Bilateral Synchronization in a Globalized World: a Global Macroeconomic Approach

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Abstract

We develop a DSGE model to evaluate the contribution of third-countries to the bilateral synchronization. Using data for 16 countries (11 European countries, China, Canada, USA, Japan and the United Kingdom) over a period from 1995 to 2016, we find that the third country effect matters for bilateral synchronization. We distinguish trade between the extensive and the extensive margin of trade. Results show that bilateral synchronization is improved by extensive margin to third countries whereas intensive margin to third countries deteriorates bilateral synchronization.

1 Introduction

Although substantial empirical evidence suggests that bilateral trade contributes positively to output comovement (Frankel and Rose, 1998; Clark and Wincoop, 2001; Imbs, 2004, Baxter and Kouparitsas, 2005, among others), theoretical models fail to fully replicate this relationship. International business cycle models are unable to generate trade effects on business cycles synchronization as strong as those observed from the data. Kose and Yi (2006) build a three-country, two asset structures, complete markets and international financial autarky model and simulate the effects of increased trade integration on business cycle correlations. The model implies an increase in output correlation for pairs of countries with stronger trade linkages but this theoretical correlation falls far short of the empirical findings. This is the so-called trade comovement puzzle in the standard international real business cycle model (IRBC). These authors

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also highlight that allowing for higher trade with third country or higher TFP shock comovement improve the models performance in reducing the gap between the empirical findings and theoretical predictions. To solve this trade comovement puzzle, Arkolakis and Ramanarayanan (2009) develop an international business cycle model in which bilateral trade in goods across two stages of production varies with trade barriers. The vertical specialization fails to solve the trade-comovement puzzle with perfect competition but helps to solve the puzzle with imperfect competition. Di Giovanni and Levchenko (2010), using firm level data and input-output matrix, show that sector pairs that experience higher bilateral trade exhibit stronger comovements. They find that vertical production linkages account for 30 percent of the total impact of bilateral trade on the business cycle correlation. Wong and Eng (2013) develop a two-country, three-processing stage New Keynesian model, which allow for both vertical trade (importing intermediates for re-exporting as intermediates) and processing trade (importing intermediates for re-exporting as final goods). According to Wong and Eng (2013), their model is able to solve the trade-comovement puzzle. Johnson (2014) develops a multi-country, multisector extension of the IRBC model that includes sector-to-sector input-output linkages both within and across countries. Results show that input trade does not solve the trade comovement puzzle: the model yields high trade-comovement correlations for goods, but near-zero correlations for services and thus low aggregate correlations. For Liao and Santacreu (2015), TFP correlations matter for the theoretical effects of trade on business synchronization. Distinguishing between intensive and extensive margins of trade, Liao and Santacreu (2015) show that the extensive margin of trade increases the correlation between trading partners aggregate productivity and therefore drives the observed output comovement. Also business cycles between countries that trade a wider variety should be highly correlated. Zlate (2016) underlines the key role played by firm entry and offshoring to explain transmission of business cycles. He develops a two-country model to examine the effect of offshoring through vertical foreign direct investments on business cycle correlation. His model distinguishes between the intensive and the extensive margins of offshoring and reveals that offshoring and its extensive margin raise output comovement across countries. More recently, Drozd et al. (2017) study the dynamic properties of trade elasticity. They argue that models will be more consistent with dynamic instead of static trade elasticity. In this paper, the objective is twofold: i) based on the model of Ghironi and Melitz (2005), we elaborate a third-country model and distnguish trade between extensive and extensive margins. Using this model, we can identify the role of trade with the rest of the World in the bilateral synchronization. And ii) we test empirically the effects of the intensive and extensive margins with the rest of the World on the synchronization between two countries. Results show that the extensive margin (between i and k) improved the synchronisation between countries i and j whereas the intensive margin has negative effects on synchronization.

2 A DSGE model with a third country effect

Our setting extends Ghironi and Melitz (2005) to the case of a three country world. As prices and wages are flexible, we solve the model in real terms. The exchange rate regime is irrelevant. There are three countries, indexed by $i = H, F, T$ for home, foreign and third countries. In what follows we describe the model from the point of view of the home countries. Similar relations apply to the other two countries. As a general principal of notation, the country appears as an exponent on variables, while the geographical origin of the goods or asset is a subscript. In each of the three country, the number of goods available for consumption obeys to

$$n^i_{D,t} = (1-\delta)(n^i_{D,t-1} + n^i_{E,t-1})$$

where $n^i_{D,t-1}$ and $n^i_{E,t-1}$ are the mass of domestic and new entrants firms, respectively. It is assumed that new entrants firms in period $t-1$ will start to produce only in period t. Moreover, an exogenous death shock with probability δ hits all incumbent firms ($n^i_{t-1} = n^i_{D,t-1} + n^i_{E,t-1}$) each period, and only a fraction $n^i_{D,t} = (1-\delta)n^i_{t-1}$ will survive and produce in the next period.

2.1 Households

The following presentation concerns home country. The model in foreign and third countries can be derived in an analogous manner.

All households are identical and allocated across the unit interval. The preferences of the j representative household are defined by a utility function, separable between consumption bundles and (the disutility of) labor, which for period t reads,

$$\frac{c^H_t(j)^{1-\sigma_c}}{1-\sigma_c} - \Xi\frac{l^H_t(j)^{1+\sigma_l}}{1+\sigma_l},$$

where $c^H_t(j)$ is number of bundles currently consumed, , $\sigma_c > 0$ is the risk aversion coefficient, $l^H_t(j)$ is the amount of labor supplied, $\sigma_l > 0$ is the curvature coefficient in disutility of labor, and $\Xi > 0$ is the coefficient of the weight of labor disutility in overall utility. A constant discount factor per period, $\beta < 1$, is used to bring future utility into present time.

The sources of household income are labor earnings, equity return and the interest service of bonds. Labor income is $w^H_t l^H_t(j)$, where the real wage w^H_t is measured in consumption bundles at the price index, $P^H_{C,t}$. Equity ownership permits households to earn average real dividend

$\widetilde{d}_t^H$ and average equity (real) value $\widetilde{v}_t^H$ of firms that are not hit by death shock and continue to produce.[1] Income is spent on purchases of bundles of consumption goods, $c_t^H(j)$; on portfolio investment x_{t+1} on incumbent firms n_t^H that is formally given by $\widetilde{v}_t^H n_t^H x_{t+1}(j)$; on net purchases of domestic $b_{H,t+1}^H(j)$, foreign bonds $b_{H,t+1}^F(j)$ and third country bonds $b_{T,t+1}^H(j)$ with real return r_t^i for $i = H, F, T$. As a result, the budget constraint of the representative household in period t becomes,

$$
\begin{aligned}
& c_t^H(j) + \widetilde{v}_t^H n_t^H x_{t+1}(j) + b_{H,t+1}^H(j) + q_{HF,t} b_{F,t+1}^H(j) + q_{HT,t} b_{T,t+1}^H(j) \\
= \; & w_t^H l_t^H(j) + (1 + r_t^H) b_{H,t}^H(j) + q_{HF,t}(1 + r_t^F) b_{F,t}^H(j) + q_{HT,t}(1 + r_t^T) b_{T,t}^H(j) + \left(\widetilde{d}_t^H + \widetilde{v}_t^H\right) n_{D,t}^H x_t(j)
\end{aligned}
$$

The first order conditions are

$$
\begin{aligned}
\beta E_t \left[\left(\frac{c_t^H(j)}{c_{t+1}^H(j)} \right)^{\sigma_c} \frac{1 + r_t^H}{1 + \pi_{t+1}^C} \right] - 1 &= 0 \\
\widetilde{v}_t^H - \frac{1}{1 + r_t} \left[(1 - \delta) E_t \left(\widetilde{d}_{t+1}^H + \widetilde{v}_{t+1}^H \right) \right] &= 0 \\
\Xi l_t^H(j)^{\sigma_l} c_t^H(j)^{\sigma_c} - w_t &= 0 \\
E_t \left[\frac{c_{t+1}^H(j)^{\sigma_c}}{1 + \pi_{t+1}^C} \left((1 + r_t^H) - \frac{q_{H,F,t+1.}}{q_{H,F,t.}} (1 + r_t^F) \right) \right] &= 0 \\
E_t \left[\frac{c_{t+1}^H(j)^{\sigma_c}}{1 + \pi_{t+1}^C} \left((1 + r_t^H) - \frac{q_{H,T,t+1.}}{q_{H,T,t.}} (1 + r_t^T) \right) \right] &= 0
\end{aligned}
$$

computed with respect to the choice variables $c_t(j)$, $b_{t+1}(j)$, $W_t(j)$, $x_t(j)$, . Intratemporal choices of consumers are as follows

$$
c_t^H = \left[\left(c_{D,t}^H \right)^{\frac{\theta - 1}{\theta}} + \left(c_{XH,t}^F \right)^{\frac{\theta - 1}{\theta}} + \left(c_{XH,t}^T \right)^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}}
$$

Associated price indexes are thus,

$$
p_t^H = \left[\left(p_{D,t}^H \right)^{1 - \theta} + \left(p_{XH,t}^F \right)^{1 - \theta} + \left(p_{XH,t}^T \right)^{1 - \theta} \right]^{\frac{1}{1 - \theta}}
$$

Thus relative demands for individual firms are,

$$
c_{D,t}^H(\omega) = \left(\rho_{D,t}^H(\omega) \right)^{-\theta} c_t^H
$$

$$
c_{XH,t}^F(\omega) = \left(\rho_{XH,t}^F(\omega) \right)^{-\theta} c_t^H
$$

[1] It is assumed that upon investing, households have no information on the probability of survival of firms. Therefore, households invest on mutual fund share of incumbent firms n_t^H (new entrants firms $n_{E,t}^H$ and already operating firms $n_{D,t}^H$). However, only survival firms $n_{D,t+1}^H = (1 - \delta) n_t^H$ will pay dividends.

$$c^T_{XH,t}(\omega) = \left(\rho^T_{XH,t}(\omega)\right)^{-\theta} c^H_t$$

with $\rho^H_{D,t}(\omega) = \left(\frac{p^H_{D,t}(\omega)}{p^H_t}\right)$, $\rho^F_{XH,t}(\omega) = \left(\frac{p^F_{XH,t}(\omega)}{p^H_t}\right)$ and $\rho^T_{XH,t}(\omega) = \left(\frac{p^T_{XH,t}(\omega)}{p^H_t}\right)$. In this study, we assume that export prices are set in the local currency of sale. Therefore, $p^F_{XH,t}$ and $p^T_{XH,t}$ represent prices of goods from foreign and third country that are labeled in home currency.

2.2 Firms

In period t, the firm of type ω produces a quantity $y_t(\omega)$ of this good using the Cobb-Douglas production function,

$$y^H_t(\omega) = A^H_t z(\omega)\, l^H_t(\omega), \tag{10}$$

where $l_t(\omega)$ is the demand for labor and capital at firm ω, A^H_t is a technology shock, $z(\omega)$ is a firm-specific productivity level. The shock A^H_t is homogeneous to all firms. Nevertheless, there is firm heterogeneity determined by $z(\omega)$, which is an individual draw from the Pareto distribution characterized by its lower bound $z_{\min} > 0$ and the shape parameter $\kappa > (\theta_p - 1)$.[2]

The profit of the firm is defined as, $p^H_C d^H(\omega) = p^H_C \left[d^H_D(\omega) + d^H_X(\omega)\right]$. The question is to compute two aspects: price setting and profits.

In nominal term a country serving only the domestic market owns nominal profit,

$$p^H_C d^H_D(\omega) = p^H_{H,t}(\omega) y^H_t(\omega) - \frac{W^H_t}{A^H_t z(\omega)} y^H_t(\omega)$$

the optimal price is thus,

$$p^H_{H,t}(\omega) = \frac{\theta}{\theta - 1}\frac{W^H_t}{A^H_t z(\omega)}$$

while if it serves the export market, due to the fact that there are two destinations,

$$p^H_C d^H_X(\omega) = \left(\frac{e_{H,F,t}.p^F_{H,t}(\omega)}{1+\tau_{H,F}} - \frac{W^H_t}{A^H_t z(\omega)}\right) y^F_{H,T,t}(\omega) + \left(\frac{e_{H,T,t}.p^T_{H,t}(\omega)}{1+\tau_{H,T}} - \frac{W^H_t}{A^H_t z(\omega)}\right) y^T_{H,T,t}(\omega) - \frac{W^H_t}{A^H_t} f^H_X$$

the optimal price is thus, in local currency

$$\begin{aligned} p^F_{H,t}(\omega) &= \frac{1+\tau_{H,F}}{e_{H,F,t.}}\frac{\theta}{\theta-1}\frac{W^H_t}{A^H_t z(\omega)} \\ p^T_{H,t}(\omega) &= \frac{1+\tau_{H,T}}{e_{H,F,t.}}\frac{\theta}{\theta-1}\frac{W^H_t}{A^H_t z(\omega)} \end{aligned}$$

[2]The probability distribution function and the cumulative distribution function of $z(\omega)$ are respectively $g(z(\omega)) = \kappa z^\kappa_{\min}/z(\omega)^{\kappa+1}$ and $G(z(\omega)) = 1 - (z_{\min}/z(\omega))^\kappa$. The shape parameter κ must be higher than $(\theta_p - 1)$ to have a well-defined average productivity.

combining the definition of profits with optimal prices, we get, in real terms, the compact expression of profits,

$$\begin{aligned} d_D^H(\omega) &= \frac{1}{\theta}\left(\rho_{H,t}^H(\omega)\right)^{1-\theta} C_t^H \\ d_X^H(\omega) &= \frac{q_{H,F,t}}{\theta}\left(\rho_{H,t}^F(\omega)\right)^{1-\theta} C_t^F + \frac{q_{H,T,t}}{\theta}\left(\rho_{H,t}^T(\omega)\right)^{1-\theta} C_t^T - \frac{w_t^H}{A_t^H} f_X^H \end{aligned}$$

2.3 Sectoral averages

Before computing national aggregates, we need to compute sectoral averages regarding prices and the supply of goods. The total number of firms is determined by the cut off condition

$$d_X^H(\omega) = 0$$

which leads to

$$z_X^H = \left(\frac{1}{\theta}\right)^{-\frac{1}{1-\theta}} \left(\frac{\theta}{\theta-1}\frac{W_t^H}{A_t^H}\right)\left[q_{H,F,t}\left(\frac{1+\tau_{H,F}}{q_{H,F,t.}}\right)^{1-\theta} C_t^F + q_{H,T,t}\left(\frac{1+\tau_{H,T}}{q_{H,T,t.}}\right)^{1-\theta} C_t^T\right]^{-\frac{1}{1-\theta}} \left(\frac{W_t}{e^{\varepsilon_t^a}} f_X^H\right)^{\frac{1}{1-\theta}}$$

Comments : novel result that accounts for the third country effect.

plugging this condition in the pareto distribution,

$$\begin{aligned} \frac{n_{D,t}^H}{n_t^H} &= \int_{z_{\min}^H}^{z_X^H} g(z)dz = 1 - \left(\frac{z_{\min}}{z_X^H}\right)^{\kappa} \\ \frac{n_{X,t}^H}{n_t^H} &= \int_{z_{\min}^H}^{\infty} g(z)dz = \left(\frac{z_{\min}}{z_X^H}\right)^{\kappa} \end{aligned}$$

As standardly computed, average productivity levels in each sector is defined as follows (harmonic average)

$$\begin{aligned} \tilde{z}_{D,t}^H &= \nabla^H z_{\min}^H \\ \tilde{z}_{Xt}^H &= \nabla^H z_X^H \end{aligned}$$

with, $\nabla^H = (\kappa/(\kappa-(\theta-1)))^{\frac{1}{\theta-1}}$ thus, average prices are given by,

$$\tilde{z}_t^H = \nabla^H \left[z_{\min}^H - \left(z_{\min}^H\right)^{1+\kappa}\left(z_X^H\right)^{-\kappa} + (z_{\min})^{\kappa}\left(z_X^H\right)^{1-\kappa}\right]$$

average real prices charged on lacal markets for home goods

$$\begin{aligned} \tilde{\rho}_{D,t}^H &= \frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H \tilde{z}_{D,t}^H} \\ \tilde{\rho}_{H,t}^F &= q_{H,F,t.}^{-1}\left(1+\tau_{H,F}\right)\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H \tilde{z}_{X,t}^H} \\ \tilde{\rho}_{H,t}^T &= q_{H,T,t.}^{-1}\left(1+\tau_{H,T}\right)\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H \tilde{z}_{T,t}^H} \end{aligned}$$

average profits,

$$\begin{aligned}
\tilde{d}^H_{D,t} &= \frac{C^H_t}{\theta}\left(\frac{\theta}{\theta-1}\frac{w^H_t}{A^H_t}\right)^{1-\theta}\left(\tilde{z}^H_{D,t}\right)^{\theta-1}\\
\tilde{d}^H_{X,t} &= \left[\frac{(1+\tau_{H,F})^{1-\theta}\,q^{\theta}_{H,F,t}}{\theta}C^F_t+\frac{(1+\tau_{H,T})^{1-\theta}\,q^{\theta}_{H,T,t}}{\theta}C^T_t\right]\left(\frac{\theta}{\theta-1}\frac{w^H_t}{A^H_t}\right)^{1-\theta}\left(\tilde{z}^H_X\right)^{\theta-1}-f^H_X\frac{w^H_t}{A^H_t}
\end{aligned}$$

sectoral labour demand

2.4 General equilibrium

$$\begin{aligned}
\tilde{l}^H_{D,t} &= (\theta-1)\frac{\tilde{\pi}^H_D}{w^H_t}\\
\tilde{l}^H_{X,t} &= \frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{\pi}^H_X}{w^H_t}+\frac{f^H_X}{A^H_t}
\end{aligned}$$

Assuming symmetry in asset holdings, in each economy, defining the aggregate outpout of country i as ..., a competitive equilibrium is defined as a sequence of quantities

$$\{Q\}^{\infty}_{t=0}=\left\{\begin{array}{c} y^H_t, y^F_t, y^T_t, c^H_t, c^F_t, c^T_t, l^H_t, l^F_t, l^T_t, n^H_t, n^F_t, n^T_t, n^H_{E,t}, n^F_{E,t}, n^T_{E,t}, n^H_{X,t}, n^F_{X,t}, n^T_{X,t},\\ z^H_{X,t}, z^F_{X,t}, z^T_{X,t}, \tilde{d}^H_{X,t}, \tilde{d}^F_{X,t}, \tilde{d}^T_{X,t}, b^H_{H,t}, b^F_{H,t}, b^T_{H,t}, b^H_{F,t}, b^F_{F,t}, b^T_{F,t}, b^H_{T,t}, b^F_{T,t}, b^T_{T,t}\end{array}\right\}$$

$$\{P\}^{\infty}_{t=0}=\left\{r_t, \tilde{\rho}^H_{D,t}, \tilde{\rho}^F_{H,t}, \tilde{\rho}^T_{H,t}, \tilde{\rho}^H_{F,t}, \tilde{\rho}^F_{D,t}, \tilde{\rho}^T_{F,t}, \tilde{\rho}^H_{T,t}, \tilde{\rho}^F_{T,t}, \tilde{\rho}^T_{D,t}, w^H_t, w^F_t, w^T_t, \tilde{v}^H_t, \tilde{v}^F_t, \tilde{v}^T_t, q_{H,F,t}, q_{H,T,t}, q_{F,T,t}\right\}$$

Aggregation

given sectoral averages, totla labour demand is defined as,

$$\begin{aligned}
l^H_t &= n^H_t(\theta-1)\frac{\tilde{d}^H_D}{w^H_t}+n^H_{X,t}\frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{d}^H_X}{w^H_t}+n^H_{X,t}\frac{f^H_X}{A^H_t}+n^H_{E,t}\frac{f^H_E}{A^H_t}\\
l^F_t &= n^F_t(\theta-1)\frac{\tilde{d}^F_D}{w^H_t}+n^F_{X,t}\frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{d}^F_X}{w^F_t}+n^F_{X,t}\frac{f^H_X}{A^H_t}+n^F_{E,t}\frac{f^H_E}{A^H_t}\\
l^T_t &= n^T_t(\theta-1)\frac{\tilde{d}^T_D}{w^H_t}+n^F_{X,t}\frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{d}^T_X}{w^F_t}+n^T_{X,t}\frac{f^H_X}{A^H_t}+n^T_{E,t}\frac{f^H_E}{A^H_t}
\end{aligned}$$

while total production devoted to consumption is,

$$\begin{aligned}
y^H_t &= n^H_t\left(\tilde{\rho}^H_{H,t}\right)^{1-\theta}C^H_t+n^H_{X,t}\left(q_{H,F,t}\left(\tilde{\rho}^F_{H,t}\right)^{1-\theta}C^F_t+q_{H,T,t}\left(\tilde{\rho}^T_{H,t}\right)^{1-\theta}C^T_t\right)\\
y^F_t &= n^F_t\left(\tilde{\rho}^F_{F,t}\right)^{1-\theta}C^F_t+n^F_{X,t}\left(q^{-1}_{H,F,t}\left(\tilde{\rho}^H_{F,t}\right)^{1-\theta}C^H_t+q_{F,T,t}\left(\tilde{\rho}^T_{F,t}\right)^{1-\theta}C^T_t\right)\\
y^T_t &= n^T_t\left(\tilde{\rho}^T_{T,t}\right)^{1-\theta}C^H_t+n^T_{X,t}\left(q^{-1}_{H,T,t}\left(\tilde{\rho}^H_{T,t}\right)^{1-\theta}C^H_t+q^{-1}_{F,T,t}\left(\tilde{\rho}^F_{T,t}\right)^{1-\theta}C^F_t\right)
\end{aligned}$$

international financial market Regarding the international financial environment, we get (assuming symmetry wrt households and countries),

$$\begin{aligned}\left(\frac{c_t^H}{c_t^F}\right)^{\sigma_c} &= q_{H,F,t}\\ \left(\frac{c_t^H}{c_t^T}\right)^{\sigma_c} &= q_{H,T,t}\\ \left(\frac{c_t^F}{c_t^T}\right)^{\sigma_c} &= q_{F,T,t}\end{aligned}$$

with, $q_{H,F,t} = \frac{e_{H,F,t}.p_{C,t}^F}{p_{C,t}^H}$ and $q_{H,T,t} = \frac{e_{H,T,t}.p_{C,t}^T}{p_{C,t}^H}$.

GDP (data consistent given sunk costs for exports and entry)

$$\begin{aligned}y_{gdp,t}^H &= y_t^H + n_{X,t}^H \frac{f_X^H}{A_t^H} + n_{E,t}^H \frac{f_E^H}{A_t^H}\\ y_{gdp,t}^F &= y_t^F + n_{X,t}^F \frac{f_X^F}{A_t^F} + n_{E,t}^F \frac{f_E^F}{A_t^F}\\ y_{gdp,t}^T &= y_t^T + n_{X,t}^T \frac{f_X^T}{A_t^T} + n_{E,t}^T \frac{f_E^T}{A_t^T}\end{aligned}$$

$$\{Q\}_{t=0}^{\infty} = \left\{ \begin{array}{c} y_t^H, y_t^F, y_t^T, c_t^H, c_t^F, c_t^T, l_t^H, l_t^F, l_t^T, n_t^H, n_t^F, n_t^T, n_{E,t}^H, n_{E,t}^F, n_{E,t}^T, n_{X,t}^H, n_{X,t}^F, n_{X,t}^T, \\ z_{X,t}^H, z_{X,t}^F, z_{X,t}^T, \tilde{d}_{X,t}^H, \tilde{d}_{X,t}^F, \tilde{d}_{X,t}^T, \tilde{d}_{D,t}^H, \tilde{d}_{D,t}^F, \tilde{d}_{D,t}^T \end{array} \right\}$$

$$\{P\}_{t=0}^{\infty} = \{r_t^H, r_t^F, r_t, \tilde{\rho}_{D,t}^H, \tilde{\rho}_{H,t}^F, \tilde{\rho}_{H,t}^T, \tilde{\rho}_{F,t}^H, \tilde{\rho}_{D,t}^F, \tilde{\rho}_{F,t}^T, \tilde{\rho}_{T,t}^H, \tilde{\rho}_{T,t}^F, \tilde{\rho}_{D,t}^T, w_t^H, w_t^F, w_t^T, \tilde{v}_t^H, \tilde{v}_t^F, \tilde{v}_t^T, q_{H,F,t}, q_{H,T,t}, q_{F,T,t}\}$$

$$\beta E_t \left[\left(\frac{c_t^H(j)}{c_{t+1}^H(j)} \right)^{\sigma_c} \left(1 + r_t^H\right) \right] - 1 = 0$$

$$\begin{aligned}\tilde{v}_t^H - \frac{1}{1+r_t^H} \left[(1-\delta)E_t\left(\tilde{d}_{t+1}^H + \tilde{v}_{t+1}^H\right)\right] &= 0\\ \Xi l_t^H(j)^{\sigma_l} c_t^H(j)^{\sigma_c} - w_t^H &= 0\end{aligned}$$

$$\begin{aligned}\tilde{v}_t^F - \frac{1}{1+r_t^F} \left[(1-\delta)E_t\left(\tilde{d}_{t+1}^F + \tilde{v}_{t+1}^F\right)\right] &= 0\\ \Xi l_t^F(j)^{\sigma_l} c_t^F(j)^{\sigma_c} - w_t^F &= 0\end{aligned}$$

$$\begin{aligned}\tilde{v}_t^T - \frac{1}{1+r_t^T} \left[(1-\delta)E_t\left(\tilde{d}_{t+1}^T + \tilde{v}_{t+1}^T\right)\right] &= 0\\ \Xi l_t^T(j)^{\sigma_l} c_t^T(j)^{\sigma_c} - w_t^T &= 0\end{aligned}$$

$$
\begin{aligned}
c_t^H(i) &= \left[\left(c_{H,t}^H\right)^{\frac{\theta-1}{\theta}} + \left(c_{F,t}^H\right)^{\frac{\theta-1}{\theta}} + \left(c_{T,t}^H\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\
c_t^F(i) &= \left[\left(c_{H,t}^F\right)^{\frac{\theta-1}{\theta}} + \left(c_{F,t}^F\right)^{\frac{\theta-1}{\theta}} + \left(c_{T,t}^F\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\
c_t^T(i) &= \left[\left(c_{H,t}^T\right)^{\frac{\theta-1}{\theta}} + \left(c_{F,t}^T\right)^{\frac{\theta-1}{\theta}} + \left(c_{T,t}^T\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{v}_t^H &= \frac{w_t^H}{A_t^H} f_E^H \\
\widetilde{v}_t^F &= \frac{w_t^F}{A_t^F} f_E^F \\
\widetilde{v}_t^T &= \frac{w_t^T}{A_t^T} f_E^T
\end{aligned}
$$

$$
\begin{aligned}
n_t^H &= (1-\delta)(n_{t-1}^H + n_{E,t}^H) \\
n_t^F &= (1-\delta)(n_{t-1}^F + n_{E,t}^F) \\
n_t^T &= (1-\delta)(n_{t-1}^T + n_{E,t}^T)
\end{aligned}
$$

$$
\begin{aligned}
n_t^H &= n_{Dt}^H + n_{X,t}^H \\
n_t^F &= n_{Dt}^F + n_{X,t}^F \\
n_t^T &= n_{Dt}^T + n_{X,t}^T
\end{aligned}
$$

$$
\begin{aligned}
z_X^H &= \left(\frac{1}{\theta}\right)^{-\frac{1}{1-\theta}} \left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right) \left[q_{H,F,t}\left(\frac{1+\tau_{H,F}}{q_{H,F,t.}}\right)^{1-\theta} C_t^F + q_{H,T,t}\left(\frac{1+\tau_{H,T}}{q_{H,T,t.}}\right)^{1-\theta} C_t^T\right]^{-\frac{1}{1-\theta}} \left(\frac{w_t^H}{A_t^H} f_X^H\right)^{\frac{1}{1-\theta}} \\
z_X^H &= \left(\frac{1}{\theta}\right)^{-\frac{1}{1-\theta}} \left(\frac{\theta}{\theta-1}\frac{w_t^F}{A_t^F}\right) \left[q_{H,F,t}^{-1}\left(\frac{1+\tau_{H,F}}{q_{H,F,t.}^{-1}}\right)^{1-\theta} C_t^H + q_{F,T,t}\left(\frac{1+\tau_{F,T}}{q_{F,T,t.}}\right)^{1-\theta} C_t^T\right]^{-\frac{1}{1-\theta}} \left(\frac{w_t^F}{A_t^F} f_X^F\right)^{\frac{1}{1-\theta}} \\
z_X^H &= \left(\frac{1}{\theta}\right)^{-\frac{1}{1-\theta}} \left(\frac{\theta}{\theta-1}\frac{w_t^T}{A_t^H}\right) \left[q_{H,T,t}^{-1}\left(\frac{1+\tau_{H,T}}{q_{H,T,t.}^{-1}}\right)^{1-\theta} C_t^H + q_{F,T,t}^{-1}\left(\frac{1+\tau_{F,T}}{q_{F,T,t}^{-1}}\right)^{1-\theta} C_t^F\right]^{-\frac{1}{1-\theta}} \left(\frac{w_t^T}{A_t^T} f_X^T\right)^{\frac{1}{1-\theta}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{n_{X,t}^H}{n_t^H} &= \left(\frac{z_{\min}}{z_X^H}\right)^{\kappa} \\
\frac{n_{X,t}^F}{n_t^F} &= \left(\frac{z_{\min}}{z_X^F}\right)^{\kappa} \\
\frac{n_{X,t}^T}{n_t^T} &= \left(\frac{z_{\min}}{z_X^T}\right)^{\kappa}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\rho}^{H}_{H,t} &= \frac{\theta}{\theta-1}\frac{w^{H}_{t}}{A^{H}_{t}\nabla^{H} z^{H}_{\min}} \\
\tilde{\rho}^{F}_{H,t} &= q^{-1}_{H,F,t.}\left(1+\tau_{H,T}\right)\frac{\theta}{\theta-1}\frac{w^{H}_{t}}{A^{H}_{t}\nabla^{H} z^{H}_{X}} \\
\tilde{\rho}^{T}_{H,t} &= q^{-1}_{H,T,t.}\left(1+\tau_{H,T}\right)\frac{\theta}{\theta-1}\frac{w^{H}_{t}}{A^{H}_{t}\nabla^{H} z^{H}_{X}}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\rho}^{F}_{F,t} &= \frac{\theta}{\theta-1}\frac{w^{F}_{t}}{A^{F}_{t}\nabla^{F} z^{F}_{\min}} \\
\tilde{\rho}^{H}_{F,t} &= q_{H,F,t.}\left(1+\tau_{H,T}\right)\frac{\theta}{\theta-1}\frac{w^{F}_{t}}{A^{F}_{t}\nabla^{F} z^{F}_{X}} \\
\tilde{\rho}^{T}_{F,t} &= q^{-1}_{F,T,t.}\left(1+\tau_{F,T}\right)\frac{\theta}{\theta-1}\frac{w^{F}_{t}}{A^{F}_{t}\nabla^{F} z^{F}_{X}}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\rho}^{T}_{T,t} &= \frac{\theta}{\theta-1}\frac{w^{T}_{t}}{A^{T}_{t}\nabla^{T} z^{T}_{\min}} \\
\tilde{\rho}^{F}_{T,t} &= q_{F,T,t.}\left(1+\tau_{F,T}\right)\frac{\theta}{\theta-1}\frac{w^{T}_{t}}{A^{T}_{t}\nabla^{T} z^{T}_{X}} \\
\tilde{\rho}^{H}_{T,t} &= q_{H,T,t.}\left(1+\tau_{H,T}\right)\frac{\theta}{\theta-1}\frac{w^{T}_{t}}{A^{T}_{t}\nabla^{T} z^{T}_{X}}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{d}^{H}_{H,t} &= \frac{c^{H}_{t}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w^{H}_{t}}{A^{H}_{t}}\right)^{1-\theta}\left(\tilde{z}^{H}_{D,t}\right)^{\theta-1} \\
\tilde{d}^{H}_{X,t} &= \left[\frac{\left(1+\tau_{H,F}\right)^{1-\theta} q^{\theta}_{H,F,t}}{\theta}c^{F}_{t} + + \frac{\left(1+\tau_{H,T}\right)^{1-\theta} q^{\theta}_{H,T,t}}{\theta}c^{T}_{t}\right]\left(\frac{\theta}{\theta-1}\frac{w^{H}_{t}}{A^{H}_{t}}\right)^{1-\theta}\left(\tilde{z}^{H}_{X}\right)^{\theta-1} - f^{H}_{X}\frac{w^{H}_{t}}{A^{H}_{t}}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{d}^{F}_{F,t} &= \frac{c^{F}_{t}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w^{F}_{t}}{A^{F}_{t}}\right)^{1-\theta}\left(\tilde{z}^{F}_{F,t}\right)^{\theta-1} \\
\tilde{d}^{F}_{X,t} &= \left[\frac{\left(1+\tau_{H,F}\right)^{1-\theta} q^{-\theta}_{H,F,t}}{\theta}c^{H}_{t} + + \frac{\left(1+\tau_{H,T}\right)^{1-\theta} q^{\theta}_{F,T,t}}{\theta}c^{T}_{t}\right]\left(\frac{\theta}{\theta-1}\frac{w^{F}_{t}}{A^{F}_{t}}\right)^{1-\theta}\left(\tilde{z}^{F}_{X}\right)^{\theta-1} - f^{F}_{X}\frac{w^{F}_{t}}{A^{F}_{t}}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{d}^{T}_{T,t} &= \frac{c^{T}_{t}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w^{T}_{t}}{A^{T}_{t}}\right)^{1-\theta}\left(\tilde{z}^{T}_{D,t}\right)^{\theta-1} \\
\tilde{d}^{T}_{X,t} &= \left[\frac{\left(1+\tau_{H,F}\right)^{1-\theta} q^{-\theta}_{H,T,t}}{\theta}c^{H}_{t} + + \frac{\left(1+\tau_{H,T}\right)^{1-\theta} q^{-\theta}_{F,T,t}}{\theta}c^{F}_{t}\right]\left(\frac{\theta}{\theta-1}\frac{w^{T}_{t}}{A^{T}_{t}}\right)^{1-\theta}\left(\tilde{z}^{T}_{X}\right)^{\theta-1} - f^{T}_{X}\frac{w^{T}_{t}}{A^{T}_{t}}
\end{aligned}
$$

$$
\begin{aligned}
l_t^H &= n_t^H (\theta-1)\frac{\tilde{d}_D^H}{w_t^H} + n_{X,t}^H \frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{d}_X^H}{w_t^H} + n_{X,t}^H \frac{f_X^H}{A_t^H} + n_{E,t}^H \frac{f_E^H}{A_t^H} \\
l_t^F &= n_t^F (\theta-1)\frac{\tilde{d}_D^F}{w_t^H} + n_{X,t}^F \frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{d}_X^F}{w_t^F} + n_{X,t}^F \frac{f_X^H}{A_t^H} + n_{E,t}^F \frac{f_E^H}{A_t^H} \\
l_t^T &= n_t^T (\theta-1)\frac{\tilde{d}_D^T}{w_t^H} + n_{X,t}^F \frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{d}_X^T}{w_t^F} + n_{X,t}^T \frac{f_X^H}{A_t^H} + n_{E,t}^T \frac{f_E^H}{A_t^H}
\end{aligned}
$$

$$
\begin{aligned}
1 &= \left[n_t^H \left(\tilde{\rho}_{H,t}^H\right)^{1-\theta} + n_{Xt}^F \left(\tilde{\rho}_{F,t}^H\right)^{1-\theta} + + n_{Xt}^T \left(\tilde{\rho}_{T,t}^H\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
1 &= \left[n_t^F \left(\tilde{\rho}_{F,t}^F\right)^{1-\theta} + n_{Xt}^H \left(\tilde{\rho}_{H,t}^F\right)^{1-\theta} + + n_{Xt}^T \left(\tilde{\rho}_{T,t}^F\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\
1 &= \left[n_t^H \left(\tilde{\rho}_{H,t}^H\right)^{1-\theta} + n_{Xt}^F \left(\tilde{\rho}_{F,t}^H\right)^{1-\theta} + + n_{Xt}^T \left(\tilde{\rho}_{T,t}^H\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}
\end{aligned}
$$

$$
\begin{aligned}
y_t^H &= n_t^H \left(\tilde{\rho}_{H,t}^H\right)^{1-\theta} C_t^H + n_{X,t}^H \left(q_{H,F,t}\left(\tilde{\rho}_{H,t}^F\right)^{1-\theta} C_t^F + q_{H,T,t}\left(\tilde{\rho}_{H,t}^T\right)^{1-\theta} C_t^T\right) \\
y_t^F &= n_t^F \left(\tilde{\rho}_{F,t}^F\right)^{1-\theta} C_t^F + n_{X,t}^F \left(q_{H,F,t}^{-1}\left(\tilde{\rho}_{F,t}^H\right)^{1-\theta} C_t^H + q_{F,T,t}\left(\tilde{\rho}_{F,t}^T\right)^{1-\theta} C_t^T\right) \\
y_t^T &= n_t^T \left(\tilde{\rho}_{T,t}^T\right)^{1-\theta} C_t^H + n_{X,t}^T \left(q_{H,T,t}^{-1}\left(\tilde{\rho}_{T,t}^H\right)^{1-\theta} C_t^H + q_{F,T,t}^{-1}\left(\tilde{\rho}_{T,t}^F\right)^{1-\theta} C_t^F\right)
\end{aligned}
$$

international

$$
\begin{aligned}
\left(\frac{c_t^H}{c_t^F}\right)^{\sigma_c} &= q_{H,F,t} \\
\left(\frac{c_t^H}{c_t^T}\right)^{\sigma_c} &= q_{H,T,t} \\
\left(\frac{c_t^F}{c_t^T}\right)^{\sigma_c} &= q_{F,T,t}
\end{aligned}
$$

$$
\begin{aligned}
(1+r_t^H) &= (1+r_t^F)E_t\left(\frac{q_{H,F,t+1}}{q_{H,F,t}}\right) \\
(1+r_t^H) &= (1+r_t^T)E_t\left(\frac{q_{H,T,t+1}}{q_{H,T,t}}\right)
\end{aligned}
$$

with, $q_{H,F,t} = \frac{e_{H,F,t}.p_{C,t}^F}{p_{C,t}^H}$ and $q_{H,T,t} = \frac{e_{H,T,t}.p_{C,t}^T}{p_{C,t}^H}$.

$$
\begin{aligned}
y_{gdp,t}^H &= y_t^H + n_{X,t}^H \frac{f_X^H}{A_t^H} + n_{E,t}^H \frac{f_E^H}{A_t^H} \\
y_{gdp,t}^F &= y_t^F + n_{X,t}^F \frac{f_X^F}{A_t^F} + n_{E,t}^F \frac{f_E^F}{A_t^F} \\
y_{gdp,t}^T &= y_t^T + n_{X,t}^T \frac{f_X^T}{A_t^T} + n_{E,t}^T \frac{f_E^T}{A_t^T}
\end{aligned}
$$

3 Simulations

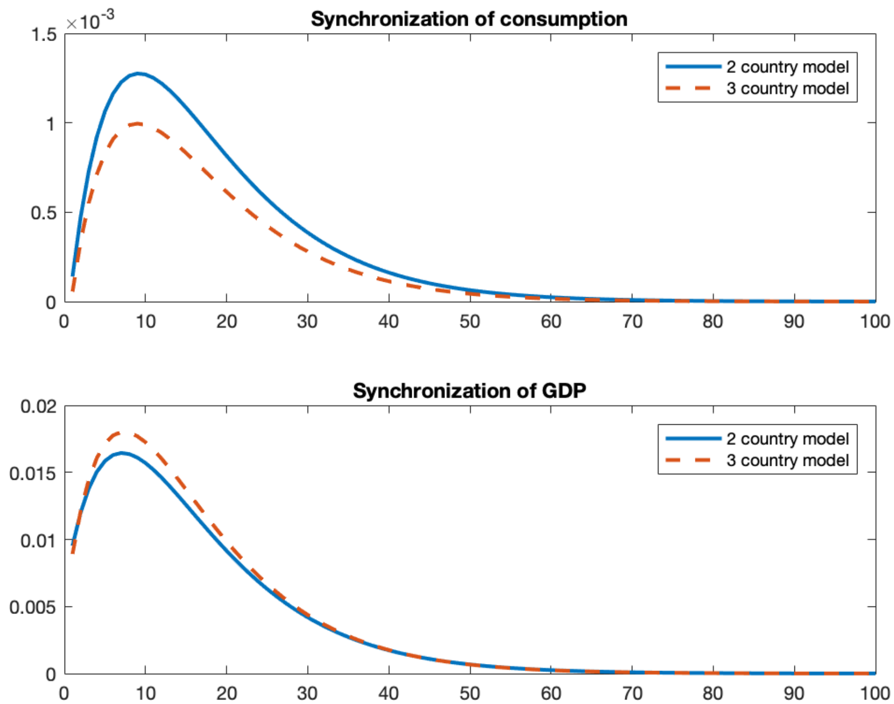


Figure 1: Synchronization between home and foreign

- We depict the "spread" between impulse response functions of home and foreign variables following home productivity shock. The lower the spread, the higher the synchronization between home and foreign variables is.
- The blue and red dashed lines are associated with spreads obtained from the two and three country model, respectively.

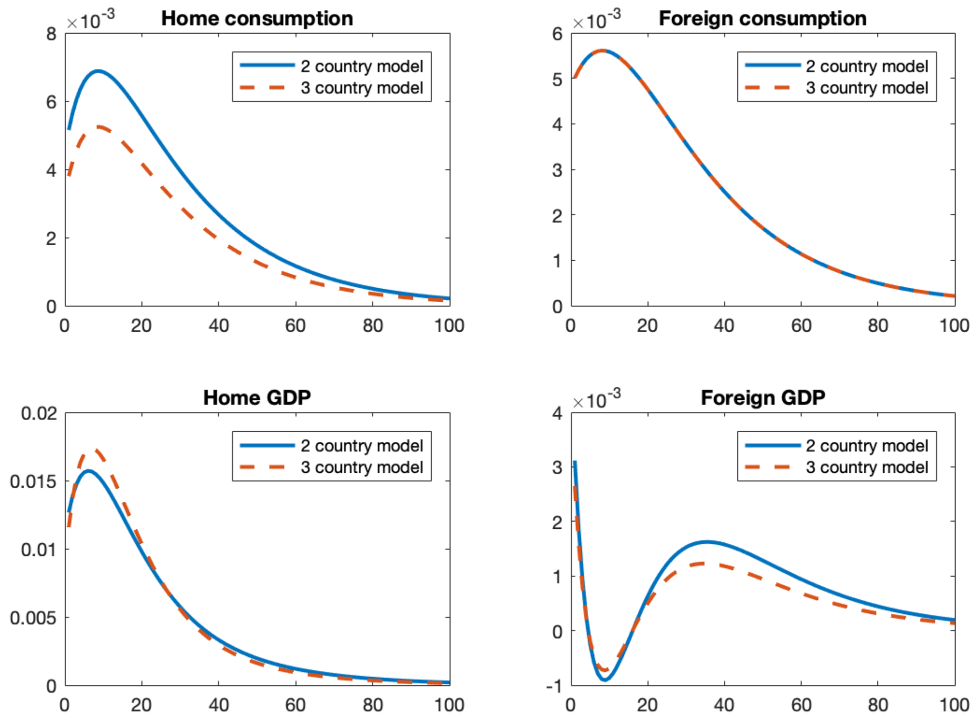


Figure 2: IRFs of aggregate variables following home productivity shock

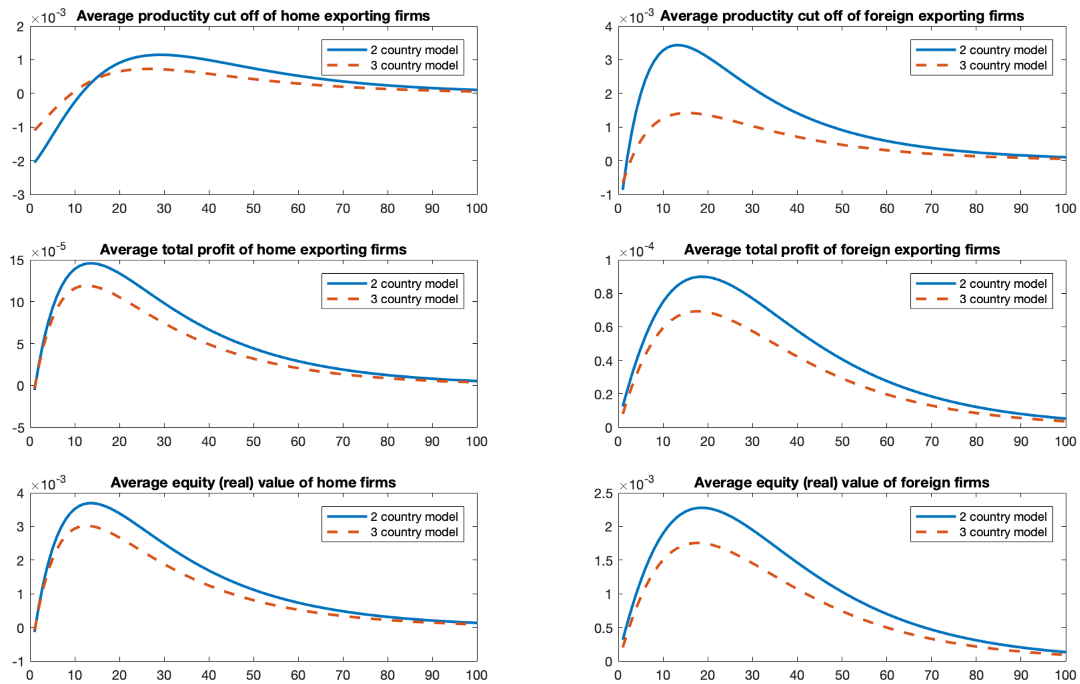


Figure 3: IRFs of productivity variables following home productivity shock

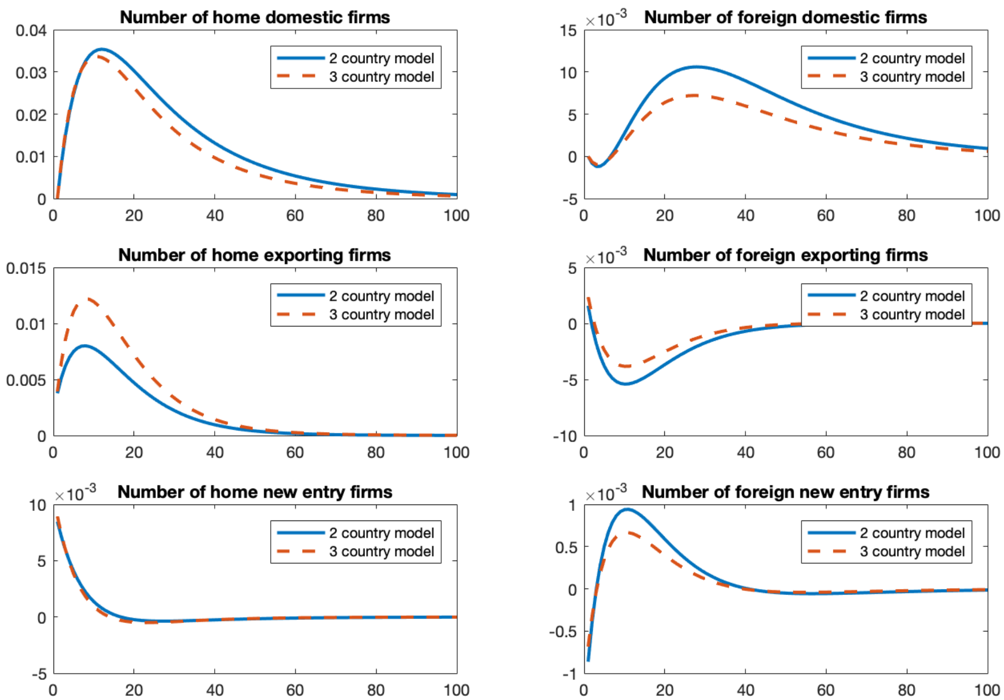


Figure 4: IRFs of number of firms following home productivity shock

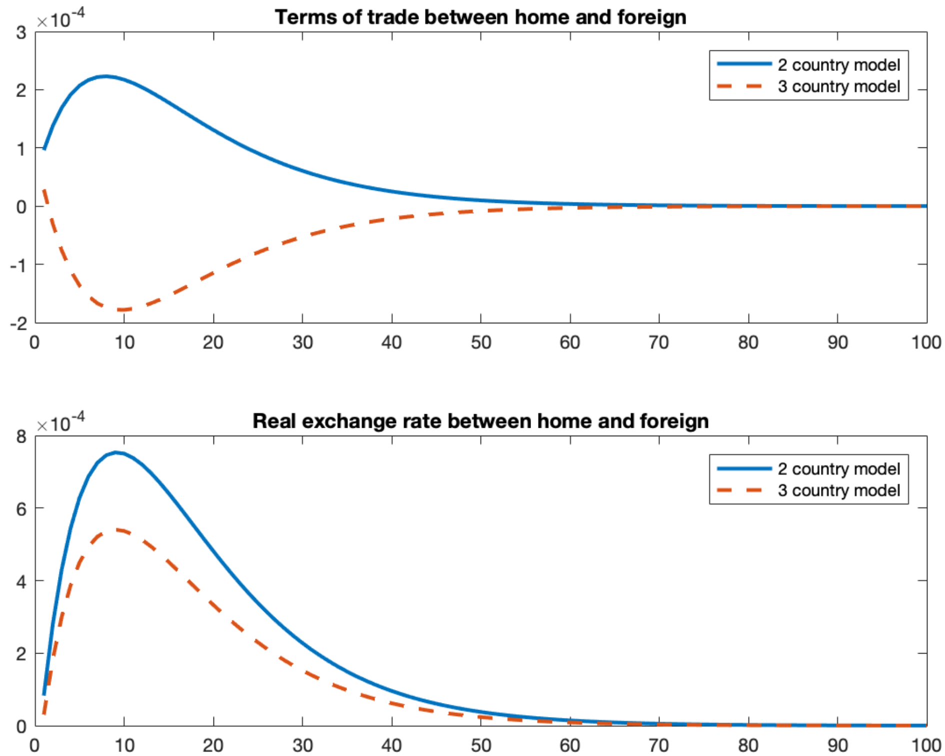


Figure 5: IRFs of terms of trade following home productivity shock

- The terms of trade is defined as the ratio between average relative prices of home and foreign exporting firms. A increase of this term is interpreted as a home country terms of trade deterioration.
- The real exchange rate is defined as units of home consumption per unit of foreign consumption. An increase of this term is interpreted as a real exchange rate depreciation of home country.

4 Empirical Motivation

Intensive and extensive margins of trade are obtained from the export decomposition using the methodology proposed by Hummels and Klenow (2005). The extensive margin of trade is defined as the ratio of country k's exports to j in I_{ij} and country k's exports to j in I, where I_{ij} is the set of observable goods in which country i has positive export to country j and I is the set of all goods:

$$EM_{ij} = \frac{\sum\limits_{m \in I_{ij}} P_{kjm} X_{kjm}}{\sum\limits_{m \in I} P_{kjm} X_{kjm}} \tag{1}$$

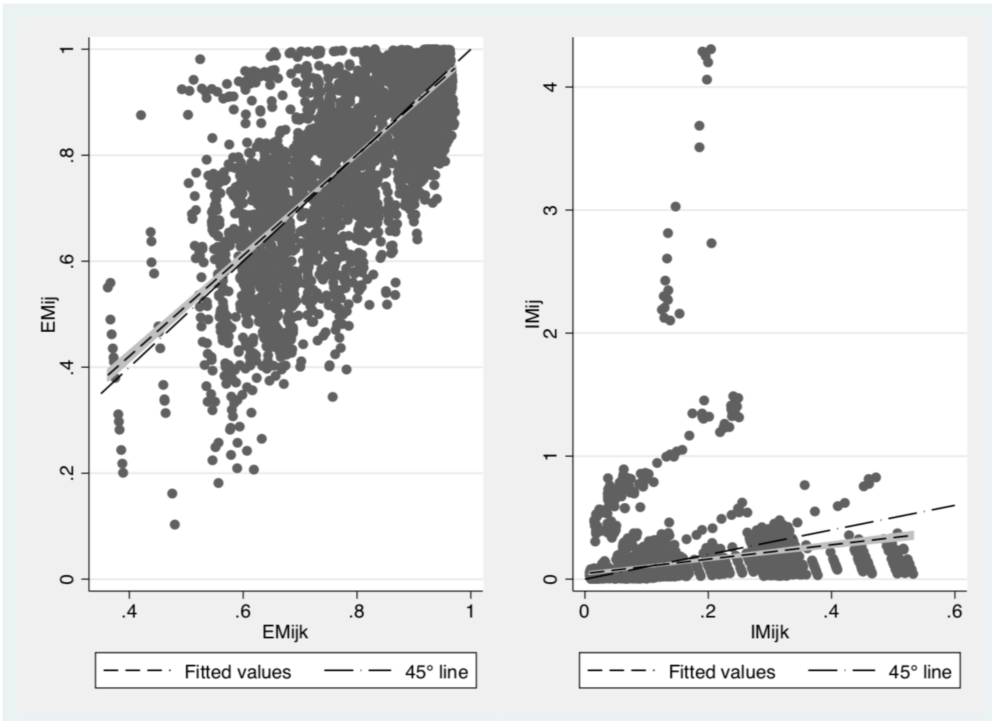
$$IM_{ij} = \frac{\sum\limits_{m \in I_{ij}} P_{ijm} X_{ijm}}{\sum\limits_{m \in I_{ij}} P_{kjm} X_{kjm}} \tag{2}$$

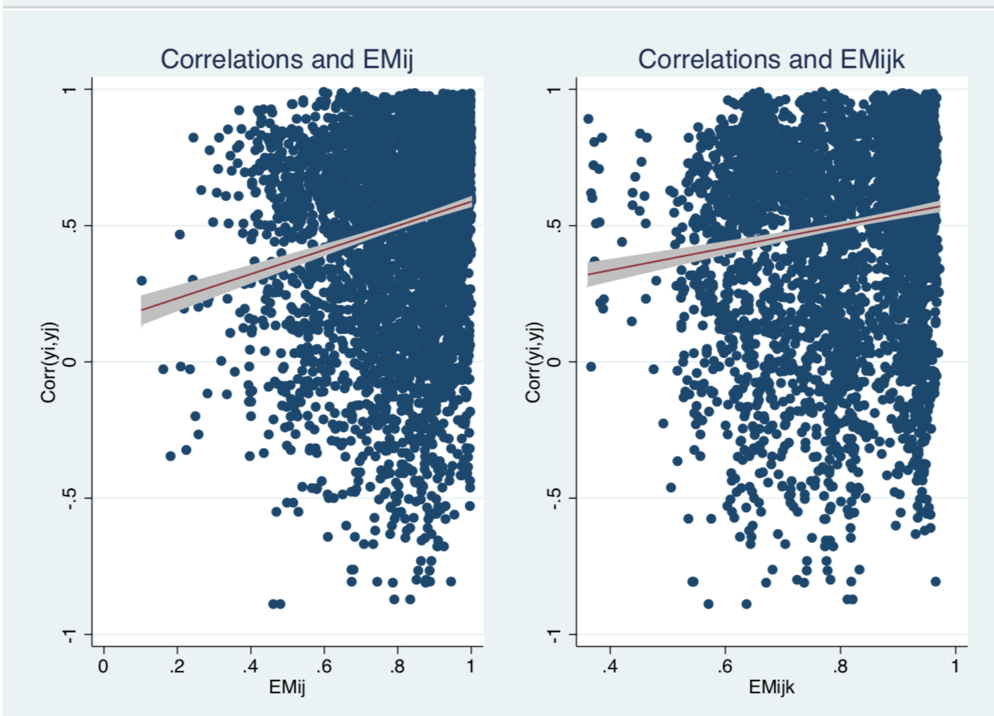
$$EM_{ij,k} = \frac{1}{n-1} \Sigma_{k \neq j} EM_{ik} \tag{3}$$

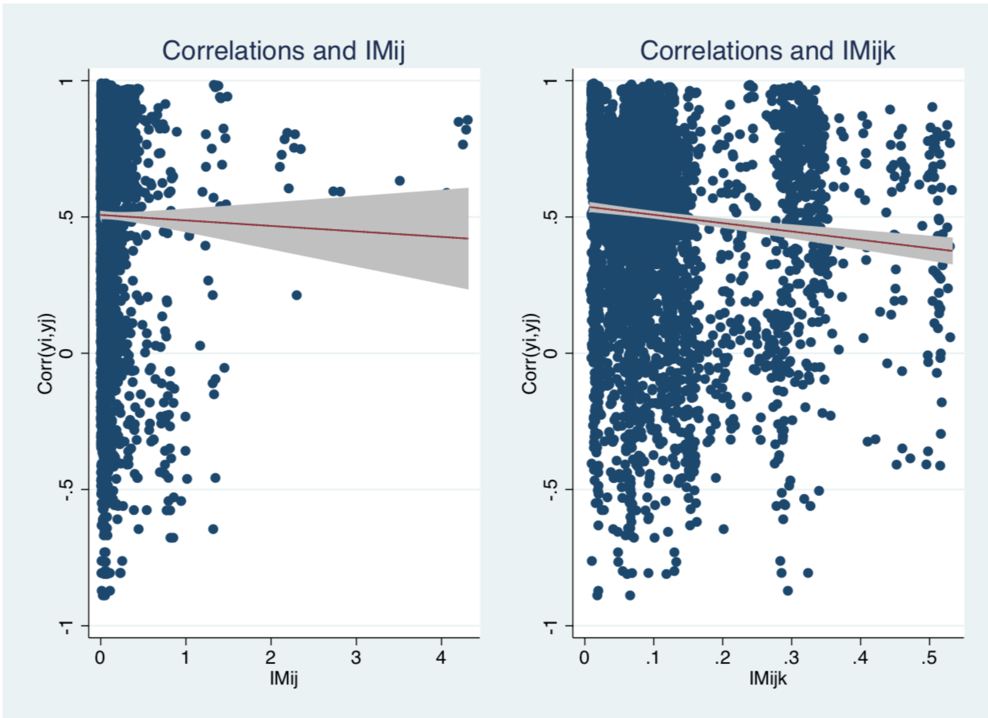
$$IM_{ij,k} = \frac{1}{n-1} \Sigma_{k \neq j} IM_{ik} \tag{4}$$

with p_{kjm} and x_{kjm} respectively the price and quantity of goods m imported by country j from the reference country k (here k is the rest of the World). The wider the variety of goods that country i export to country j, the higher the EM_{ij} is. The intensive margin of trade compares nominal exports for country i and k in a common set of varieties. It is constructed as the ratio of country i's nominal shipments to country j to the country k's nominal shipments to country j in the same set of goods. IM_{ij} is higher when country i exports high quantity of each product category to j.

Data concern 16 countries; Austria, BelgiumLuxembourg, Canada, China, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Portugal, Spain, the United Kingdom, the United States, over the period 1995-2016. For trade data, we use the BACI database (CEPII). This database covers 5017 six-digit U.N HS product codes.


EMij
EMijk
Fitted values
45° line
IMij
IMijk
Fitted values
45° line


Correlations and EMij
Corr(yi,yj)
EMij
Correlations and EMijk
Corr(yi,yj)
EMijk


Correlations and IMij
Correlations and IMijk
Corr(yi,yj)
IMij
IMijk
1
.5
0
-.5
-1
0
1
2
3
4
.1
.2
.3
.4
.5

5 Estimations

Table 1: OLS estimations

Corr(yi, yj)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
OT	0.0115***							
	(0.00409)							
EMij		0.202***	0.276***				0.407***	
		(0.0382)	(0.0245)				(0.0305)	
EMijk		0.723***		0.291***				0.943***
		(0.0582)		(0.0340)				(0.0457)
IMij		0.0221***			0.00504		-0.0402***	
		(0.00635)			(0.00459)		(0.00563)	
IMijk		-0.173***				-0.0456***		-0.158***
		(0.00894)				(0.00577)		(0.00773)
Constant	0.543***	0.318***	0.569***	0.569***	0.521***	0.384***	0.475***	0.294***
	(0.0147)	(0.0172)	(0.00799)	(0.00942)	(0.0153)	(0.0164)	(0.0153)	(0.0162)
Observations	4,410	4,410	4,410	4,410	4,410	4,410	4,410	4,410
R-squared	0.002	0.115	0.028	0.016	0.000	0.014	0.039	0.101

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 2: Panel estimations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Corr(yi, yj)	RE	RE	RE	RE	RE	RE	RE	RE	RE	RE	FE
OT	0.0416***										
	(0.00537)										
EMij		0.202***	0.306***				0.267***		0.279***		0.202***
		(0.0361)	(0.0310)				(0.0343)		(0.0326)		(0.0361)
EMijk		0.485***		0.415***				0.666***	0.207***		0.460***
		(0.0811)		(0.0749)				(0.0762)	(0.0783)		(0.0828)
IMij		0.0164**			0.0389***		0.0169***			0.0339***	0.0148**
		(0.00646)			(0.00589)		(0.00647)			(0.00587)	(0.00655)
IMijk		-0.190***				-0.167***		-0.205***		-0.157***	-0.201***
		(0.0184)				(0.0182)		(0.0182)		(0.0183)	(0.0202)
Constant	0.643***	0.204***	0.576***	0.597***	0.625***	0.0624	0.619***	0.108*	0.615***	0.192***	0.163***
	(0.0493)	(0.0645)	(0.0453)	(0.0477)	(0.0491)	(0.0662)	(0.0431)	(0.0589)	(0.0488)	(0.0707)	(0.0582)
Obs.	4,410	4,410	4,410	4,410	4,410	4,410	4,410	4,410	4,410	4,410	4,410

Standard errors in parentheses

*** p¡0.01, ** p¡0.05, * p¡0.1

6 Conclusion

Following Ghironi and Melitz (2005), we have developed a 3-country model. Bilateral synchonization (between $Home$ and $Foreign$) is higher in the 3-country model than in the 2-country model. Estimations explain this result with the positive impact of the extensive margin with the rest of the world on the bilateral synchronization. Following Liao and Santacreu (2015), the extensive margin facilitates TFP spillovers and so synchronization.

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7 Appendix

current account

$$tb_t^H = n_{X,t}^H \left(q_{H,F,t} \left(\tilde{\rho}_{H,t}^F\right)^{1-\theta} C_t^F + q_{H,T,t} \left(\tilde{\rho}_{H,t}^T\right)^{1-\theta} C_t^T \right) - n_{X,t}^F \left(\tilde{\rho}_{F,t}^H\right)^{1-\theta} C_t^H - n_{X,t}^T \left(\tilde{\rho}_{T,t}^H\right)^{1-\theta} C_t^H$$

the variety effect

$$\begin{aligned} 1 &= \left[n_t^H \left(\tilde{\rho}_{H,t}^H\right)^{1-\theta} + n_{Xt}^F \left(\tilde{\rho}_{F,t}^H\right)^{1-\theta} + + n_{Xt}^T \left(\tilde{\rho}_{T,t}^H\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\ 1 &= \left[n_t^F \left(\tilde{\rho}_{F,t}^F\right)^{1-\theta} + n_{Xt}^H \left(\tilde{\rho}_{H,t}^F\right)^{1-\theta} + + n_{Xt}^T \left(\tilde{\rho}_{T,t}^F\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\ 1 &= \left[n_t^H \left(\tilde{\rho}_{H,t}^H\right)^{1-\theta} + n_{Xt}^F \left(\tilde{\rho}_{F,t}^H\right)^{1-\theta} + + n_{Xt}^T \left(\tilde{\rho}_{T,t}^H\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \end{aligned}$$

$\tilde{\pi}_{X,t}^H$

thus we can compute sectoral averages

given setoral averages, we get

$$\begin{aligned} \frac{q_{H,F,t}}{\theta} \left(\rho_{H,t}^F(\omega)\right)^{1-\theta} C_t^F + \frac{q_{H,T,t}}{\theta} \left(\rho_{H,t}^T(\omega)\right)^{1-\theta} C_t^T &= \frac{W_t}{e^{\varepsilon_t^a}} f_X^H \\ \frac{q_{H,F,t}}{\theta} \left(\frac{1+\tau_{H,F}}{q_{H,F,t.}} \frac{\theta}{\theta-1} \frac{W_t^H}{A_t^H z(\omega)} \right)^{1-\theta} C_t^F + \frac{q_{H,T,t}}{\theta} \left(\frac{1+\tau_{H,T}}{q_{H,T,t.}} \frac{\theta}{\theta-1} \frac{W_t^H}{A_t^H z(\omega)} \right)^{1-\theta} C_t^T &= \frac{W_t}{e^{\varepsilon_t^a}} f_X^H \\ \frac{1}{\theta} \left(\frac{\theta}{\theta-1} \frac{W_t^H}{A_t^H z(\omega)} \right)^{1-\theta} \left[q_{H,F,t} \left(\frac{1+\tau_{H,F}}{q_{H,F,t.}} \right)^{1-\theta} C_t^F + q_{H,T,t} \left(\frac{1+\tau_{H,T}}{q_{H,T,t.}} \right)^{1-\theta} C_t^T \right] &= \frac{W_t}{e^{\varepsilon_t^a}} f_X^H \\ \left(\frac{1}{\theta}\right)^{-\frac{1}{1-\theta}} \left(\frac{\theta}{\theta-1} \frac{W_t^H}{A_t^H} \right) \left[q_{H,F,t} \left(\frac{1+\tau_{H,F}}{q_{H,F,t.}} \right)^{1-\theta} C_t^F + q_{H,T,t} \left(\frac{1+\tau_{H,T}}{q_{H,T,t.}} \right)^{1-\theta} C_t^T \right]^{-\frac{1}{1-\theta}} \left(\frac{W_t}{e^{\varepsilon_t^a}} f_X^H \right)^{\frac{1}{1-\theta}} &= z_X^H \end{aligned}$$

we borrow the standard asumption from trade theory, y assuming that the distriution of firms follows a pareto distribution.

the distribution offirms is as follows. thus sectorial averages are then macroeconomic variables are defined after aggregation in table. fincial aspects are left open

the number en firm entry is defined by the condition

$$\widetilde{v}_t^H = f_E^H \frac{w_t^H}{A_t^H}$$

the sectorial organization depends on the cut off point

$$\pi_X^H(\omega) = 0$$

average prices

$$
\begin{aligned}
p^H_{H,t}(\omega) &= \frac{\theta}{\theta-1}\frac{W^H_t}{A^H_t\Delta^H z^H_{\min}} \\
p^F_{H,t}(\omega) &= \frac{1+\tau_{H,F}}{e_{H,F,t.}}\frac{\theta}{\theta-1}\frac{W^H_t}{A^H_t\Delta^H z^H_X} \\
p^T_{H,t}(\omega) &= \frac{1+\tau_{H,T}}{e_{H,T,t.}}\frac{\theta}{\theta-1}\frac{W^H_t}{A^H_t\Delta^H z^H_X}
\end{aligned}
$$

thus, real prices in terms of

$$
\begin{aligned}
\frac{p^H_{H,t}(\omega)}{p^H_{C,t}} &= \frac{\theta}{\theta-1}\frac{W^H_t/p^H_{C,t}}{A^H_t\Delta^H z^H_{\min}} \\
\frac{p^F_{H,t}(\omega)}{p^F_{C,t}} &= \frac{1+\tau_{H,F}}{\frac{e_{H,F,t.}p^F_{C,t}}{p^H_{C,t}}}\frac{\theta}{\theta-1}\frac{W^H_t/p^H_{C,t}}{A^H_t\Delta^H z^H_X} \\
\frac{p^T_{H,t}(\omega)}{p^T_{C,t}} &= \frac{1+\tau_{H,T}}{\frac{e_{H,T,t.}p^T_{C,t}}{p^H_{C,t}}}\frac{\theta}{\theta-1}\frac{W^H_t/p^H_{C,t}}{A^H_t\Delta^H z^H_X}
\end{aligned}
$$

Thus, real prices are as follows,

$$
\begin{aligned}
\tilde{\rho}^H_{H,t} &= \frac{\theta}{\theta-1}\frac{w^H_t}{A^H_t\Delta^H z^H_{\min}} \\
\tilde{\rho}^F_{H,t} &= q^{-1}_{H,F,t.}\left(1+\tau_{H,F}\right)\frac{\theta}{\theta-1}\frac{w^H_t}{A^H_t\nabla^H z^H_X} \\
\tilde{\rho}^T_{H,t} &= q^{-1}_{H,T,t.}\left(1+\tau_{H,T}\right)\frac{\theta}{\theta-1}\frac{w^H_t}{A^H_t\Delta^H z^H_X}
\end{aligned}
$$

average productivity

$$
\begin{aligned}
\tilde{z}^H_t &= \frac{n^H_{D,t}}{n^H_t}\nabla^H z^H_{\min} + \frac{n^H_{X,t}}{n^H_t}\nabla^H z^H_X \\
&= \left[1-\left(\frac{z_{\min}}{z^H_X}\right)^{\kappa}\right]\nabla^H z^H_{\min} + \left(\frac{z_{\min}}{z^H_X}\right)^{\kappa}\nabla^H z^H_X \\
&= \nabla^H\left[z^H_{\min} - \left(z^H_{\min}\right)^{1+\kappa}\left(z^H_X\right)^{-\kappa} + \left(z_{\min}\right)^{\kappa}\left(z^H_X\right)^{1-\kappa}\right]
\end{aligned}
$$

thus, average profits, are defined as,

$$
\begin{aligned}
\tilde{\pi}_{D,t}^{H} &= \int_{z_{\min}^{H}}^{\infty} \pi_D^H(\omega)dG(\omega) = \int_{z_{\min}^{H}}^{\infty} \frac{1}{\theta}\left(\rho_{H,t}^{H}(\omega)\right)^{1-\theta} C_t^H dG(\omega) \\
&= \int_{z_{\min}^{H}}^{\infty} \frac{1}{\theta}\left(\rho_{H,t}^{H}(\omega)\right)^{1-\theta} C_t^H \kappa z_{\min}^{\kappa} z^{-\kappa-1}dz = \int_{z_{\min}^{H}}^{\infty} \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H z}\right)^{1-\theta} C_t^H \kappa z_{\min}^{\kappa} z^{-\kappa-1}dz \\
&= \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} C_t^H \kappa z_{\min}^{\kappa} \int_{z_{\min}^{H}}^{\infty} z^{\theta-1-\kappa-1}dz \\
&= \frac{C_t^H}{\theta} z_{\min}^{\kappa}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} \kappa \left[\frac{z^{\theta-1-\kappa}}{(\theta-1)-\kappa}\right]_{z_{\min}^{H}}^{\infty} \\
&= \frac{C_t^H}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta}\left(\tilde{z}_{D,t}^{H}\right)^{\theta-1}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\pi}_{X,t}^{H} &= \int_{z_X^H}^{\infty} \pi_X^H(\omega)dG(\omega) = \int_{z_X^H}^{\infty}\left[\frac{q_{H,F,t}}{\theta}\left(\rho_{H,t}^{F}(\omega)\right)^{1-\theta} C_t^F + \frac{q_{H,T,t}}{\theta}\left(\rho_{H,t}^{T}(\omega)\right)^{1-\theta} C_t^T - \frac{w_t^H}{A_t^H} f_X^H\right] dG(\omega) \\
&= \int_{z_X^H}^{\infty} \frac{q_{H,F,t}}{\theta}\left(\rho_{H,t}^{F}(\omega)\right)^{1-\theta} C_t^F \; dG(\omega) \\
&\quad + \int_{z_X^H}^{\infty} \frac{q_{H,T,t}}{\theta}\left(\rho_{H,t}^{T}(\omega)\right)^{1-\theta} C_t^T \; dG(\omega) \\
&\quad - \int_{z_X^H}^{\infty} \frac{w_t^H}{A_t^H} f_X^H \; dG(\omega)
\end{aligned}
$$

$$
\begin{aligned}
\int_{z_X^H}^{\infty} \frac{q_{H,F,t}}{\theta}\left(\rho_{H,t}^{F}(\omega)\right)^{-\theta} C_t^F \; dG(\omega) &= \frac{q_{H,F,t}}{\theta}\int_{z_X^H}^{\infty}\left(q_{H,F,t.}^{-1}\left(1+\tau_{H,F}\right)\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H z}\right)^{1-\theta} C_t^F \;\kappa z_{\min}^{\kappa} z^{-\kappa-1}dz \\
&= \frac{\left(1+\tau_{H,F}\right)^{1-\theta} q_{H,F,t}^{\theta}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} C_t^F \;\kappa z_X^{\kappa}\int_{z_X^H}^{\infty} z^{\theta-1-\kappa-1}dz \\
&= \frac{\left(1+\tau_{H,F}\right)^{1-\theta} q_{H,F,t}^{\theta}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} C_t^F \;\kappa z_X^{\kappa}\left[\frac{z^{\theta-1-\kappa}}{\theta-1-\kappa}\right]_{z_X^H}^{\infty} \\
&= \frac{\left(1+\tau_{H,F}\right)^{1-\theta} q_{H,F,t}^{\theta}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} C_t^F \;\frac{\kappa}{\kappa-(\theta-1)} z_X^{\theta-1} \\
&= \frac{\left(1+\tau_{H,F}\right)^{1-\theta} q_{H,F,t}^{\theta}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} C_t^F \;\tilde{z}_X^{\theta-1} \\
\int_{z_X^H}^{\infty} \frac{q_{H,T,t}}{\theta}\left(\rho_{H,t}^{T}(\omega)\right)^{-\theta} C_t^T \; dG(\omega) &= \frac{\left(1+\tau_{H,T}\right)^{1-\theta} q_{H,T,t}^{\theta}}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t^H}{A_t^H}\right)^{1-\theta} C_t^T \;\tilde{z}_X^{\theta-1} \\
\int_{z_X^H}^{\infty} \frac{w_t^H}{A_t^H} f_X^H \; dG(\omega) &= \frac{w_t^H}{A_t^H} f_X^H \kappa z_X^{\kappa}\int_{z_X^H}^{\infty} z^{-\kappa-1}dz = \frac{w_t^H}{A_t^H} f_X^H \kappa z_X^{\kappa}\left[\frac{z^{-\kappa}}{-\kappa}\right]_{z_X^H}^{\infty} = f_X^H\frac{w_t^H}{A_t^H}
\end{aligned}
$$

thus, for the exporting sector, we get,

$$
\begin{aligned}
\tilde{\pi}_{X,t}^{H} &= = \frac{(1+\tau_{H,F})^{1-\theta}\, q_{H,F,t}^{\theta}}{\theta} \left(\frac{\theta}{\theta-1} \frac{w_t^H}{A_t^H} \right)^{1-\theta} C_t^F \ \tilde{z}_X^{\theta-1} \\
&\quad + \frac{(1+\tau_{H,T})^{1-\theta}\, q_{H,T,t}^{\theta}}{\theta} \left(\frac{\theta}{\theta-1} \frac{w_t^H}{A_t^H} \right)^{1-\theta} C_t^T \ \tilde{z}_X^{\theta-1} - f_X^H \frac{w_t^H}{A_t^H}
\end{aligned}
$$

collecting terms, we get, sectoral profits as follows,total profits is thus given by,

$$
n_t^H \tilde{\pi}_t^H = n_{D,t}^H \tilde{d}_{D,t}^H + n_{X,t}^H \tilde{d}_{X,t}^H
$$

labour demand

$$
\begin{aligned}
d_D^H(\omega) &= \frac{1}{\theta} \left(\rho_{H,t}^H(\omega) \right) y_{D,t}^H(\omega) \\
&= \frac{1}{\theta} \left(\frac{\theta}{\theta-1} \frac{w_t^H}{A_t^H z(\omega)} \right) y_{D,t}^H(\omega) \\
&= \frac{1}{\theta} \left(\frac{\theta}{\theta-1} \frac{w_t^H}{A_t^H z(\omega)} \right) A_t^H z(\omega) l_{D,t}^H(\omega) \\
&= \frac{1}{\theta-1} w_t^H l_{D,t}^H(\omega)
\end{aligned}
$$

thus,

$$
\begin{aligned}
l_{D,t}^H(\omega) &= (\theta-1) \frac{d_D^H(\omega)}{w_t^H} \\
\tilde{l}_{D,t}^H &= (\theta-1) \frac{\tilde{d}_D^H}{w_t^H}
\end{aligned}
$$

secteur exportateur

$$
\pi_X^H(\omega) = \frac{q_{H,F,t}}{\theta} \left(\rho_{H,t}^F(\omega) \right) y_{F,t}^H + \frac{q_{H,T,t}}{\theta} \left(\rho_{H,t}^T(\omega) \right) y_{T,t}^H - \frac{w_t^H}{A_t^H} f_X^H
$$

since,

$$
\begin{aligned}
\rho_{H,t}^F(\omega) &= q_{H,F,t.}^{-1} \left(1+\tau_{H,F}\right) \frac{\theta}{\theta-1} \frac{w_t^H}{A_t^H z(\omega)} \\
\rho_{H,t}^T(\omega) &= q_{H,T,t.}^{-1} \left(1+\tau_{H,T}\right) \frac{\theta}{\theta-1} \frac{w_t^H}{A_t^H z(\omega)}
\end{aligned}
$$

we get,

$$\begin{aligned} l^H_{X,t}(\omega) &= \frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\pi^H_X(\omega)}{w^H_t} + \frac{f^H_X}{A^H_t} = l^H_{F,t}(\omega) + l^H_{T,t}(\omega) \\ \tilde{l}^H_{X,t} &= \frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{\pi}^H_X}{w^H_t} + \frac{f^H_X}{A^H_t} \end{aligned}$$

$$\begin{aligned} l^H_t &= n^H_t \tilde{l}^H_{D,t} + n^H_{X,t}\tilde{l}^H_{D,t} + n^H_{E,t}\frac{f^H_E}{A^H_t} \\ l^H_t &= n^H_t\,(\theta-1)\,\frac{\tilde{\pi}^H_D}{w^H_t} + n^H_{X,t}\frac{(\theta-1)}{(1+\tau_{H,F})}\frac{\tilde{\pi}^H_X}{w^H_t} + n^H_{X,t}\frac{f^H_X}{A^H_t} + n^H_{E,t}\frac{f^H_E}{A^H_t} \end{aligned}$$