A New Asymmetric Copula with Sign-Reversible Correlation

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Abstract

This paper proposes a new asymmetric copula using the bivariate split-normal distribution. The novelty of this copula is that it can change correlation signs of the upper and lower tails of the distribution independently. The new copula is applied to the stock and government bond price returns of the five peripheral EU countries and Germany during the EU sovereign crisis. Their upper and lower tail correlation coefficients are estimated by rolling maximum likelihood method. It finds that the peripheral countries had a strong asymmetry, namely positive lower-tail correlation and negative or near zero upper tail correlation of the stock-bond distribution, in the early stage of the crisis. It also finds that the signs of their correlations changed from negative to positive in the crisis and stay still positive after the crisis. In contrast, Germany had no sign-reversal or asymmetry in correlation.

Keywords: Copula, Asymmetry, Rolling Estimation, Split-Normal Distribution.

1 Introduction

Asymmetry of financial asset returns has been extensively analyzed by copula in econometrics. Recently, some authors reported that financial asset returns of the peripheral EU countries showed correlation sign changes and strong asymmetry at the same time. This situation cannot

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be analyzed by conventional asymmetric parametric copulas, such as the Joe-Clayton copula, since Kendall's correlation coefficient of this copula is always positive, as shown by Li and Kang (2018).

Then we propose a novel copula that can have change correlation sign in the upper and lower tail area independently. Up to our knowledge, no conventional parametric copula does not have this property.[1] It is constructed from a split normal distribution, namely a bivariate distribution consisting of two halved bivariate normal density functions with different correlation coefficients connected on the negative 45 degree line.

We applies this copula to the stock and bond price returns of the five peripheral euro countries (Greece, Ireland, Italy, Portugal, Spain). The correlation coefficients of the underlying distribution are estimated by the rolling maximum likelihood method.

The rest of the paper is organized as follows. In the next section the literature review is presented. The dependence structure of stock and bond price returns is defined in Section 3. The algorithm of the state space estimation is presented in Section 4. The results of the empirical analysis are presented in Section 5.

Hereafter, the two correlation coefficients of these underlying distributions are referred to as upper and lower tail correlation coefficients, when there is no fear of confusion.

1.1 Split-Normal Copula

Copula, first proposed by Sklar(1959), is joint distribution function with uniform marginal distribution.Copula can be constructed by

$$\text{Copula} : C(w, v) = F(F_X^{-1}(w), F_Y^{-1}(v)), \quad 0 < w < 1, \quad 0 < v < 1, \tag{1}$$

and its density function by

$$\text{Copula Density Function: } c(w, v) = \frac{f_{X,Y}(F_X^{-1}(w), F_Y^{-1}(v))}{f_X(F_X^{-1}(w)) f_Y(F_Y^{-1}(v))}, \tag{2}$$

where $F(x, y)$ is the distribution function of random variables X and Y, $f_{X,Y}(x, y)$ is the density function, and $F_X(x)$ is the marginal distribution function and $F_X^{-1}(w)$ is its inverse. An important property of copula is that same copula C is derived from any strictly monotonic transformations of X and Y. We use this invariance property in deriving the new copula. For the proof, see Nelson (2006, p.25).

We here construct a novel copula, from bivariate split normal distribution. This distribution is defined by the two normal density functions with different correlation coefficients continuously connected on the $x = -y$ line, which is a special case of the multivariate split normal distribution

[1]Chang() showed that copula with flexible dependence can be obtained by mixture of parametric copula. This is another promising line of research.

investigated by Villani and Larson (2006). The density function is formally defined as

$$\text{Split Normal Density: } f_{X,Y}(x,y) = \begin{cases} a_U \times \phi(x,y,\rho_U,\omega_U^2) & \text{if} \quad y > -x \\ a_L \times \phi(x,y,\rho_L,\omega_L^2) & \text{if} \quad y \leq -x \end{cases}, \tag{3}$$

where ϕ is the bivariate normal density of random variables X and Y with correlation ρ and both with mean zero and variance ω^2, defined by

$$\phi(x,y,\rho,\omega^2) = \frac{1}{2\pi\omega^2\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)\omega^2}\right). \tag{4}$$

We have assumed that X and Y have the same variance ω_U in the upper tail in (3) and ω_U in the lower tail, and hence the joint distribution is symmetric with respect to the 45 degree line. We do not consider the case where the joint distribution of X and Y id not line-symmetric.

Then, without loss of generality, we can assume that $\omega_U = 1$ for the sake of normalization, using the copula invariance property; we have the same copula function even if the random variables X and Y in (3) are divided by ω_U and hence the variance of the upper-left area is normalzed to unity. The weight constants a_L and a_U should satisfy the condition that $f_{X,Y}(x,y)$ in (3) is a bivariate density function, namely:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)dxdy = 1, \; f_{X,Y}(x,y) \geq 0. \tag{5}$$

This condition is reduced to the equation

$$0.5a_U + 0.5a_L = 1, \tag{6}$$

since

$$\int_{-\infty}^{\infty}\int_{-y}^{\infty} \phi(x,y,\rho_U,\omega_U)dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{-y} \phi(x,y,\rho_L,\omega_L)dxdy = 0.5. \tag{7}$$

The heights of the two parts of the density functions are matched on the boundary $x+y=0$ by adjusting a_U and a_L so that the two parts are connected continuously. The condition that the two density functions are connected continuously on the boundary $y=-x$ is

$$\frac{a_U}{2\pi\omega_U^2\sqrt{1-\rho_U^2}} \exp\left[-\frac{x^2}{(1-\rho_U)\omega_U^2}\right] = \frac{a_L}{2\pi\omega_L^2\sqrt{1-\rho_L^2}} \exp\left[-\frac{x^2}{(1-\rho_L)\omega_L^2}\right], -\infty < x < \infty,$$

which is reduced to

$$a_L/a_U = \frac{(1-\rho_U)\sqrt{1-\rho_L^2}}{(1-\rho_L)\sqrt{1-\rho_U^2}}, \quad (1-\rho_U)\omega_U^2 = (1-\rho_L)\omega_L^2. \tag{8}$$

Then we can drop one parameter using conditions (6), and two parameters using (8). Then the values of (ω_L, a_L, a_U) are determined by the values of ρ_U and ρ_L, noting that $\omega_L = 1$.

The figures of the lower row of Figure 2 show the contours of the split normal distribution with $\rho_U = -0.534$ and $\rho_L = 0.862$ and the derived distribution (2) with standard normal marginals, respectively. The correlation coefficients are those of Greece in May 2010 in the crisis period. Note that asymmetry was enlarged in the crisis as the lower-tail correlation coefficient became positive and that of the upper-area remained negative.

Figure 2 shows contours of the split normal distribution (3) with $\rho_U = -0.612$ and $\rho_L = -0.298$ and the derived distribution (2), respectively. The correlation coefficients were estimated using the data from Greece in September 2007; the x-axis and y-axis variables were transformed so that their marginal distributions are standard normal. Note that the correlation coefficients were both negative and the bond was a safer asset in the pre-crisis period.

1.2 Numerical Evaluation

We estimate ρ_U and ρ_L in the copula density for the split normal distribution in (3) in using rolling maximum likelihood method.

We evaluate $F_X(x)$, $f_X(x) = F'_X(x)$, and $F_X^{-1}(w)$ numerically, since no analytical expression of (1). We can obtain $F_Y^{-1}(v) = F_X^{-1}(w)$ and $f_Y(v) = f_X(w)$ from the symmetry of $f_{X,Y}(x, y)$;

First, decompose $F_X(x)$ as

$$F_X(x) = P(X < x, 0 < X + Y) + P(X < x, 0 > X + Y), \tag{9}$$

where X and Y in the first term on the right hand side follow the upper-tail area of the density function (3) and X and Y in the first term on the right hand side follow the lower tail area of the density function (3).

These probabilities can be evaluated as follows: First,since $P(X+Y > 0) = 0.5 \times a_U$ from(3) we have

$$P(X < x, X + Y > 0) = P(X < x, X + Y > 0|X + Y > 0) \times 0.5 \times a_U.$$

Noting that the conditional distribution of X and Y given $X + Y > 0$ is that of normal distribution with zero means, variances ω_U^2, and correlation coefficients ρ_U. We have that

$$\begin{aligned} P(X < x, X + Y > 0|X + Y > 0) &= P(Z_1 < x, Z_1 + Z_2 > 0|Z_1 + Z_2 > 0) \\ &= \frac{P(Z_1 < x, Z_1 + Z_2 > 0)}{P(Z_1 + Z_2 > 0)} \end{aligned}$$

where Z_1, Z_2 are normally distributed random variables, globally, with zeros means, variances ω_U^2, and correlation coefficients ρ_U. Then, noting that $P(Z_1 + Z_2 > 0) = 0.5$ from the symmetry of the distribution of Z_1 and Z_2, we have

$$P(X < x, X + Y > 0) = P(Z_1 < x, Z_1 + Z_2 > 0) \times a_U.$$

Analogously, we have

$$P(X < x, X + Y < 0) = a_L \times P(Z_1 < x, 0 > Z_1 + Z_2),$$

where Z_1 and Z_2 are normally distributed with zero means, variances ω_L^2, and covariance $\rho_L \omega_L^2$,

Then we can evaluate $F_X(x)$ for given x applying the multivariate normal probability algorithm pmvnorm of Genz et al. (2018) in the mvtnorm package of **R** to the first and second terms of (9). We then derive $F_X(x), f_X(x)$, and $F_x^{-1}(\cdot)$ as smooth functions by spline interpolation. The procedure is as follows: first, evaluate $(x_i, F_X(x_i))$ at $i = 1, \ldots, n$ for sufficient large n for accuracy. In this paper we use $n = 100$. Second, obtain smooth approximation of $F_X(x)$ by applying spline interpolation with increasing constraint to $(x_i, F_X(x_i))$. The density function $f_X(x)$ is obtained as an analytically derivative of $F_X(x)$. The quantile function $F_X^{-1}(\cdot)$ is obtained analogously from $(F_X(x_i), x_i)$.

2 Empirical Analysis

2.1 Stock-Bond Correlation

In econometrics correlations between financial returns have been researched extensively by copula. The two-parameter Joe-Clayton copula was estimated by Patton (2006) in the time series settings. Christoffersen et al. (2012) estimated dynamic asymmetric correlations in large cross sections generalizing the dynamic conditional correlation (DCC) model of Engle (2002). Okimoto (2008) used Markov switching model and copula to analyze international equity markets. Yoshiba (2013) used rolling estimation of the copula parameters.

This sign-reversal of Stock-Bond correlation coefficients in the EU crisis was reported recently by Dufour et al. (2017) and Ohmi and Okimoto (2016). They found that the stock-bond correlations changed signs from negative to positive in the crisis and remained positive after it. Yoshiba (2013) reported that the stock-bond correlations had sign change and correlation asymmetry in the financial crisis using nonparametric copula.

However, the stock-bond correlation during the EU sovereign debt crisis has not been fully analyzed by copula, because the correlation-sign changes cannot be analyzed by conventional parametric copulas. The sign of correlation of the parametric copulas, such as the Joe-Clayton copula, is always positive, as shown by Li and Kang (2018). They cannot express the situation that the upper-tail and lower-tail have different sign. [2]

[2]Chang (2019) showed that mixture copulas can express asymmetric correlation structure with sufficient flexibility. This is another promising line of research.

Then, in order to analyze the asymmetry of the stock bond-correlation in the EU crisis, this paper proposes a novel two-parameter asymmetric copula, whose upper and lower tail correlation coefficients can change value and sign independently.

2.2 Model

We assume that the marginal distribution of the stock and bond returns follow the AR(p)-GARCH(1,1) model with Student's t-distribution, namely

$$R_j(t) = \mu_j + \sum_{i=1}^{p_j} \gamma_i R_j(t-i) + e_j(t),\ e_j(t) = \sigma_j \epsilon_j(t),\ \epsilon_j(t) \sim t(\lambda_j) \text{ independently}, \tag{10}$$

$$\sigma_j^2(t) = \alpha_{j0} + \alpha_{j1}(t-1) + \beta_{j1}\sigma_j^2(t-1),\ \ j = \text{Stock}, \quad \text{Bond}$$

It is assumed that $w = F_t(\epsilon_{Stock}, \lambda_{Stock})$ and $v = F_t(\epsilon_{Bond}, \lambda_{Bond})$, where $F_t(\cdot, \lambda)$ is t-distribution function with degrees of freedom λ, has split-normal copula density, defined by

$$c(w, v) = \frac{f_{X,Y}(F_X^{-1}(w), F_Y^{-1}(v))}{f_X(F_X^{-1}(w)) f_Y(F_Y^{-1}(v))},$$

where $f_{X,Y}(x, y), F_X(x)$, and $F_X^{-1}(w)$ are density, margincal distribution, and inverse distribution functions, respectively, of the split-normal distribution defined by (3).

2.3 Data

We estimate the model using weekly data of the stock price indices and 10-year government bond yields which are denoted by $S(t)$ and $Y(t)$, downloaded from the website of **investing.com**. Table 1 summarizes the data we use in the paper.

The sample period from August 2006 to August 2018 contains the EU crisis from the beginning to the end. The EU crisis emerged when the underreported Greek government debt became evident in November 2009 and the crisis started to calm down when the Outright Monetary Transaction was announced by the European Central Bank in 2012. Ireland and Spain exited from the bailout program around the end of 2013, although the financial markets in some countries were still turbulent.

The estimation of our model is executed as follows:

1. Calculate the returns on stock and bond prices by

$$\text{Stock Price Returns: } R_{Stock}(t) = \log S(t) - \log S(t-1), \tag{11}$$

$$\text{Bond Price Returns: } R_{Bond}(t) = \log B(t) - \log B(t-1), \tag{12}$$

Table 1: Summary Statistics of Financial Returns

Country	Assets	Mean	SD	Min	Max	Skew	Kurt	Kendall's τ
Greece	FTSE ATHEX 20	-0.256	4.690	-23.160	20.747	-0.368	2.837	0.206
	10 Year Gov. Bond	0.017	7.550	-61.953	161.233	9.913	231.988	
Ireland	FTSE Ireland	-0.026	3.878	-37.103	22.979	-1.271	14.061	0.011
	10 Year Gov. Bond	0.048	2.029	-15.061	21.381	0.592	24.936	
Italy	FTSE MIB	0.087	4.546	-24.360	91.854	9.923	208.396	0.143
	10 Year Gov. Bond	0.007	1.303	-5.824	10.447	0.758	8.246	
Portugal	PSI-ALL Share GR	0.016	2.487	-20.528	7.677	-1.192	6.772	0.135
	10 Year Gov. Bond	0.038	2.487	-18.871	16.597	-0.107	15.053	
Spain	IBEX 35	-0.017	3.134	-23.827	11.823	-0.775	4.646	0.059
	10 Year Gov. Bond	0.040	1.403	-6.545	10.521	1.004	10.961	
Germany	DAX	0.073	3.206	-24.347	14.942	-0.707	5.481	-0.277
	10 Year Gov. Bond	0.046	0.953	-3.606	3.676	-0.178	0.641	

Note: SD, Skew, and Kurt stand for standard deviation, skewness, and Kurtosis, respectively, Calculated from percentage weekly returns of stock indices and 10-year government bond prices from January 2004 to August 2018.

where $S(t)$ is the stock price, $Y(t)$ is the 10-year bond yield, and the bond price $B(t)$ is constructed by

$$\text{Bond Price: } B(t) = \frac{1}{(1+Y(t))^{10}}. \tag{13}$$

2. Estimate GARCH models (10) and obtain standardized residuals as $\hat{\epsilon}_{Stock}(t)$ and $\hat{\epsilon}_{Bond}(t)$ for $\epsilon_{Stock}(t)$ and $\epsilon_{Bond}(t)$, respectively. Their estimates are shown in Table 2.

3. Transform the GARCH residuals to uniformly distributed variables $W = F_t(\hat{\epsilon}_{Stock}, \hat{\lambda}_{Stock})$ and $V = F_t(\hat{\epsilon}_{Bond}, \hat{\lambda}_{Bond})$, where $\hat{\lambda}_{Stock}$ and $\hat{\lambda}_{Bond}$ are estimated degrees of freedom of t-distributions.

2.4 Stock and Government Bond Markets

Figures 2-7 show the estimates of the upper- and lower-tail correlation coefficients estimated by the rolling maximum likelihood method with 100 weeks rolling window.

In the pre-crisis period, the signs of stock-bond correlation of all the countries were negative or around zero. The five peripheral countries had near-zero or negative upper-tail correlations and large positive lower-tail correlations (> 0.5). This large asymmetry suggests an outflow of money from these financial markets.

Table 2: Summary Statistics of AR(p)-GARCH(1,1) Estimation

Country	Assets	AR(p)	GARCH(1,1)			df
		p	α_0	α_1	β_1	
Greece	Stock	0	0.272	0.101	0.888	9.41
			(0.155)	(0.028)	(0.031)	(2.55)
	Bond	0	0.081	0.185	0.814	3.72
			(0.026)	(0.028)	(0.024)	(0.38)
Ireland	Stock	0	0.140	0.063	0.925	5.04
			(0.095)	(0.023)	(0.016)	(0.85)
	Bond	0	0.069	0.126	0.853	3.74
			(0.062)	(0.044)	(0.033)	(0.54)
Italy	Stock	4	1.391	0.352	0.571	5.230
			(0.492)	(0.084)	(0.080)	(0.822)
	Bond	4	0.038	0.092	0.883	6.909
			(0.030)	(0.025)	(0.028)	(1.550)
Portugal	Stock	0	0.102	0.118	0.875	5.243
			(0.059)	(0.030)	(0.025)	(0.948)
	Bond	0	0.026	0.100	0.897	5.439
			(0.028)	(0.028)	(0.025)	(1.076)
Spain	Stock	1	0.089	0.075	0.917	9.281
			(0.065)	(0.019)	(0.018)	(2.206)
	Bond	1	0.025	0.085	0.900	8.133
			(0.033)	(0.034)	(0.038)	(2.032)
Germany	Stock	0	0.451	0.156	0.800	8.016
			(0.200)	(0.046)	(0.057)	(1.737
	Bond	0	0.014	0.060	0.925	20.991
			(0.021)	(0.024)	(0.028)	(11.287)

Note: The order of AP(p) is chosen by minimizing AIC. Standard errors are shown in parentheses. df stands for degrees of freedom.

The stock-bond correlations of the periferal EU countreis are were still positive after 2014, which suggests that the government bonds of these countries were not safe assets.

In contrast, Germany[3] had negative or near-zero correlations throughout the sample period, unlike the peripheral EU countries.

3 Simulation Study

In order evaluate the precision of the rolling maximum likelihood estimation of the split-normal copula, we conduct a small Monte Carlo experiment. Artificial data is generated from the split-normal copula using the correlation coefficients estimated in the previous section in the data generating process. the time-varying upper- and lower-tail correlations of Spain and Germany estimated in the empirical analysis detailed in the previous section. we use grid width 0.04 in the grid-search maximum likelihood estimation. This width is larger than that used in the empirical analysis, for the sake of saving computational time. Figures 8 and 9 shows the mean of the rolling maximum likelihood estimates and their 95 percent confidence intervals. Table 3 shows that the standard deviation and the root mean squared error from the moving average of correlation coefficients are very close so that the rolling maximum estimator is virtually an unbiased estimator of the moving average.

The average of the root mean squared errors around the moving average correlations range from 0.16 to 0.23 when the length of rolling windows is 100, and from 0.15 to when the length of rolling windows is 200.

4 Conclusion

This paper proposed a novel copula and it successfully identified the sign-reversal and asymmetry of the time-varying stock-bond correlation in the EU sovereign crisis. In all the peripheral countries the stock-bond correlation coefficients changed the sign from negative as soon as the crisis started. The mid-crisis period is characterized by the asymmetry of the stock-bond correlation, namely, the positive lower-tail correlation and the negative or near zero upper tail correlation of the stock-bond distribution. This asymmetry cannot be expressed by the other parametric copulas.

We believe that this method can be applied to other asset returns, such as commodities and real estate, in crisis periods and it is interesting to know whether we can find the asymmetry and sign-reversal of the stock-bond correlation in other financial crises.

[3]The stock-bond correlations of the Netherlands and France, which are core EU countries, had a similar pattern to that of Germany.

Table 3: Summary of Simulation

100 Week Rolling Window

Country	Upper Tail				Lower Tail			
	bias	bias^m	Ave. rmse_U	Ave. rmse_U^m	Ave. bias	Ave. bias^m	Ave. rmse_U	Ave. rmse_U
Greece	0.094	0.019	0.240	0.212	0.060	0.015	0.179	0.163
Ireland	0.107	0.014	0.251	0.215	0.068	0.023	0.226	0.210
Italy	0.058	0.023	0.239	0.229	0.058	0.018	0.192	0.180
Portugal	0.074	0.019	0.215	0.195	0.061	0.020	0.211	0.199
Spain	0.066	0.010	0.211	0.193	0.074	0.019	0.214	0.196
Germany	0.055	0.009	0.169	0.155	0.058	0.008	0.176	0.160

200 Week Rolling Window

Country	Upper Tail				Lower Tail			
	bias	bias^m	rmse	rmse^m	bias	bias^m	rmse	rmse^m
Greece	0.101	0.017	0.180	0.135	0.097	0.010	0.153	0.110
Ireland	0.214	0.016	0.275	0.152	0.080	0.014	0.168	0.136
Italy	0.078	0.010	0.168	0.141	0.091	0.009	0.151	0.111
Portugal	0.084	0.013	0.163	0.130	0.105	0.011	0.173	0.125
Spain	0.133	0.027	0.206	0.145	0.134	0.016	0.197	0.134
Germany	0.078	0.019	0.141	0.112	0.102	0.032	0.161	0.119

Note: Time-average of the biases and root mean squared errors of the rolling maximum likelihood ratio estimators are shown for each countries. The reported bias and root mean squared errors are defined as follows:

$$rmse_j(t) = \sum_{i=1}^{N} (r_j^{(i)}(t) - \rho_j(t))^2/N,\ \ j = Upper, Lower$$

$$rmse_j^m(t) = \sum_{i=1}^{N} (r_j^{(i)}(t) - \bar{\rho}_j(t))^2/N,$$

$$\text{bias}_j(t) = |\bar{r}_j(t) - \rho_j(t)|,\ \ \text{bias}_j^{\text{m}}(t) = |\bar{r}_j(t) - \bar{\rho}_j(t)|,$$

where N is the number of iterations of the experiment. $r_U^{(i)}(t)$ is the upper tail correlation estimate in the i-th iteration at date t and $\rho_U(t)$ is the upper tail correlation coefficient estimated in in the previous section and was used in the artificial data generation process.
where
$\bar{r}_j(t) = \sum_{i=1}^{N} r_j^{(i)}(t),\ \ \bar{\rho}_L(t) = \sum_{i=t-W/2+1}^{t+W/2} \rho_L(t)$ is W-week moving average of correlations.

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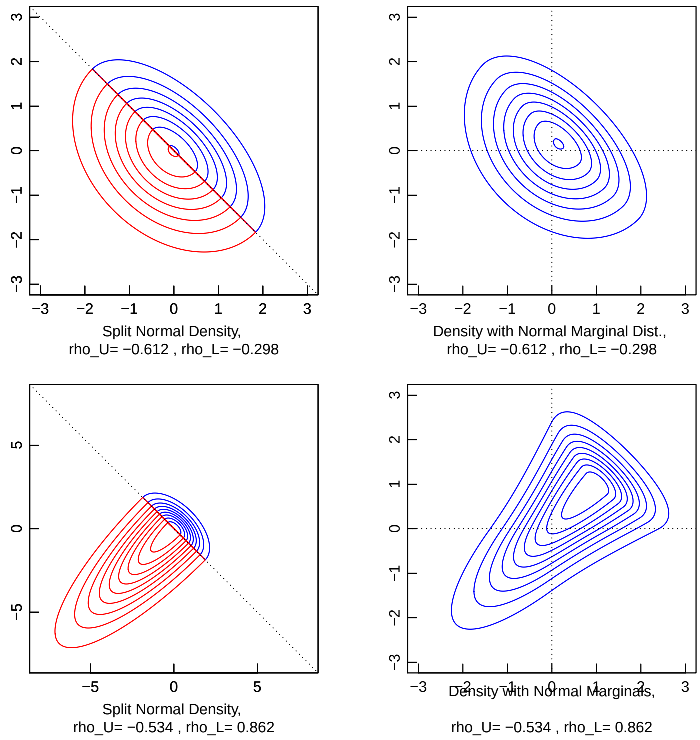

Figure 1: Contours of split normal densities are shown in the left column and contours of transformed distribution with standard normal marginal in the right column. In the upper row $\rho_U = -0.612$ and $\rho_L = -0.298$ and in the lower row $\rho_U = -0.532$ and $\rho_L = 0.862$. The parameter values were those of Greece in September 2007 and May 2010, respectively.

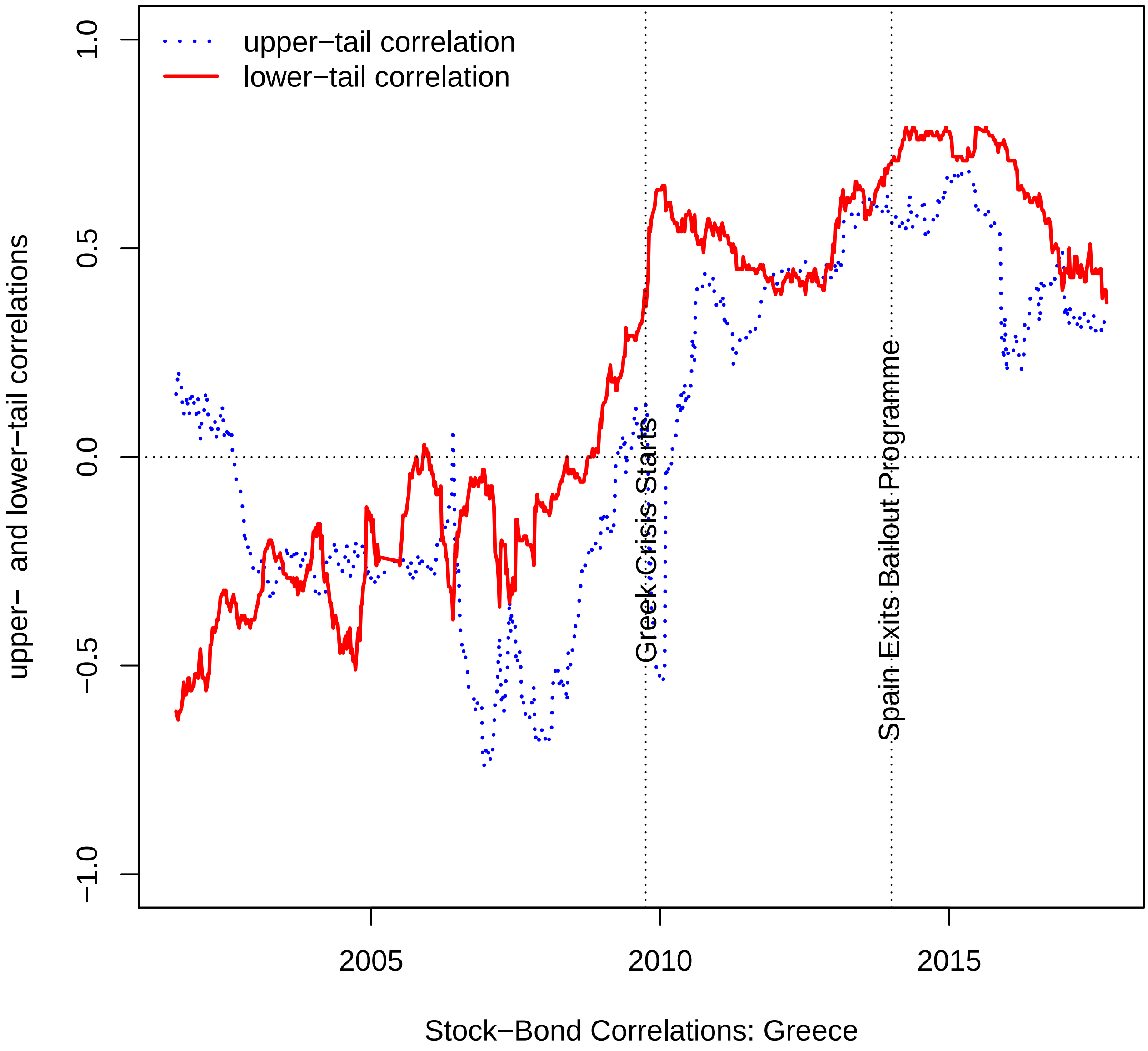

Figure 2: Upper- and Lower-Tail Correlation of Stock-Bond distribution: Greece

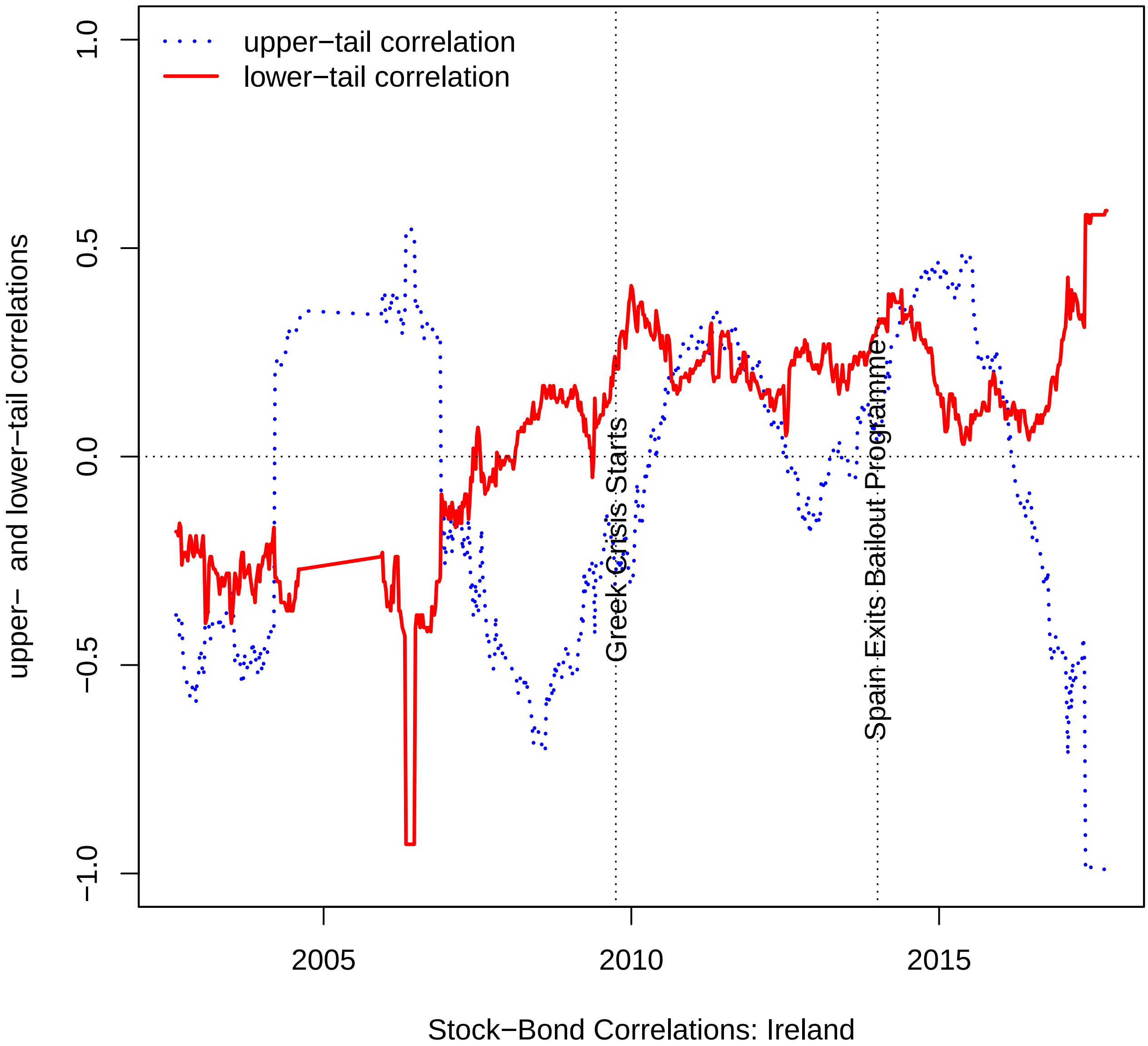

Figure 3: Upper- and Lower-tail correlation of Stock-Bond distribution: Ireland

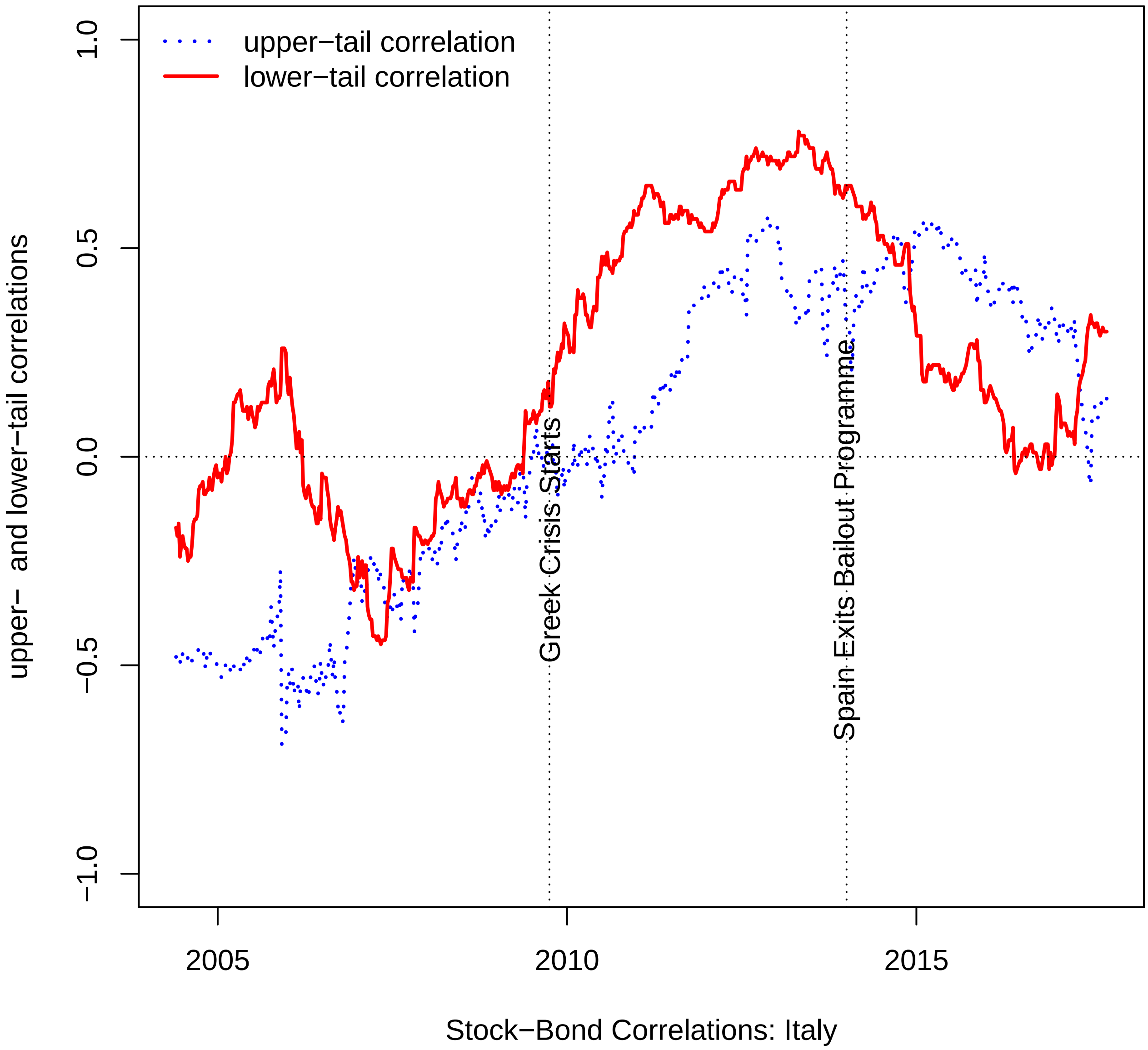

Figure 4: Upper- and Lower-tail correlation of Stock-Bond distribution: Italy

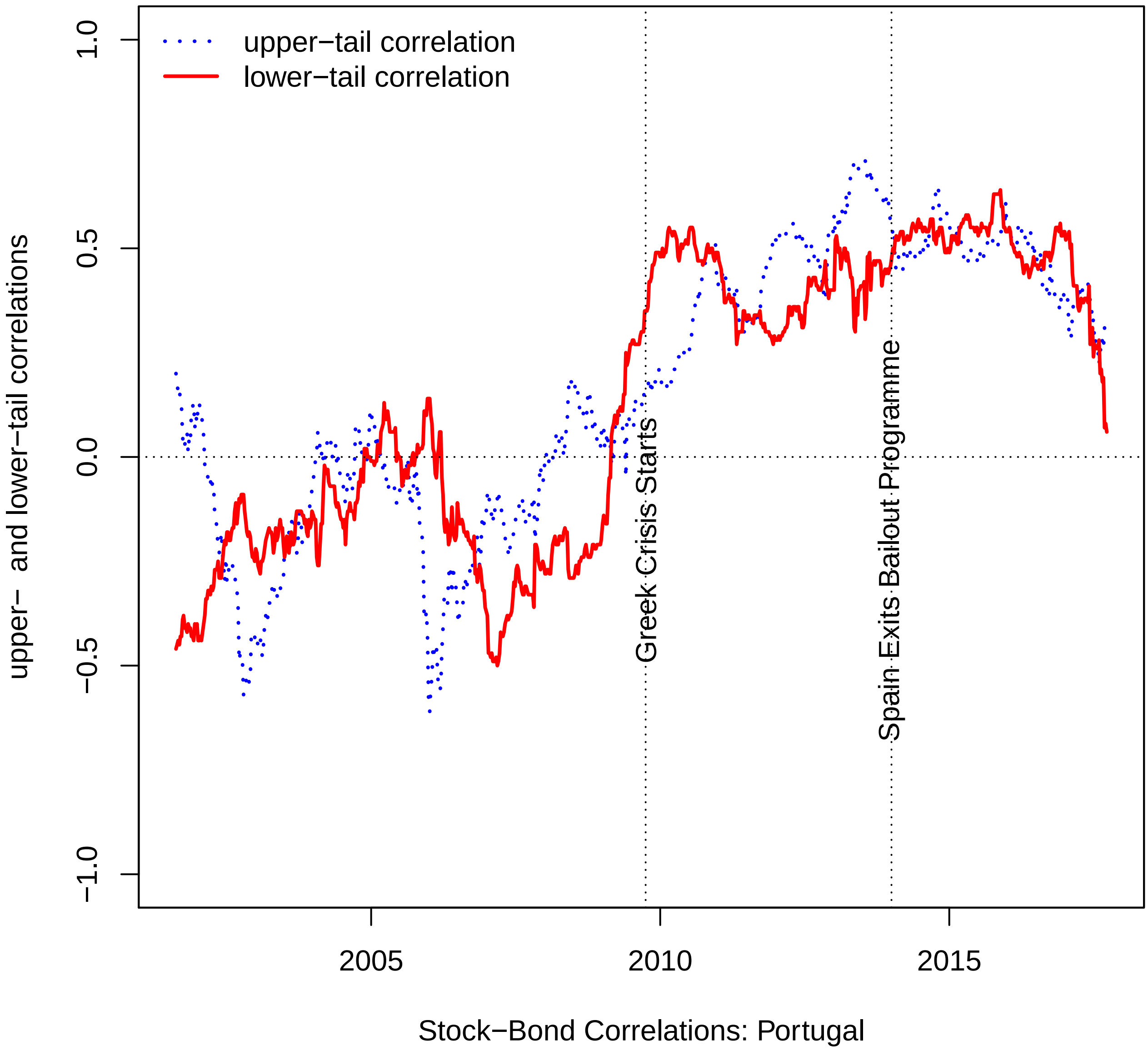

Figure 5: Upper- and Lower-tail correlation of Stock-Bond distribution: Portugal

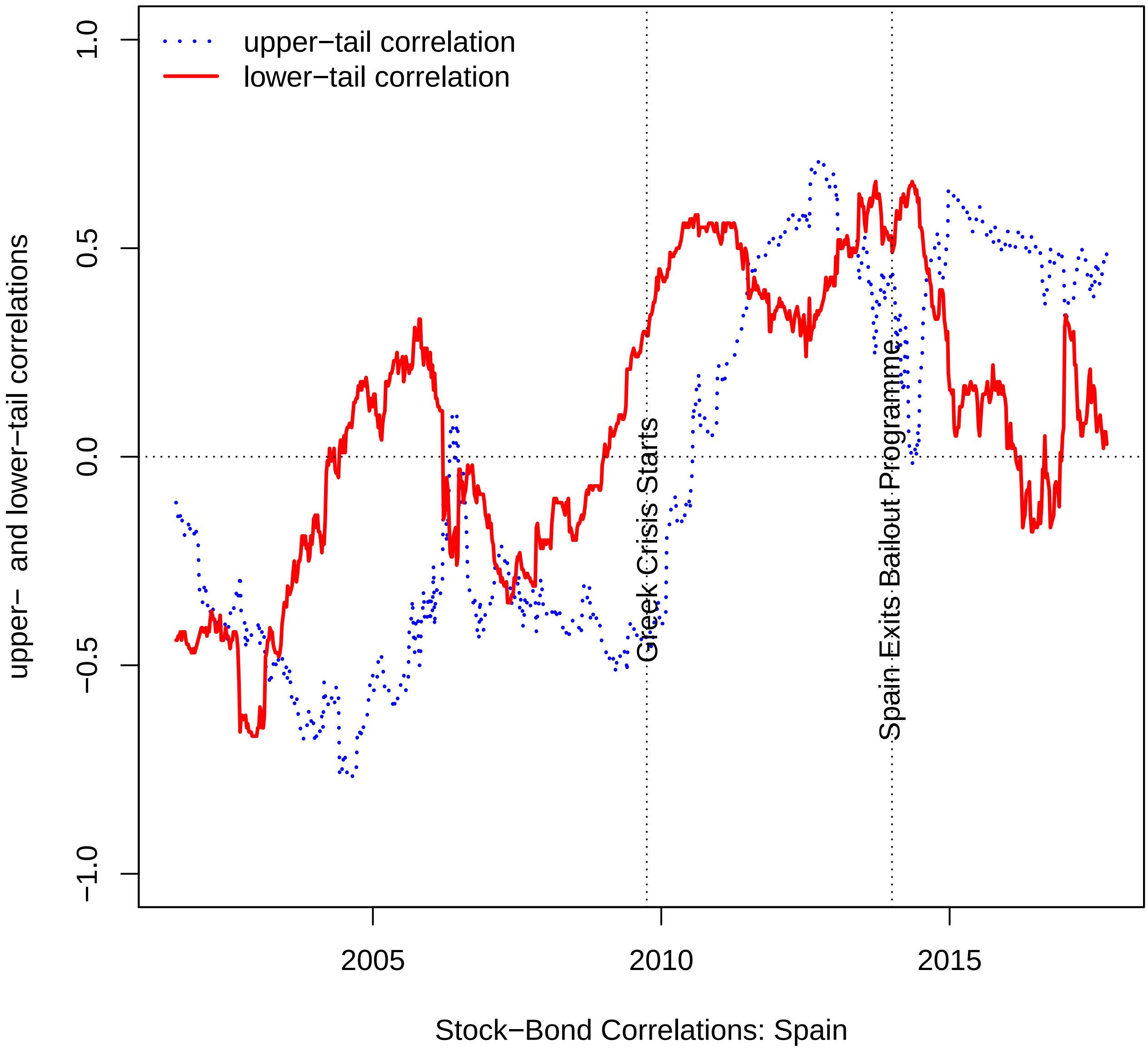

Figure 6: Upper- and Lower-tail correlation of Stock-Bond Distribution: Spain

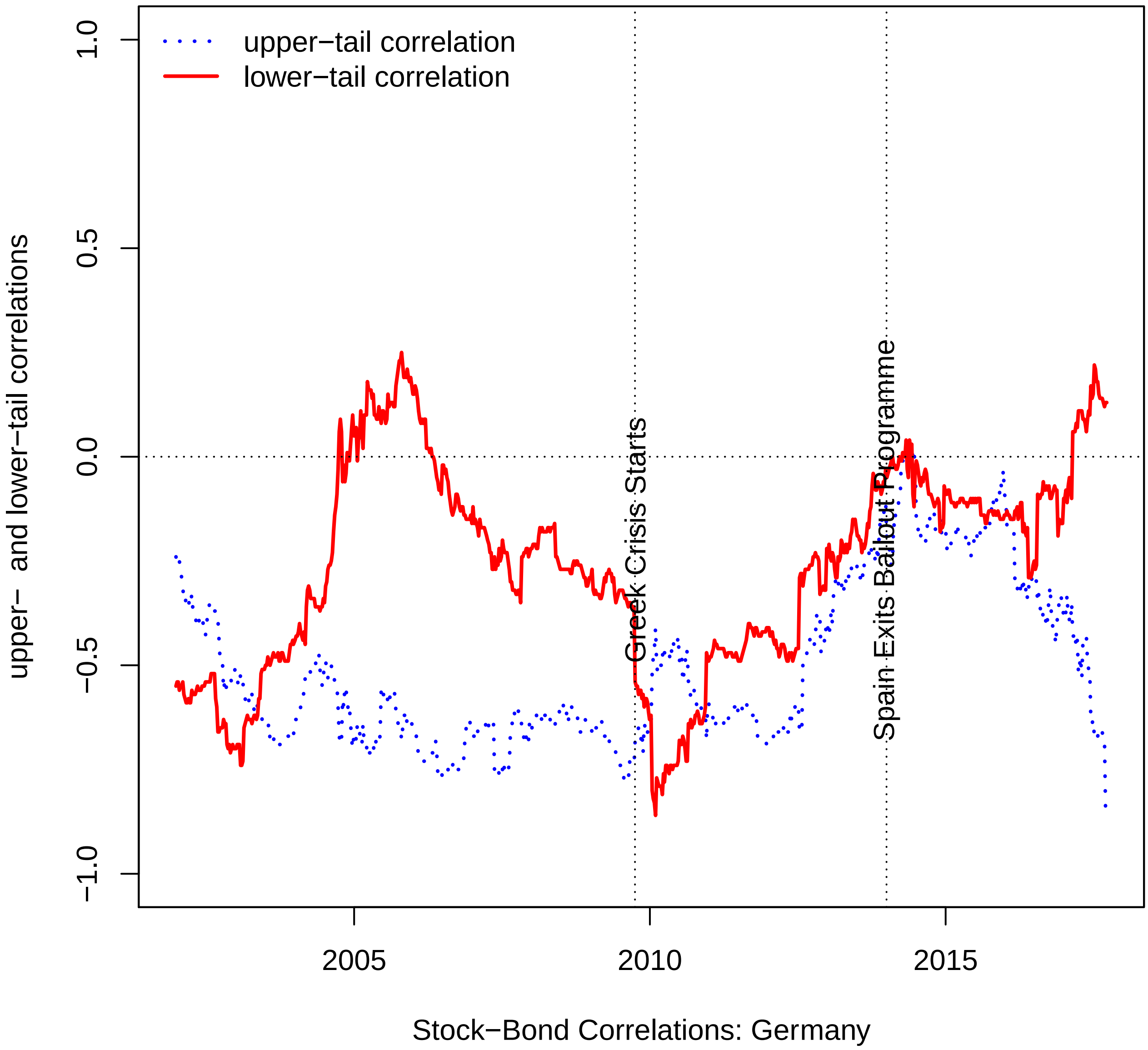

Figure 7: Upper- and Lower-tail correlation of Stock-Bond distribution: Germany

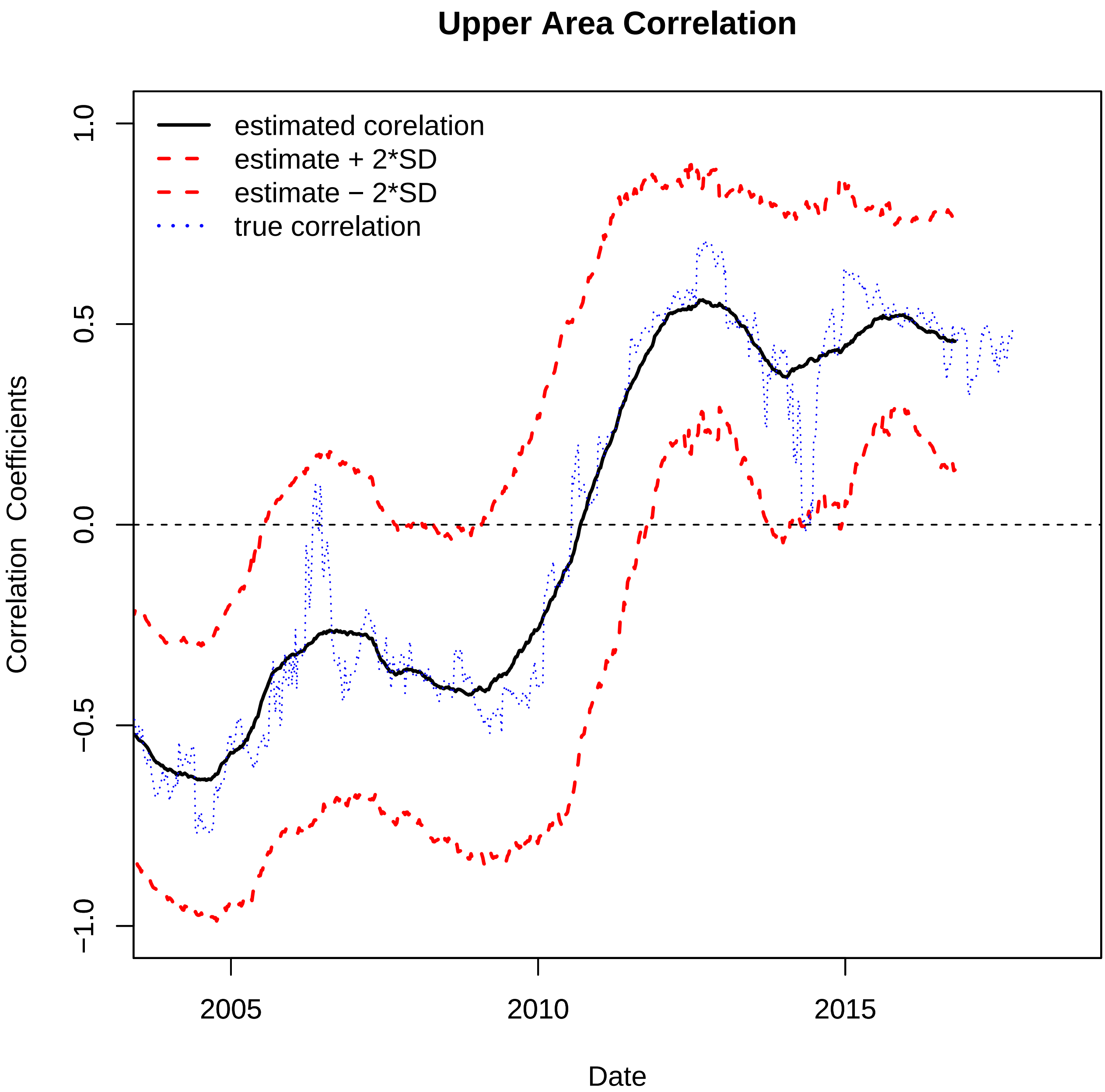

Figure 8: Simulation Experiment of Estimated Upper Tail correlation Coefficients and Confidence Interval. Number of iterations was 100.The estimated upper and lower tail correlations of Spain were used in the data generation process

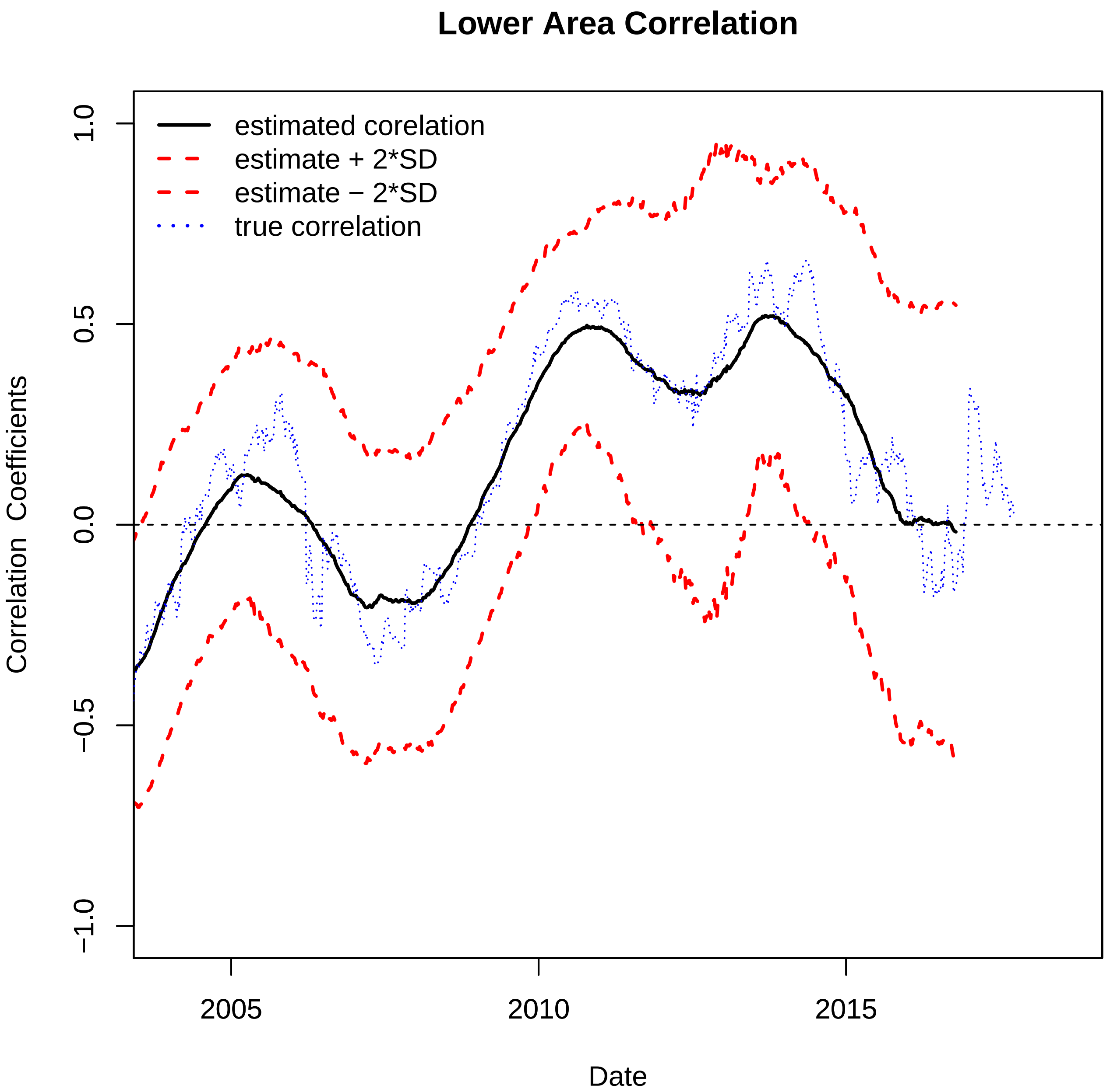

Figure 9: Simulation Experiment of Estimated Lower Tail correlation Coefficients and Confidence Intervals Number of iterations is 100.The estimated upper and lower tail correlations of Spain were used in the data generation process

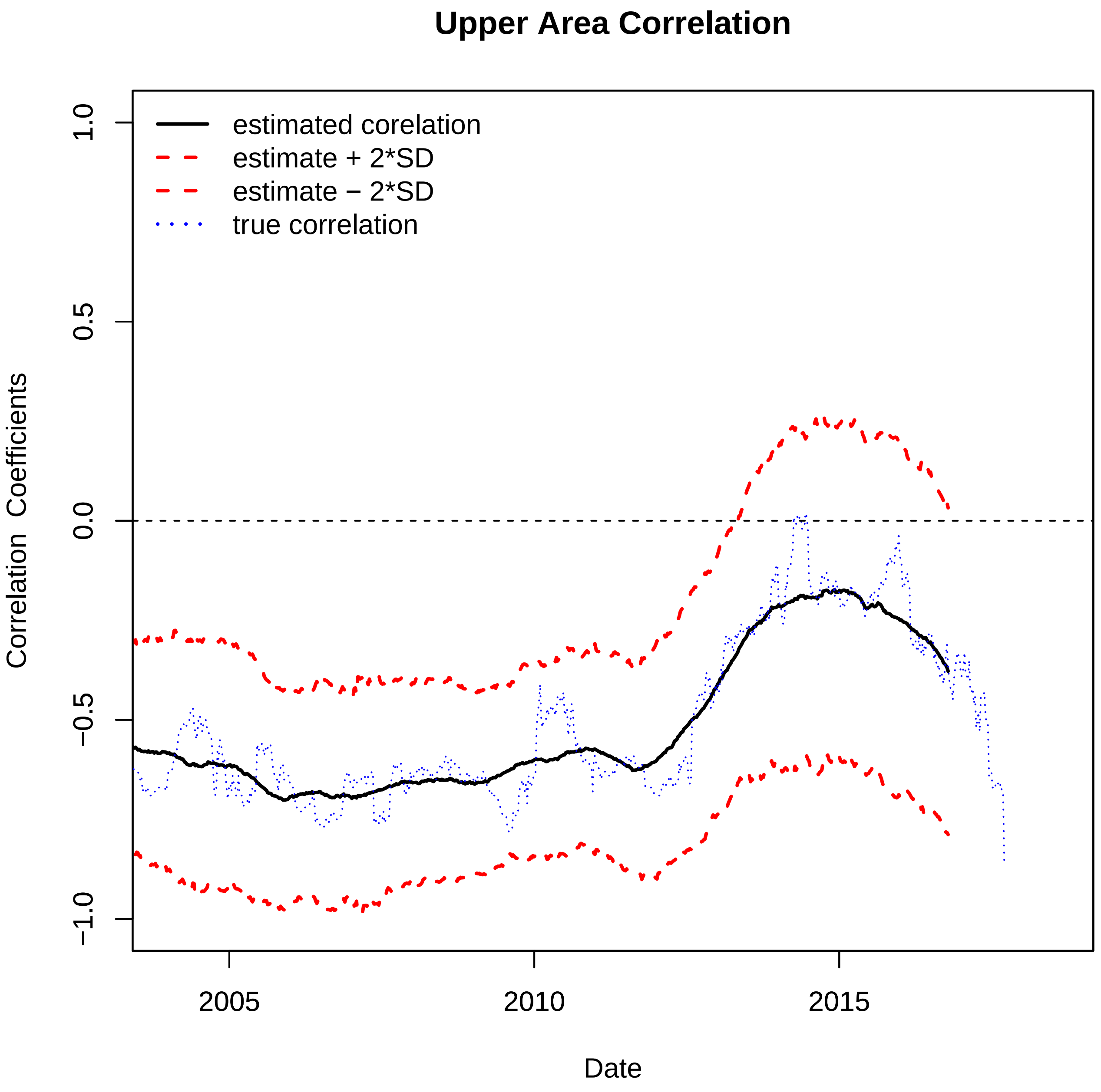

Figure 10: Simulation Experiment of Estimated Upper Tail correlation Coefficients and Confidence Interval. Number of iterations was 200.The estimated upper and lower tail correlations of Germany were used in the data generation process

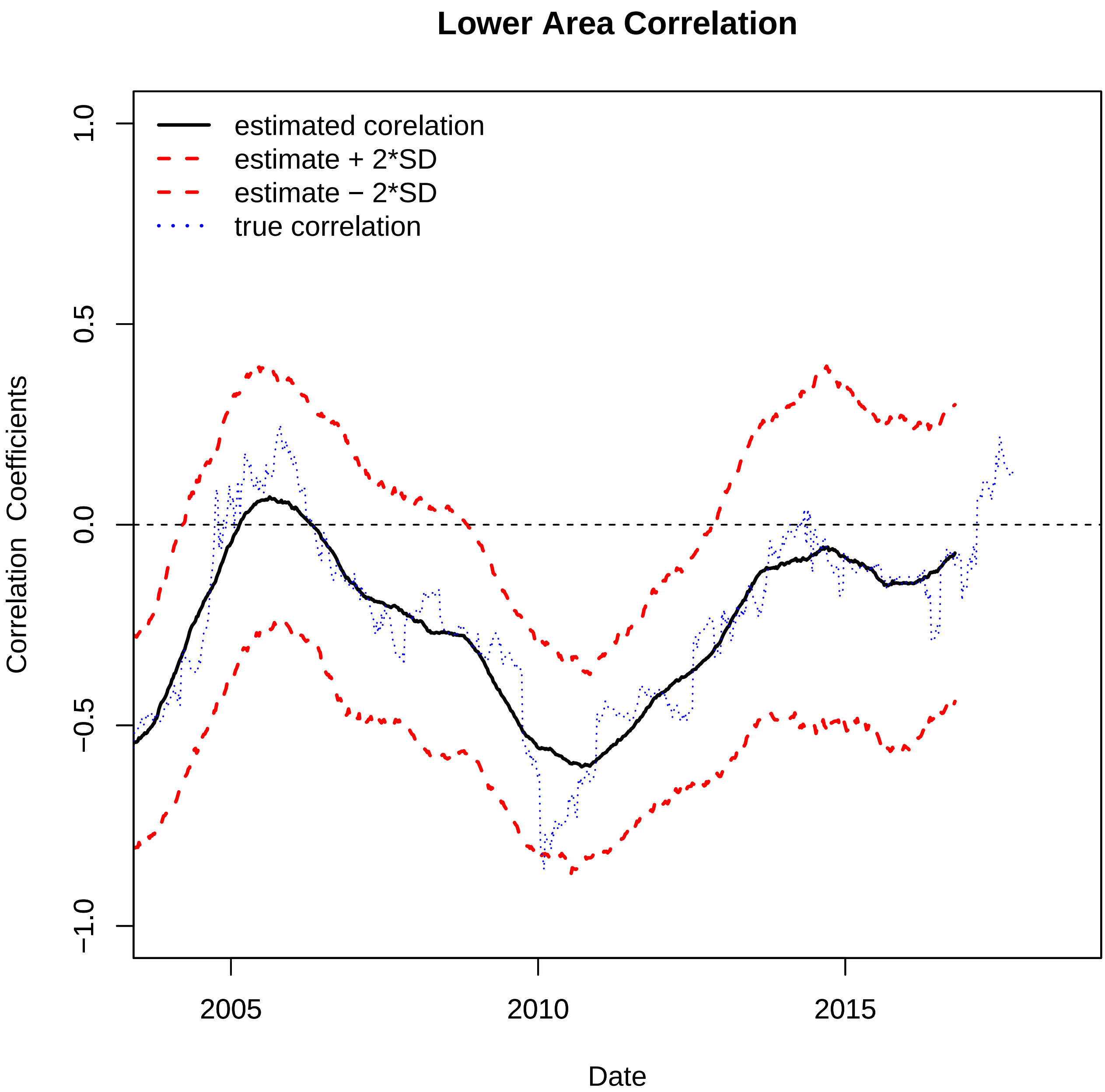

Figure 11: Simulation Experiment of Estimated Lower Tail correlation Coefficients and Confidence Intervals Number of iterations is 200.The estimated upper and lower tail correlations of Germany were used in the data generation process