima would apparently agree with those reasons. In order for the excess demand approach to become very familiar to the academic circle, we would have to do a sort of extraordinary work for publicity and sales effort. Morishima had special courage to write a new style of econ text (1984) *Economics for Countries of Poor Resources: A New Text* , in which he adopted the Hicks-Morishima approach again. Unfortunately, history repeated itself： the above approach received a rather dull response. Since I regard myself as an "academic son" of Morishima and as an "academic grandson" of Hicks, I ought to be strong in the neglected stream of academia, arguing again usefulness and applicability of the Hicks-Morishima Approach.

### 2.1 Are A Pair of Two Goods Substitutive or Complementary ?

Homo sapience is a complicated animal, a more complicated than a money conscious economic man usually described by some economists. As Soseki Natsume (1867-1916), a famous Japanese writer, once remarked:

> A cool-headed man with too much intelligence tends to quarrel with a neighbor. A warm-hearted man with much compassion tends to forget his own position. A strong-willed man with too much attachment tends to enter a dead end. So, a human society may not be a very good place to live. (Soseki, 1906)

In reality, there seem to be many diversified markets which are more or less interlocked with each other. If we take a simple example from a female clothes market, it is true that skirt and jeans markets are mutually interrelated. Their relations, however, are much more complicated that a man in the street would usually imagine. The demand for female cloths may be influenced by so many possible factors. First of all, a young stylish girl may want to be dressed in the height of fashion: indeed, being behind the fashion would be the most terrible thing to do. If wearing slim jeans, but not wide skirt, becomes more fashionable than before, then there would be so many young girls who want to change their fashion style from skirt to jeans. Besides, if skirt becomes more expensive due to an increased cost of fabric, then there would be a tendency that the sale of jeans as a good substitute of skirt will rise. Moreover, the overall demand for cloths, jeans or skirt, is determined by the purchasing power of buyers. When the economic conditions become improved (or

worsened), people want to spend more (or less) money for clothing, so that the sales of skirt and jeans will overall tend to go up (or down).

This is not all of the story. Let us recall that we live in the world of risk and uncertainty. If we consider people's forecasts and expectations, the demand and supply of a good should get much more complicated. .

For instance, even if the price of a skirt has risen "this week", no one can exactly tell what will happen to that price "next week." If a young girl expects it to rise higher next week, she has a good incentive to buy it this week. Besides, even if her income stays sluggish this month, yet she expects it to go up significantly the next month, she might feel rich and affluent even now, possibly showing positive buying behavior. Similar reasoning will be applicable to the supply side, not only the demand side. In general, if the trader expects the sale to be sluggish this year, but to be increasing next year, he is likely to positively respond to his optimistic forecast.

To sum up, human beings are complicated, and so are markets. In fact, a skirt market and a jeans market are so intricate that they cannot simply be analyzed. It is the Hicks-Morishima approach that can shed a nice analytical light on the entangled interdependence of several markets. In historical perspective, that approach was originated in Chapter 5 of Hicks (1939, 1946), and later highly developed in Morishima (1984) in many possible ways. In our regret, however, time seems to pass too slowly to recognize a masterpiece: indeed, the magnificent Hicks-Morishima approach has been long undervalued or neglected. As I have said repeatedly, it is my sincere desire to revive the approach once more and hopefully attach some new findings and flavors to it.

## 2.2 A Simple Two-Good Model : The Starting Point

At the starting point of discussion, let us begin our market model analysis with a simple case of two goods, $x_1$ and $x_2$. More specifically, let us suppose that $x_1$ stands for tea and $x_2$ coffee. Further assume that the price of $x_1$ and the one of $x_2$ are respectively denoted by $p_1$ and $p_2$

It is natural to assume that the amount of demand for $x_1$ is dependent not only on $p_1$, but also on $p_2$. Since tea and coffee are presumably in competitive relations, they are formally regarded as "substitutive" in microeconomic theory. Therefore the demand function, $D_1$, for good 1 may be formulated as follows:

$$D_1: \quad x_1 = -p_1 + 0.5p_2 + 3 \qquad (1)$$

It is supposed here that there are two different routes by which the amount of demand of coffee, $x_1$, rises. The first route is the *direct* one by means of a fall in $p_1$, the price of coffee per se. This may be called the "own effect" of a price change. The second route is the *indirect* one in the sense that the price rise of tea as a substitute of coffee causes the change of demand from tea to coffee. This may be named the "cross effect" of a price change. As is clearly seen in Eq. (1), it is supposed that the first own effect overpowers the second cross effect.

Concerning the supply side, we assume that its direction of change is exactly the opposite of the demand side. More specifically, let us consider the following linear supply function:

$$S_1 \quad : \quad x_1 = p_1 - 0.5\,p_2 - 1 \qquad (2)$$

Let the price of coffee rise. Then, we obtain the following two effects. First, the distributor intends to increase the supply of coffee in the market . This is the "own effect" of a more expensive coffee. Second, when the price of tea falls, the distributor not only wishes to sell tea less, but also he tends to sell coffee more. This is the "cross effect" or the "spill over" effect over the two markets. Besides, it is supposed here that the first own effect overwhelms the second cross one.

We are now in a position to introduce the concept of an excess demand function. Note that excess demand means "demand minus supply": namely, $E_1 = D_1 - S_1$. Thus, in the light of Eqs. (1) and (2), we can easily derive the excess demand function for $x_1$ as follows:

.

$$E_1 = D_1 - S_1 \; : \quad x_1 = -2\,p_1 + p_2 + 4 \qquad (3)$$

This demonstrates that the amount of excess demand for good 1 is *inversely* proportional to its price (the own effect), whereas the former amount is in *direct* proportion to the price of good 2 (the cross effect). Besides, understandably, the own effect overpowers the cross one.

Market equilibrium of good 1 is realized when its demand and supply are just equal, namely its excess demand completely vanishes. It is at this equilibrium point that the price of good 1 ceases to change. On the one hand, if the demand for good 1 exceeds its supply, then the excess demand occurs, implying that the price of good 1 must go up. On the other, if the demand is less than the supply, then the excess

supply occurs, so that the price has to fall。 This is a famous adjustment process a la Leon Walrus, which may be summarized as follows:

$E_1 > 0$ : $D_1 > S_1$ (disequilibrium) $\longrightarrow$ $p_1$ goes up.
$E_1 = 0$ : $D_1 = S_1$ (equilibrium) $\longrightarrow$ $p_1$ does not change.
$E_1 < 0$ : $D_1 < S_1$ (disequilibrium) $\longrightarrow$ $p_1$ goes down.

Let us take a look at Fig. 1. In the left chart (A), the straight line on the price plane represents the excess demand function: $-2p_1 + p_2 + 4 = 0$. (Note that this function can also be rewritten as $p_2 = 2p_1 - 4$.) Since the two goods are substitutes, the line is positively sloped. Moreover, reflecting the fact that the own effect overpowers the cross effect, the slope of the straight line for $p_1$-axis should be steeper than the one for $p_2$-axis , so that the former slope must exceed 45 degree.

Suppose that a pair of prices $(p_1, p_2)$ lies right on the straight line $E_1$. Then, $p_1$ does not change at all, and maintains the same value as before. Clearly, the excess demand line $E_1 = 0$ divides the whole price plane into two areas. In the *left* area where $E_1 > 0$, $p_1$ rises in a direction indicated by the right arrow (→). In contrast, in the *right* area where $E_1 < 0$, $p_1$ falls in an opposite direction indicated by the left arrow (←).

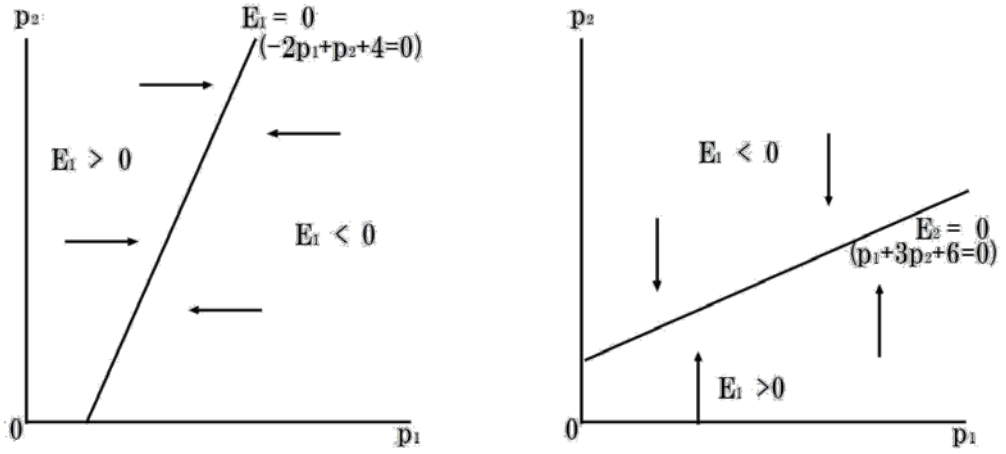
A similar argument will apply for good 2, or coffee. So, we can formulate the demand , the supply and the excess demand functions in the following way:

$D_2$ : $x_2 = 0.5p_1 - 2p_2 + 4$ (4)
$S_2$ : $x_2 = -0.5p_1 + p_2 - 2$ (5)
$E_2 = D_2 - S_2$ : $x_2 = p_1 - 3p_2 + 6$ (6)

This teaches us that the amount of excess demand for good 2 is inversely proportional to its price $p_2$ (the own effect), whereas the former amount is in direct proportion to $p_1$ , the price of good 1 (the cross effect). Besides, as before, the own effect overpowers the cross one.

The Walrasian adjustment process for good 2 may be formulated in an analogous fashion:

$E_2 > 0$ : $D_2 > S_2$ (disequilibrium) $\longrightarrow$ $p_2$ rises.
$E_2 = 0$ : $D_2 = S_2$ (equilibrium) $\longrightarrow$ $p_2$ does not change.
$E_2 < 0$ : $D_2 < S_2$ (disequilibrium) $\longrightarrow$ $p_2$ falls.




(A) Excess demand for $x_1$ : changes in $p_1$ and $p_2$

(B) Excess demand for $x_2$ : changes in $p_1$ and $p_2$

**Fig. 1 The Relation between the Two Markets**

Let us take a look at Fig. 1 again. In the right chart diagram (B), the straight line $E_2 = 0$ , or $p_1 - 3p_2 + 6 = 0$, represents the excess demand for good 2. Note that

this line may be rewritten as $p_1 = 3p_2 - 6$. This is the case in which the two goods are substitutes and each own effect is dominating, so that the excess demand line is positively sloped and its "slope for $p_2$- axis" must exceed 45 degree. The directions of price adjustment $p_2$ are indicated by arrows in the figure: namely, $p_2$ rises in the lower area, but falls in the upper area.

### 2.3 General Equilibrium of Two Markets: The Simplest Case

We are now ready to get into the world of "general equilibrium" with the two goods, in which the demand and supply of each good are just equal and the changes of prices stop moving. In other words, general equilibrium looks like a stationary state.

Now, let us carefully investigate what happens if the two chart diagrams (A) and

(B) of Fig. 1 as well as the corresponding sets of arrows are superimposed. Then,

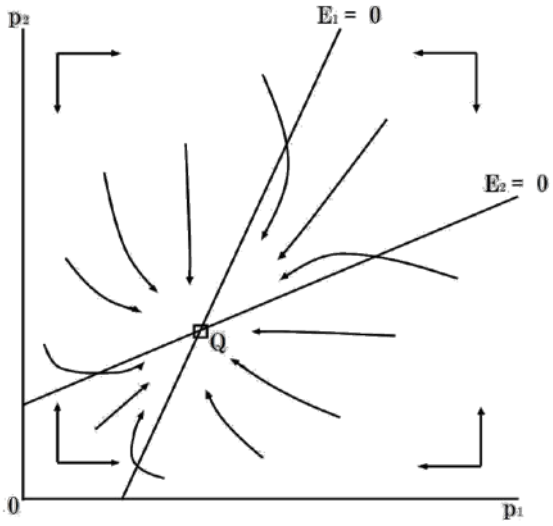



**Fig. 2 The existence and stability of the two-good markets: An ideal state**

we will immediately see that a very charming float diagram like Fig. 2 appears. At the intersection point $Q$ of the two excess demand lines, $E_1 = 0$ and $E_2 = 0$, the following two equations must be satisfied : [5)]

$$-2p_1 + p_2 + 4 = 0 \quad (7)$$

$$p_1 - 3p_2 + 6 = 0 \quad (8)$$

Solving Eqs. (7) and (8) for $p_1$ and $p_2$ simultaneously, we can see that a pair of coordinates of the intersection point $Q$ is shown by $Q = (18/5, 16/5)$. Therefore, the equilibrium prices of $p_1$ and $p_2$ are respectively 18/5 (= 3.6 ) and 16/5 (= 3.2).

To sum up, the intersection point $Q$ indicates the point of "general equilibrium" of the two markets. Remarkably, it should be a stable point as well. Note that any price movements may be shown by the arrows of the pair ($p_1$ , $p_2$ ). In whatever direction the price pair may move in Fig. 2, it is eventually destined to converge at the

equilibrium point $Q$.

## 3 Many Non-Normal Cases: Instability, Multiple-Equilibrium, and Nonexistence

### 3.1 The Saddle-Point Equilibrium: Substitutable Goods yet Dominating Cross Effects

. As the saying goes, saying is one thing but doing is really another. All the foregoing analyses have just dealt with the ideal, utopian two-good world in which the only one point of general equilibrium uniquely exists and its stability is also guaranteed. One might wonder if such utopian world really exists. When we are getting into the real world as it is, it is far from perfect, with a lot of irregularities. For instance, general equilibrium might not exist at all. There would be more than one equilibrium. Besides, many other "irregular situations" including unstable points and/or saddle points might occur.

Suppose that there is a market equilibrium with substitutable goods. Then, the most important question to ask is whether and to what extent the stability of the equilibrium is guaranteed. Interesting enough, the equilibrium is not always stable. In fact, we will see that the cross effects of price changes act as disturbing factors of instability. In fact, if the cross effects are strong enough to outweigh the own effects, then the equilibrium point will become an instable saddle point. To clarify such an interesting point, let us focus on the case that the excess demand functions are of the following forms:

$$E_1 \quad : \quad E_1 = -p_1 + 2p_2 - 4 \tag{9}$$

$$E_2 \quad : \quad E_2 = 3p_1 - p_2 - 6 \tag{10}$$

The excess demand lines of good 1 and 2 are depicted in Fig. 3. Since we are dealing with substitutable goods, it follows that both lines should be positively sloped. There is an important point we should not miss, however. Now, the cross effects are strong enough to outweigh the own effects. Indeed, in Eq. (9) above, the absolute value 2 of the coefficient of $p_2$ is greater than the one 1 of the coefficient of $p_1$, whereas in Eq. (10), the absolute value 3 of the coefficient of $p_1$ exceed the one 1 of the coefficient of $p_2$. As a result, in sharp contrast to the idealistic case discussed above, the slope of $E_1$-line is steeper for $p_2$-axis, whereas the one of $E_2$-line is steeper for $p_1$-axis.

Let compare Fig. 4 with Fig. 3. Then, we clearly understand that the positional

relationship of the lines $E_1 = 0$ and $E_2 = 0$ has been reversed. Consequently, such tly

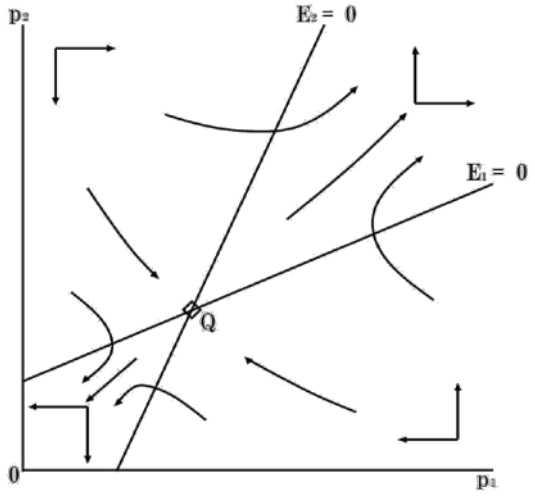


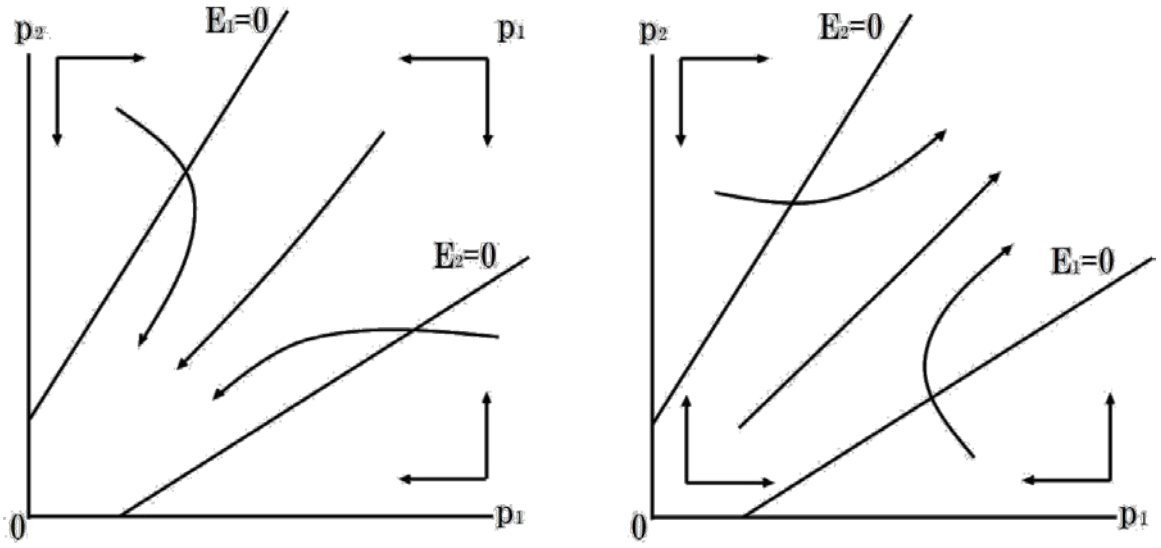

**Fig. 3 The equilibrium point is an unstable saddle point: The cross effect is so strong**

reversal of locations would greatly change the adjustment processes of prices.

The coordinates of equilibrium point is Q = (16/5, 18/5). Apart from the previous discussion, however, we can no longer say that this point is a stable point in its true sense. In fact, it should be a saddle point that is unstable on almost all paths. As is seen in Fig. 3, except only for the two stable paths, one path going down from the North-West to Q, and another going up from the South-East to Q, all other possible paths are not stable at all. This means that the saddle point $Q$ should substantially be unstable.

### 3.2 Non-existence of the two-market equilibrium

What we should consider next is the emergence of "non-normal situations." Since all of them are of vital importance, they respectively require for very careful investigations. As the first example, let us examine the case in which the excess demand functions are written as follows:




(A) The prices are declining to zero (B) The prices are expanding forever

Fig. 4 Non-existence of equilibriums

$$E_1 : \quad E_1 = -2p_1 + p_2 - 2 \tag{11}$$

$$E_2 : \quad E_2 = p_1 - 2p_2 - 2 \tag{12}$$

Let us take a look at Fig. 5. As is seen in Chart (A), the two excess demand lines, $E_1 = 0$ and $E_2 = 0$, never intersect at all in the positive coordinates of the price space. If we dared to solve for $p_1$ and $p_2$ in Eqs. (11) and (12), we would obtain a pair of NEGATIVE prices $(p_1, p_2) = (-2, -2)$, which should not economically be meaningful. Eventually, as is shown in Chart (A), both prices, $p_1$ and $p_2$, tend to eventually decline toward zero along the directions of arrows. [5)]

If we interchange the positions of the two lines, $E_1 = 0$ and $E_2 = 0$ in the left chart (A), then we immediately obtain those lines in the right chart (B). Then, we have to deal with the following set of equations:

$$E_1 : \quad E_1 = -p_1 + 2p_2 + 4 \tag{11}$$

$$E_2 : \quad E_2 = 3p_1 - p_2 + 6 \tag{12}$$

In Chart (B), there exist no intersections of the two lines in the positive price quadrant. If we dared to simultaneously solve $E_1 = 0$ and $E_2 = 0$, then we would find $(p_1, p_2) = (-16/5, -18/5)$, a pair of negative and economically meaningless prices. As a result, there are no general equilibrium solutions, and the prices eventually tend to rise forever as indicated by arrows.

### 2.3 The Case of Complementary Goods : A Completely New Situation

In the above, we have limited our investigation to the case of competitive or substitutable goods. Looking at the real world, however, we can eyewitness many other cases in which some goods may not be mutually competitive but rather move in the same direction. For instance, while tea and coffee are substitutes, tea and sugar are complementary, and so are coffee and sugar.

As the first example of complementary goods, let us discuss the following excess demand functions of the two goods:

$$E_1 : E_1 = -2p_1 - p_2 + 4 \quad (13)$$

$$E_2 : E_2 = -p_1 - 3p_2 + 6 \quad (14)$$

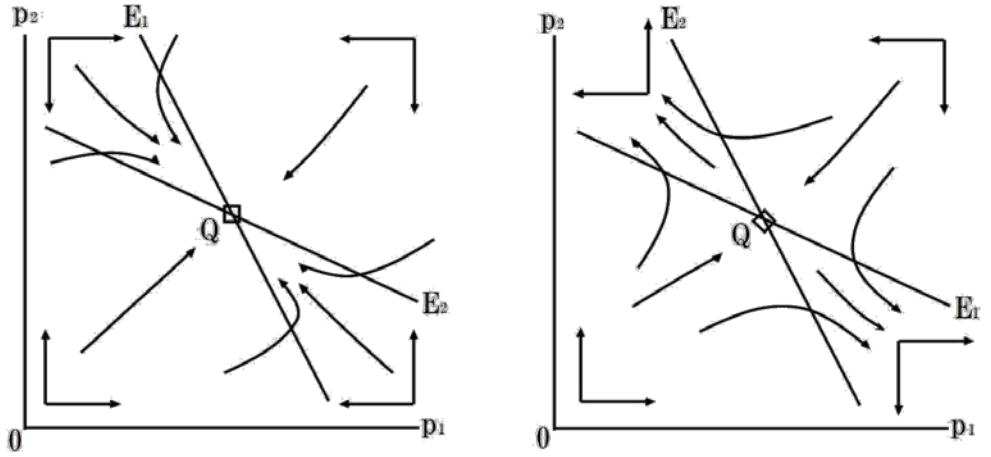
Let take a close look at Eq. (13) and (14). The, we will see that in the right-hand sides of those equations, the values of coefficients of $p_1$–term and $p_2$–term are both negative. This means that both the excess demands for $x_1$ and $x_2$ have to increase when $p_1$ falls, or when $p_2$ falls. This implies that the two goods are complementary, not substitutes. Besides, we assume here that the own effects overpowers the cross effects. Therefore, we would rightly conjecture that the equilibrium point $Q$, if it exists, must be stable. Surely, such conjecture will turn out be right by looking at the left chart (A) of Fig. 5.

Concerning Fig. 5, there is one more point to watch. We must make sure that the two excess demand lines, $E_1$ and $E_2$, are now NEGATIVELY sloped. Since the own effects dominate the cross effects in Chart (A), it follows that both the slope of $E_1$ for $p_1$–axis and the slope of $E_2$ for $p_2$–axis should be steeper.

We will turn to the second case which is represented by the following pair of excess demand functions:

$$E_1 : E_1 = -p_1 - 3p_2 + 6 \quad (15)$$

$$E_2 : \qquad E_2 = -2p_1 - p_2 + 4 \qquad (16)$$




(A) Point Q is stable: The own effects are dominating

(B) Point Q is unstable: The cross effects are dominating

Fig. 5 The two goods are complementary

Now, the cross effects are now more powerful than the own effects. As can easily be seen in Chart (B), the slope of $E_1$ for $p_2$–axis (not for $p_1$–axis) is steeper while the one of $E_2$ for $p_1$–axis ( not for $p_2$–axis ) is steeper. Therefore, the equilibrium point $Q$ is an almost unstable saddle point.

In conclusion, the power relation between the own effects and the cross effects plays a vital role in the determination of the stability of an equilibrium point. Whether the goods are substitutes or complementary, the dominance of the own effects over the cross effects clearly guarantees the stability of an equilibrium. On the contrary, if the cross effects dominate the own effects, then an equilibrium point becomes an almost unstable saddle point.

### 2.4 The Possibility of Multiple Equilibriums: The Strange Cases of Non-Linearity

In the above, we have maintained the special assumption that all the relevant

functions including the demand, the supply, and excess demand functions are LINEAR.

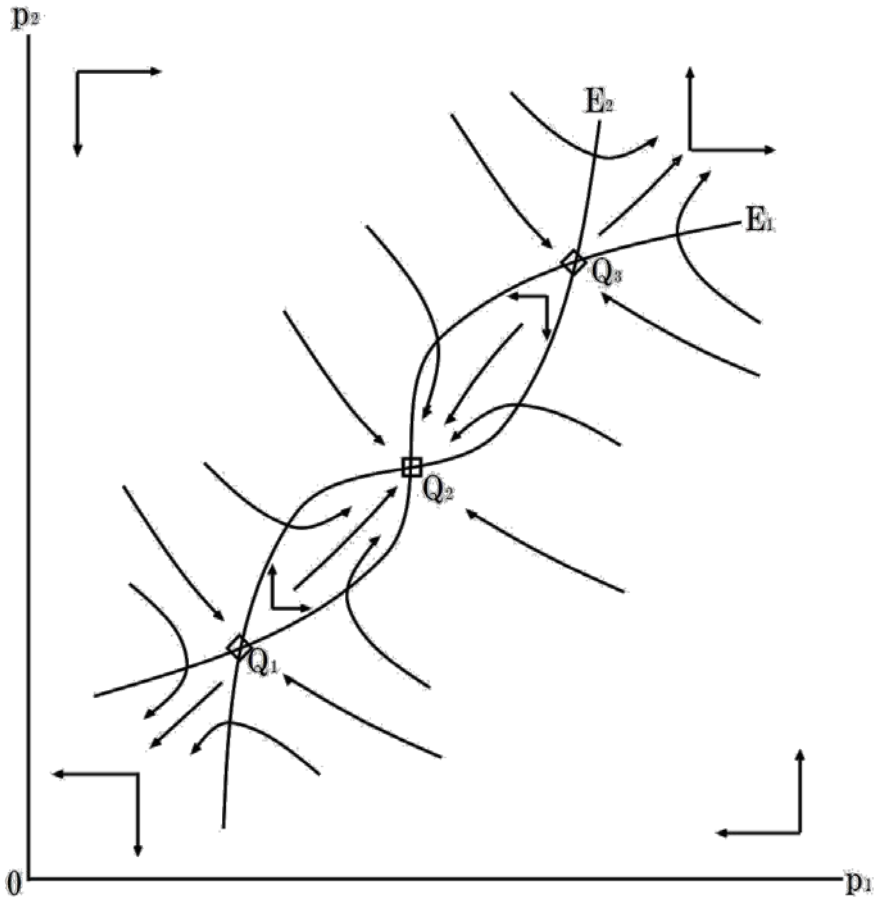



**Fig. 6 Multiple equilibrium: non-linear excess demand curves**

We never live in such a linear world, however. We should say that everything must be NON-LINEAR, with the result that a lot of non-normal phenomena emerge.

For instance, if the two excess demand curves are not linear, there is a possibility that multiple equilibriums emerge. Seeing is believing! Let us take a close look at seemingly strange diagram like Fig. 6 where the curve $E_1$ is S-shaped and the curve $E_2$ inverse S-shaped. Here, we have assumed that while the two goods remain to substitutes, the power relations between the own effects and the cross effects are no longer fixed, but rather very changeable: therefore, the own effects are dominant in some situations, yet the cross effects dominant in other situations.

Under such changeable situations, it is naturally expected that the intersection of the two curves $E_1$ and $E_2$ is not uniquely determined. In Fig. 6, there are THREE intersections, $Q_1$, $Q_2$ and $Q_3$. Among those three points, only the middle point $Q_2$ is stable because the cross effects overwhelm the cross effects in its neighborhood. However, the remaining two points $Q_2$ and $Q_3$ should not be stable at all since the cross effects are supposed to effectively overpower the own effects at those neighborhoods. It is my strong belief that the working of our real economy looks like Fig. 6.

If the two goods are complementary, those NON-LINEAR CURVES should be negatively sloped. Here again, we expect to see the sequence of stable and unstable intersections. To save the space, a detailed analysis of this and many other complicated situations will be omitted here.

. .

## 3 Comparative Statics : The Impact of Parameter Changes on Equilibriums

Let us turn our attention to the issue of comparative statics. This is a very important problem that was first intensively discussed by J.R. Hicks and later greatly developed by Michio Morishima.

First of all, let us assume that the two goods are substitutes like tea and coffee. Let us take a look at Fig. 7. In the left chart (A), the straight lines $E_1$ and $E_2$ respectively represent the excess demand lines of good 1 (tea) and good 2 (coffee). More specifically, let us consider the following set of equations:

$$E_1 \quad : \quad E_1 = -2p_1 + p_2 + 4 \tag{15}$$

$$E_2 \quad : \quad E_2 = p_1 - 3p_2 + 6 \tag{16}$$

Let put $E_1 = 0$ and $E_2 = 0$. Then, by solving for $p_1$ and $p_2$, we find a pair of equilibrium prices: in fact, the coordinates of equilibrium point $Q$ : $(p_1^Q, p_2^Q) = (3.6, 3.2)$.

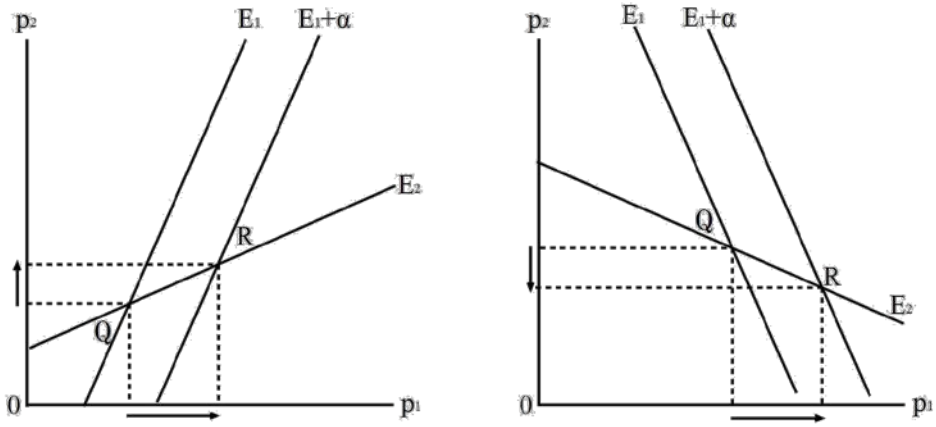
Now suppose that because of some reasons, the popularity of tea suddenly rises among the people. Then, the excess demand curve $E_1$ has to shift to the right. This implies that regarding the constant term of Eq, (15) above, it has to change its value from 4 to $(4 + \alpha)$:

$$\text{A change in } E_1 : \quad E_1 + \alpha = -2p_1 + p_2 + 4 + \alpha \tag{17}$$

Let us take a look at Fig. 7. In Chart (A), the new equilibrium point $R$ is found at

the intersection of the two lines, $E_1 = 0$ and $E_2 = 0$. For convenience, let us specify $\alpha = 2$. Then, the coordinates of $R$ are calculated as $(p_1^R, p_2^R) = (4.8, 3.6)$.

.




(A) The goods are substitutable (B) The goods are complimentary

Fig. 7 Comparative statics: changes in excess demand

### 3.1 Hicksian Three Laws of Price Changes: The Case of Substitutable Goods

In his classic *Value and Capital,* Hicks (1939, 1946) succeeded in deriving the very famous three laws which stood for the core of his comparative static results. It was also remarkable to see that those Hicksian laws were later developed a great deal by his successor, Morishima (1964). Ii is the Hicksian laws that we will intensively investigate in this and following subsections.

We are most interested in comparing the two equilibrium points, the old point $Q$ and the new point $R$. We are expected to obtain several results of great importance. First of all, we would like to pay attention to comparison of the following two relations:

$$p_1^R = 4.8 > 3.6 = p_1^Q \quad (18)$$

$$p_2^R = 3.6 > 3.2 = p_2^Q \quad (19)$$

To begin with, let us suppose that because of some reasons, tea becomes very popular among the people, whence its demand suddenly increases. To take an example,

the opening of the Great Britain Exhibition in Kyoto may produce such a sudden surge for the popularity of tea in Japan. Then, its immediate effect we can expect to have is that tea will be more expensive than before in response to the demand increase. Not only that, there would also be many other repercussions such as its indirect impact on other markets including the coffee market. Besides, those influences will invoke the third and forth rounds of repercussions as well. Those direct and indirect effects would make our repercussion calculations very hard and complicated. In order to make sure of this point, if we patiently compute the *proportional rates of increase* of the prices (in terms of per cent), then we will be able to derive the following equations:

$$(p_1{}^R - p_1{}^Q) \,/\, p_1{}^Q = (4.8-3.6) \,/\, 3.6 = 1/3 \qquad (20)$$

$$(p_2{}^R - p_2{}^Q) \,/\, p_2{}^Q = (3.6-3.2) \,/\, 3.2 = 1/8 \qquad (21)$$

Clearly, the ratio $(p_1{}^R - p_1{}^Q) / p_1{}^Q$ is greater than the ratio $(p_2{}^R - p_2{}^Q) / p_2{}^Q$, implying that the rising rate of tea exceeds the one of coffee.

We must bear in mind that whether or not the excess demand functions are linear, all the comparative static results derived above generally hold. J.R. Hick first obtained all those results which Michio Morishima rightfully called the "Hicksian Three Laws of Price Changes." They are really outstanding consequenses of the Hicks-Morishima approach, which can be summarized as the following theorem.

**Theorem 1 (Hicks-Morishima on substitutable goods)**

Assume that the two goods, $x_1$ and $x_2$, are substitutable and that the own effects overpower the cross effects. Then the point of general equilibrium, if it exists, must be stable. When the excess demand for good 1, $E_1$, increases, the following three properties must hold:

(i) The price of good 1, $p_1$, will rise.

(ii) The price of good 2, $p_2$, will rise too.

(iii) The price of good 1 will rise proportionately more than the price of good 2; namely, the price ratio $(p_1/p_2)$ will rise.

### 3.2 The Case of Complementary Goods: Hicksian Three Laws Continued

Now, let us turn to the case of complementary goods Let us assume that goods 1 and 2 respectively represent tea and sugar. Then, as Chart (B) of Fig. 7 above tells us, the two excess demand lines, $E_1$ and $E_2$, are both downward sloped. If we suppose

that the own effects outweigh the cross effects, then the old intersection point $Q$ becomes a stable point.

Now, suppose that a sudden tea boom happens among the people, resulting in a drastic increase in tea demand. This is the first direct effect of tea price rise. Moreover, such rise will lead to a sequence of secondary repercussions since it will dampen the demands for both tea and sugar. Recall here that complementary goods have to go in the same direction. This is the second indirect effect. If we employ a similar reasoning, then we would also take account of third and fourth round of repercussions that must have less influence than the second round.

In Chart (B), let us shift the excess demand function to the right, from $E_1$ to $E_1 + \alpha$. Then, since such shift changes the equilibrium point in a right and downward direction, from $Q$ to $R$, it will have to cause a big rise in $p_1$ together with a small change in $p_2$. Consequently, in absolute value terms, the rising rate of $p_1$ will exceed the falling rate of $p_2$. We omit a more detailed calculation here. [6)]

The final results for the case of complementary goods will be summarized as the following theorem:

**Theorem 2 (Hicks-Morishima on complementary goods)**

Suppose that the two goods, $x_1$ and $x_2$, are complementary, and that the own effects overpower the cross effects. Then, the point of general equilibrium, if it exists, must be stable. When the excess demand for good *1*, $E_1$, increases, the following three properties must hold:

(i) The price of good 1, $p_1$, will rise.

(ii) The price of good, $p_2$, will fall a bit.

(iii) The rising rate of good 1 will be greater than the falling rate of good *2*;

Namely, the price ratio ($p_1/p_2$) will rise a great deal.

### 4.3 A Very Big Change of Excess Demand: Price Instability and Downward Spiral

In the above, we have derived several comparative static results. We have to bear in mind that those results were applicable only to the case of LINER excess demand functions. Introduction of NON-LINEARITY, however, would lead to have considerable modifications. For instance, we could expect to have an extraordinary situation in which a slight change in outside parameter would result in a dramatic break of the whole price system. Such "crush phenomenon" of price system will be our next target of investigation.

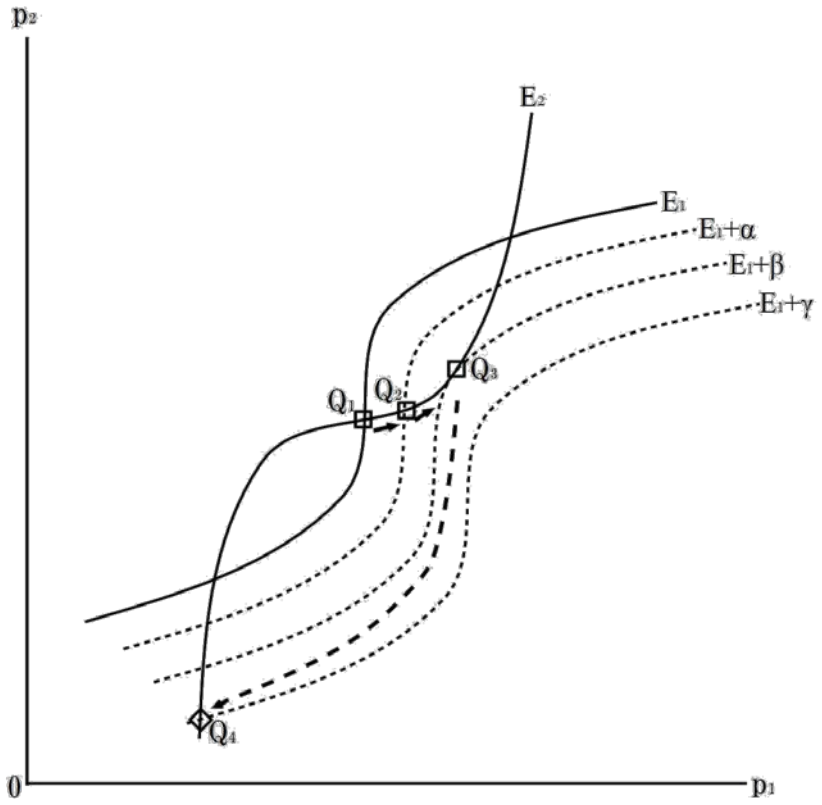



**Fig. 8　A big change in excess demand:　The prices may break suddenly**

Let us tale a careful look at Fig. 8.　Our starting situation in this figure is the same as the one of Fig. 6.　Clearly, the center-located intersection point $Q_1$ of the two curves $E_1$ and $E_2$ is stable in the small but not stable in the large.　Let us suppose an interesting situation where the excess demand for good 1 gradually increases.　More specifically, let us assume continuous shifts of the curve $E_1$ to the right, to $E_1 + \alpha$, and to $E_1 + \beta$, and further to $E_1 + \gamma$.

Let increments of excess demand be $\alpha$ and $\beta$.　Then, the new equilibrium points respectively become $Q_2$ and $Q_3$, which must imply rises in $p_1$ and $p_2$.　Note that those two new points $Q_2$ and $Q_3$ are in the neighborhood of the old point $Q_1$ : therefore, the former two points are stable.　Our situation would change drastically,

however, when the increment of excess demand becomes very large, exactly as large as $\gamma$ . Then, subject to such a new burst of excess demand, an equilibrium point would have to nosedive from $Q_3$ far down to $Q_4$ , as is clearly indicted by the sequence of small arrows (→→) in Fig. 8 . The newest point $Q_4$ is no longer stable, thus there would no guarantee that it will stay there. This looks like a "collapse of bubble", or a very unstable chaos.

To sum up, when the excess demand functions become non-linear, the performance of the market of several markets becomes correspondingly complicated, with the possible results of unstable and discontinuous shifts of equilibrium points. There might be the cases in which a sort of bubbles occur and collapse. To solve such complicated and chaotic situations, we would need to establish an entirely new field such as "nonlinear dynamics." We must keep in mind that a nonlinear world is a weird place: we never know what will happen next.

## 6 Morishima Towards a New Economicssons: Concluding Remarks

This paper may be regarded as an hommage to my two great mentors, Professors John W. Hicks and Michio Morishima. Although the most of their books and papers have been respected as the classic works in the economics profession, some of their works nevertheless have been given less respect and more or less ignored. I believe that the Hicks-Morishima approach to interdependence of several markets on the basis of excess demand approach constitutes one of those unlucky treatment. So, in this paper, I have attempted to do my utmost to mend such asymmetric tendency.

The Hicks-Morishima approach was adopted and developed by Morishima in his book (1984). For this book, Morishima gave an interesting subtitle "An Introduction to New Economics." Because it was far more than "an introduction" and contained "several new and original results," it seemed to me that the subtitle per se was "too modest, and even cheating." In fact, Morishima himself made several remarks :

> The approach of this book differs to a considerable degree from standard ecnomics. ... This book attempts to analyze the price mechanism in accordance with reality and at the same time to introduce students directly to the major problems of economics — an analysis of the way in which the real economy operates and the best way to bring about a change in direction in this operation.
>
> (Morishima, 1984, Preface)

Morishma's later book (1999) could be thought of as one of his "last swans," in which he greatly lamented the recent Japanese society as one approaching at a "dangerous stage." Here again, he made some intriguing remarks:

> This book is the one I have constantly wanted to write for a long time. In fact, it is a sort of a "Grand Integral Social Science" in the sense that it contained economics, sociology, education, history and many other related fields. It could rightly be called an "Academic Symphony." .. Let us take a careful look at many non-economic fields such as the worlds of politics, religion, and ideology. Then you will see that many main actors such as political, religious , and military bosses may play very large roles in our society. Unfortunately, economics and other related fields are so powerless to investigate those important problems. So, I should say that the existing social science remains in an under-developed state, thus lacking its very essential core." (Morishima, 1999, Appendix)

Honestly speaking, the right way toward the "Grand Integral Social Science" a la Morishima seems to be too far for me to take. Even a right direction for the road has not been found yet. A few years ago, when I published my book (2010) *The Economic Thought of Risk and Uncertainty* written in Japanese, I sent one copy to Professor Tetsuya Nose, a close friend of Morishima. Nose immediately replied to me, "This looked like 'an academic symphony' a la Morishima! " This was really the highest praise on my academic work.

I sincerely hope that this paper would serve very well as one small step toward Morishima's dream of establishing the Grand Integral Social Science. I strongly believe in the following golden saying: "Life is an adventure! Life is really a challenge! "

## Footnotes

* I am indebted to financial help from several research programs for Japanese Ministry of Education. I am also grateful to Dr. Masashi Tajima for helpful technical assistance.

1) Before the Second World War, Kiyoshi Oka and Chiujiro Nakaya met and became close friends in Paris. As was fully described in Takase (2008), Nakaya was then Oka's "true friend for life." Oka used to intimately call Nakaya "Chiu-san" whereas Nakaya gave a strange nickname "Doyasu" to Oka.

2) In September 1972, I submitted my Ph.D. dissertation to the University of Rochester, with Professors Lionel W. McKenzie and James Friedman being thesis advisers. As we seen in Sakai (1972), the thesis consisted of the two parts. The first part carefully discussed axiomatic approaches to consumption theory, the second part mathematical foundations of production theory. I would like to point out that Chapter 3 in Part 2, titled *An Axiomatic Approach to Input Demand Theory,* was based on my joint work with Dr. Masayoshi Hirora with further refinements. Very fortunately, a still further revision of this Chapter was published in *International Economic Review,* which was first published by Michio Morishima of Osaka University and Lawrence Klein of the University of Pennsylvania.

3) More exactly speaking, Professor Morishima's invited lecture was titled *Marx in the Light of Modern Economic Theory.* It was nothing but a Walrus memorial lecture at the North American Meeting of Econometric Society. Although it was first read at Hilton Hotel, New York, in 28 December 1973, it was later published as a leading article of *Econometrica.* It was really lucky that I myself was right there as an attentive audience. This constitutes one of never-to-be-forgotten memories in my long life.

4) Independent of Morishima's nostalgia, I would like to record here my own nostalgia relating to Mr. Tsukuba. Let us remind the reader that when Kyuya Fukada, a famous writer and mountain climber, decided to exclusively select "One Hundred Famous Mountains in Japan", he set the following three criterions. The first criterion for his selection was based on the dignity of a mountain. Mental or psychological dignity was more important than mere physical height. The second criterion was related to the history of a mountain, and the third to the characteristic traits. For instance, Mt. Tsukuba is not a very high mountain, definitely lower than 1000 meter. However, it has had a very intimate connection with human lives in the very early

stages of civilization. Indeed, so many people used to say "Mt. Tsukuba in the East, Mt. Fuji in the West." Therefore, Fukada thought that Mt. Tsukuba satisfied the three criterions for "Selective One Hundred Mountain Club." I have myself lived at the foot of Mt. Tsukuba, and even climbed to the mountaintop with Professor Leo Hurvicz, a close friend of McKenzie and a Nobel Prize winner. .

5) A group of chart diagrams like Fig. 2 were first introduced by Hicks (1939). Morishima who admired Hicks so much intended to make the chart diagrams revived, with the aim of establishing his own "new economics." To our regret, however, such revival of the Hicks-Morishima approach has not been very successful so far. It is our real intention that we give the approach one more chance.

6) As I mentioned before, the exact derivation of Propositions One and Two was the main theme of the Young Morishima. For that purpose, he literally racked his brains. In this paper, I have intended to limit my analysis on a simple yet useful case of *linear* excess demand functions. Besides, following the Morishima spirits, I have often made use of many eye-catching graphical approaches. I can say, however, that the extension of my analysis to the more general nonlinear cases should not be so difficult, requiring for a bit more calculation and patience.

7) In this paper, I have intentionally avoided to refer to the theories of marginal utility and marginal productivity. I believe that this is definitely in line with the Morishima spirits. For those fifty years, I myself have been striving for establishing a "new economics in the new century."

## References


Abe, K. (1962) *A Woman in Sand Dune*, in Japanese. Shincho Publishing Company, Tokyo.

Fukuda, K. (1964) *One Hundred Famous Mountains in Japan, in Japanese.* Shincho Publishing Company, Tokyo.

Hicks, J. R (1939, 2nd ed. 1946) *Value and Capital*, Oxford University Press.

Hicks, J. R. (1953-54) "A Reply," *Review of Economic Studies*, Vol.21.

Hicks, J. R. (1977) *Economic Perspectives*, Oxford University Press.

Hicks, J. R. (1984) *The Economics of John R. Hicks*, Blackwell.

Kato, K. and Ito, M. (1992) "Economic Power without Proper Economics," in Japanese. Monthly Journal *This Is Yomiuri,* August Issue.

Komoro, N. (2004) *Economics Super Stars: Seminar on the History of Economic Thought,* in Japanese. Diamond Publishing Company, Tokyo.

Morishima, M. (1953-54) "A Note on a Point in *Value and Capital*," *Review of Economic Studies*, Vol.21.

Morishima, M. (1953-54) "A Rejoinder," *Review of Economic Studies*, Vol. 21.

Morishima, M. (1974) "Marx in the Light of Modern Economic Theory," *Econometrica*, Vo.42, No.4.

Morishima, M. (1984) *The Economics of Industrial Society, Translated by Anthony, Clark and Hunter.* Cambridge University Press..

Morishima, M. (1993) *Modern Economics as Economic Thought,* NHK Pamphlet, in Japanese. Japan Air Association Publishing Company.

Morishima, M. (1999) *Relying on your Intellect Makes you Harsh and Rigid*, in Japanese. Iwanami Publishing Company, Tokyo.

Morishima, M. (1999) *Why Will Japan Collapse ?* in Japanese. Iwanami Publishing Company, Tokyo.

Oka, K. (1963) *Ten Science Essays in Spring Evenings,* Mainichi Newspaper Publishing Company, Tokyo.

Nakaya, C. (1938) *On the Stone Age of Ancient Japan : A Revised Synopsis*、in Japanese. Youtoku Publishing Company, Tokyo.

Sakai, Y. (1972) *Axiomatic Foundations of Consumption and Production Theories.* Ph.D. dissertation thesis, University of Rochester.

Sakai, Y. (1973) “An Axiomatic Approach to Input Demand Theory." *International Economic Review*, Vol.14, No.3.

Sakai, Y. (2010) *The Economic Thought of Risk and Uncertainty,* in Japanese. Minerva Publishing Company., Kyoto.

Sakai, Y. (2011) "The Comparative Theory of Economic Systems: Werner Sombart versus John R. Hicks." *Hikone Ronso,* Vol. 386.

Natsume, S. (1906) *Kusamakura*, or Sleeping on a Bed of Grass. Iwanami Publishers, Tokyo.

Takase, M. (2008) *Kiyoshi Oka: an Outstanding Poet of Mathematics,* in Japanese. Iwanami Publishing Company, Tokyo.