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Interconnection of Fiscal Policies on Sustainability of Public Debt

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Abstract

This paper investigates the interconnection between certain fiscal policies in achieving a sustainable level of public debt. The fiscal policies that are investigated relate to the consumption tax rate, the income tax rate, and to public spending. The paper focuses on the critical level of public debt-to-GDP ratio, for which if the ratio exceeds this level at time 0, then it diverges to $+\infty$ as time passes. The paper theoretically examines how the critical level depends on the fiscal policies, and reveals some merits of consumption taxation. As the consumption tax rate increases, so income taxation and cutting public spending become more effective in sustaining public debt.

Key Words: sustainability of public debt, fiscal policies, consumption tax, income tax, public spending, balanced growth path

JEL Classification Numbers: E62, H6

1 Introduction

High accumulations of public debt significantly constrain the fiscal management policies of many countries. Some governments try to implement fiscal reforms in response to high levels of public debt. For example, currently there is much debate about Japan's plan to increase its consumption tax rate to address its public debt.

This paper studies the role that fiscal policies play in achieving sustainable public debt, using an AK-type economic growth model. The paper explicitly derives the critical initial public debt-to-gross domestic product (GDP) ratio that is compatible with a balanced growth path (BGP). If the ratio exceeds this critical level at time 0, then it diverges to $+\infty$ as time passes. If the public debt-to-GDP ratio is at the critical level at time 0, then it remains constant as time passes: this can be interpreted as the public debt being sustainable. This paper highlights how the critical level of the initial public debt-to-GDP ratio depends on fiscal policies. The relevant fiscal policies relate to the consumption tax rate, the income tax rate, and to public spending. The study investigates the combined-effects of these policies on the sustainability of public debt. The results reveal some advantages of consumption taxation in maintaining sustainable levels of public debt. Given the high levels of accumulated public debt observed in the real world, it is important to examine how a country's fiscal policies are interconnected in achieving the sustainability of such debt.

This paper reveals that a slight cut in public spending enhances the marginal effect of the consumption tax rate on the sustainability of public debt. A decrease in public spending stimulates consumption, thereby increasing the government's revenue from consumption tax. Consequently, if a government cuts public spending, there is then an increase in the marginal contribution of the consumption tax rate in improving fiscal health. This increase occurs even if the consumption tax rate is fixed. Similarly, a slight increase in the income tax rate enhances the marginal effect of the consumption tax rate on the sustainability of public debt. Accordingly, consumption and income taxes are complementary for enhancing the sustainability of public debt. The paper also shows that if a government raises the consumption tax rate, then there is an increase in the marginal effect of raising the income tax rate and of cutting public spending to increase the critical level of public debt-to-GDP ratio. This consequence is a merit of consumption taxation.

Many previous studies examine the sustainability of public debt. Using an overlapping generations model with endogenous growth, Bräuninger (2005) theoretically studies the relationship between a deficit ratio and economic growth rates. Yakita (2008) and Arai (2011) introduce public capital formation into a Bräuninger-type model. Teles and Mussolini (2014) demonstrate that a high public debt-to-GDP ratio limits the effects of fiscal policies on economic growth. These studies find that there is a critical public debt-to-GDP ratio for sustainable public debt. If the public debt is greater than the critical level at the initial time point, then it is not sustainable. Greiner (2011b) demonstrates that a critical level of public debt that is sustainable exists, if the primary surplus-to-GDP ratio is independent of the public debt-to-GDP ratio. This critical level crucially depends on the difference between the real interest rate and the economic growth rate. Kondo (2007, 2012) and Kondo and Kitaura (2009) explicitly derive the critical level of public debt; however, they assume a lump-sum tax. Hence, they do not investigate the interaction between the consumption and income tax rates for the sustainability of public debt.

Many macroeconomic papers that investigate public debt focus on the relevant tax systems. Schmitt-Grohe and Uribe (1997) demonstrate that the balanced-budget rule may be a source of expectation-driven economic fluctuations. They use a dynamic model in which public expenditures are fully financed by income tax. Contrastingly, Giannitsarou (2007) shows that if a government exclusively uses consumption tax for expenditures, then such fluctuations disappear. Schmitt-Grohe and Uribe (1997) and Giannitsarou (2007) assume a constant tax rate. Nourry *et al.* (2013) and Greiner and Bondarev (2015) introduce a state-dependent consumption tax to their economic growth models, where the consumption tax rate is a function of the consumption level. Although these macroeconomic papers investigate the local indeterminacy of equilibrium paths around a steady state, the sustainability of public debt is not a main theme. Schmitt-Grohe and Uribe (1997), Giannitsarou (2007) and Nourry *et al.* (2013) assume a balanced budget, and Greiner and Bondarev (2015) postulate Bohn's rule (1998) that ensures the sustainability of the public debt.[1] Kamiguchi and Tamai (2012) and Greiner (2011a, 2013) also assume that a government's fiscal policies obey Bohn's

[1] Bohn (1998) proposes a fiscal rule under which public debt is sustainable in the sense that the initial level of public debt is sufficiently covered by the sum of the present value of the primary surpluses that will be obtained from now to the future. Under the rule, the primary surplus-to-GDP ratio is required to increase if the public debt-to-GDP ratio increases.

rule. Many empirical studies accept that national governments tend to abide by Bohn's rule. However, it is somewhat unclear how, and to what extent, the fiscal policies affect the sustainability of public debt under Bohn's rule.[2] This current paper uses an economic model that considers income and consumption tax in addition to public spending. The paper explicitly derives the critical initial level of public debt-to-GDP ratio that is compatible with BGP. The paper theoretically investigates how this level depends on fiscal policy.

The remainder of this paper is organized as follows. Section 2 constructs a model economy on which the following analysis is based. Section 3 analyzes an equilibrium path. A BGP is especially highlighted. The main result is presented in Section 4. Section 5 briefly concludes the paper.

2 Model

This section constructs a model economy on which the analyses are based. This economy has an infinite time horizon, and has a household sector, a productive sector and a government.

2.1 Household

The household sector consists of many identical households. The representative household maximizes a discounted integral of instantaneous utilities from now to the infinite future. An instantaneous utility function is represented by a utility function $u(C(t)) = \log C(t)$, where $C(t)$ is the consumption level at time t. The household inelastically supplies a labor service L^S to a representative firm. The wage rate is denoted by $w(t)$. At each time point, the household purchases goods for consumption and saves some income for future consumption. Holding a real asset bears real interest. The real interest rate is denoted by $r(t)$. The total real asset is denoted by

$$W(t) \equiv B(t) + K(t), \tag{1}$$

where $B(t)\,(\in \mathbb{R})$ and $K(t)\,(\geq 0)$ are the public debt and the physical capital, respectively.[3] Their initial levels $B(0)$ and $K(0)$ are given for the house-

[2]Since Hamilton and Flavin (1986), many empirical studies investigate whether public debt is sustainable. For relatively recent results, see Curtaşu (2011) and Fincke and Greiner (2012).

[3]The symbol $\mathbb{R}$ stands for the set of real numbers.

hold. The consumption behavior and the income of the household are levied by a government. The tax rate for consumption and for income are denoted by $\tau_C > 0$ and $\tau_I \in (0, 1)$, respectively. The feasible paths of consumption and saving must be subject to budget constraints and to the no Ponzi-game (NPG) condition. The representative household's behavior is summarized as the following maximizing problem:

$$\max_{C(t),\, W(t)} \int_0^{\infty} e^{-\rho t} \log C(t)\, dt \tag{2}$$

subject to

$$(1 + \tau_C) C(t) + \dot{W}(t) = (1 - \tau_I) \left(w(t) L^S + r(t) W(t) \right), \tag{3}$$

$$\lim_{t \to \infty} e^{-(1-\tau_I) \int_0^t r(s) ds} W(t) = 0 \tag{4}$$

where $\rho > 0$ stands for a subjective discount rate of the representative household. According to convention, the time derivative is represented by the dot symbol: $(\dot{\ }) = d/dt$. The time index is often omitted in what follows if no ambiguity arises.

2.2 Firm

The productive sector is represented by a representative firm. The firm produces output Y, by using labor service L^D and physical capital K. The production process is influenced by an economy-wide capital level $\overline{K}$. Although the firm takes $\overline{K}$ as given, the study assumes that $\overline{K} = K$ in an equilibrium. The production function is given by

$$Y = AK^{\alpha} \left(\overline{K} L^D \right)^{1-\alpha} \tag{5}$$

where $\alpha \in (0, 1)$ and $A\,(> 0)$ are constant parameters that represent a capital share and a total factor productivity, respectively. The representative firm maximizes static profit subject to its technology. Its behavior is summarized as the following maximizing problem:

$$\max_{K,\, L^D \geq 0} AK^{\alpha} \left(\overline{K} L^D \right)^{1-\alpha} - rK - wL^D \tag{6}$$

2.3 Government

The government obtains revenues from consumption tax and from income tax, and by issuing public bonds B^G for any $t \geq 0$. The flow budget constraint is given by

$$\dot{\left(B^G\right)} = rB^G - (T - G) \tag{7}$$

where

$$T = \tau_C C + \tau_I (wL + rW) \tag{8}$$

is the total tax revenue, and G is public spending. The initial level of the public debt $B^G(0)$ is given for the government. Equation (7) can be regarded as an accounting identity. For the sake of simplicity, this paper assumes that the public spending is wasteful. The paper also assumes that the government uses a constant fraction of gross domestic product (GDP) Y for its public spending, i.e.,

$$G(t) = gY(t) \tag{9}$$

for any $t \geq 0$, where $g = (0, 1)$ is a constant. In addition to (7), the government must be subject to the inter-temporal budget constraint:

$$\lim_{t\to\infty} e^{-(1-\tau_I)\int_0^t r(s)ds} B^G(t) = 0. \tag{10}$$

Condition (10) means that the present value of the public debt must converge to 0 in the remote future. This condition is equivalent to the government plans to raise money by levying tax from now to the future to just compensate their expenditure plan $\{G(t)\}$, and repay the accumulated debt at the initial time $B^G(0)$ (see Greiner (2011a, 2013)).

2.4 Market Clearing Conditions

In an equilibrium, the market of the good, the labor service and the public bond must be cleared simultaneously for any $t \geq 0$. That is, it must hold that

$$Y = C + \dot{K} + G, \quad L^S = L^D, \quad B^G = B, \tag{11}$$

for any $t \geq 0$. In what follows, an equilibrium path is highlighted, and labor service and public debt in the equilibrium are simply denoted by L and B, respectively. From (4) and (10), it holds that in an equilibrium

$$\lim_{t\to\infty} e^{-(1-\tau_I)\int_0^t r(s)ds} K(t) = \lim_{t\to\infty} e^{-(1-\tau_I)\int_0^t r(s)ds} B(t) = 0. \tag{12}$$

Further, as mentioned in Section 2.2, in an equilibrium it is required that the average economy-wide level of capital $\overline{K}$ is equal to the capital level of the representative firm K:

$$\overline{K} = K. \tag{13}$$

Consequently, it holds from (5) that

$$Y = AKL^{1-\alpha}. \tag{14}$$

3 Equilibrium

This section analyzes an equilibrium path defined in Section 2. The section focuses on the balanced growth path (BGP). Based on the examinations in this section, the paper investigates an initial level of public debt to capital (and GDP) ratio that is compatible with the BGP in the next section.

3.1 Equilibrium Path

In an equilibrium: (i) given paths of prices $\{w(t), r(t)\}_{t\geq 0}$, fiscal policies (τ_C, τ_I, g) and the initial conditions $(K(0), B(0))$, the representative household maximizes the discounted integral of utilities (2), subject to the budget constraints (3) and (4); (ii) given paths of prices $\{w(t), r(t)\}_{t\geq 0}$ and the average economy-wide level of capital $\left\{\overline{K}(t)\right\}_{t\geq 0}$, the firm maximizes its profits (6) for any $t \geq 0$; (iii) given the path of real interest rate $\{r(t)\}_{t\geq 0}$ and the initial level of the public debt $B^G(0)$, the government determines policy variables $\left((\tau_C, \tau, g), \ \left\{B^G(t)\right\}_{t>0}\right)$ subject to (7) and (10); and (iv) the markets are simultaneously cleared (11), and the average economy-wide level of capital $\overline{K}$ coincides with the physical capital level employed by the representative firm K.

Note that as first order conditions of the maximizing problem of the representative firm (6), it holds in the equilibrium that

$$r = \alpha A L^{1-\alpha}, \tag{15}$$
$$w = (1-\alpha) AKL^{-\alpha}. \tag{16}$$

The condition $\overline{K} = K$ is used to derive the first equation.

Next, a consumption path in the equilibrium is investigated. A (present-value) Hamiltonian associating with the maximizing problem (2)–(4) is defined by

$$\mathcal{H} = e^{-\rho t} \log C + \lambda \left[(1 - \tau_I) (wL^s + rW) - (1 + \tau_C) C \right],$$

where λ is a Hamilton multiplier. From conditions $\partial \mathcal{H} / \partial C = 0$ and $\partial \mathcal{H} / \partial W = -\dot{\lambda}$, it holds that

$$\frac{\dot{C}}{C} = (1 - \tau_I) \alpha A L^{1-\alpha} - \rho. \tag{17}$$

Thus, it holds that

$$C(t) = C(0) e^{\theta t}, \tag{18}$$

where

$$\theta \equiv (1 - \tau_I) \alpha A L^{1-\alpha} - \rho \tag{19}$$

is an economic growth rate on a BGP.

Remark 1. Among the fiscal policies (τ_C, τ_I, g), only the income tax rate τ_I influences the economic growth rate on the BGP θ. A high income tax rate implies that the interest income is heavily taxed. As a result, savings shrink, and the economic growth rate decreases.

The dynamics of capital stock can be derived from the first equation of the market clearing conditions (11):

$$\dot{K} = (1 - g) A K L^{1-\alpha} - C. \tag{20}$$

Using (18), I obtain

$$\dot{K} = (1 - g) A K L^{1-\alpha} - C(0) e^{\theta t}.$$

Thus, it holds that

$$K(t) = e^{(1-g)AL^{1-\alpha} t} \left[K(0) - C(0) \int_0^t e^{\left[\theta - (1-g)AL^{1-\alpha}\right] s} ds \right].$$

An easy calculation yields

$$\begin{aligned} K &= e^{(1-g)AL^{1-\alpha} t} \left[K(0) - C(0) \frac{1}{(1-g) A L^{1-\alpha} - \theta} \right] \\ &\quad + \frac{C(0)}{(1-g) A L^{1-\alpha} - \theta} e^{\theta t}. \end{aligned} \tag{21}$$

In Assumption 1, assume that the denominator that appears in (21) $(1-g)\,AL^{1-\alpha}-\theta$ is positive. It is clear from (21) that the capital path is on the BGP if and only if the initial level of consumption is chosen at

$$\begin{aligned} C(0) &= \left[(1-g)\,AL^{1-\alpha}-\theta\right]K(0) \\ &= \left[(1-g)\,AL^{1-\alpha}-(1-\tau_I)\,\alpha AL^{1-\alpha}+\rho\right]K(0), \end{aligned} \tag{22}$$

given $K(0)>0$. The second equation of (22) is obtained from the definition of θ in equation (19).

Here, the following assumption is made.

Assumption 1. $0 \leq \theta\,(\equiv (1-\tau_I)\,\alpha AL^{1-\alpha}-\rho) < (1-g)\,AL^{1-\alpha}$, where θ is the parameter defined in (19).

The first inequality $0 \leq \theta$ means that the economic growth rate θ is non-negative on the BGP, which is satisfied if ρ is sufficiently near 0. The second inequality $\theta < (1-g)\,AL^{1-\alpha}$ guarantees that the economic variables (C, K) are positive on a BGP (see (22)). Note that the conditions (12) are satisfied on the BGP since $\rho > 0$ (see (12), (15) and (19)).

Some remarks are given with regard to the effects of the public spending g (Remark 2) and the income tax rate τ_I (Remark 3) on the consumption path or tax revenue of the government.

Remark 2. Assume that the government cut public spending g at time 0. Then, the supply of the goods that is available for households (and firms) increases. Thus, the initial level of consumption increases, which is captured in (22), while the growth rate of consumption on the BGP does not change (see (19)). Thus, the revenues of the government from the consumption tax from now onwards are expected to increase, even if a consumption tax rate is fixed.

Remark 3. The income tax rate τ_I affects the consumption path in a more complex manner than in the case of public spending g. Assume that a government unexpectedly increases the income tax rate at $t=0$. Then, as mentioned in Remark 1, the household saving level and the economic growth rate both decrease. How do households change their consumption path? Instead of saving, the initial level of consumption increases (see (22)). Contrastingly, the growth rate of consumption decreases, because the economic growth rate decreases as a result of the depressed levels of savings (see (19)). Briefly, if the income tax rate increases, then the consumption level initially increases, but its growth rate decreases.

3.2 Balanced Growth Path

On the BGP, it holds from (18), (22), (21) and (14) that

$$\begin{aligned} C(t) &= C(0)e^{\theta t} \\ &= \left[(1-g)AL^{1-\alpha} - (1-\tau_I)\alpha AL^{1-\alpha} + \rho\right]K(0)e^{\theta t}, \\ K(t) &= K(0)e^{\theta t}, \\ Y(t) &= AK(0)L^{1-\alpha}e^{\theta t}, \end{aligned} \tag{23}$$

given $K(0) > 0$.

Using the BGP conditions in (23), consider a dynamic path of the public debt. Substituting (8) and (9) into (7) yields

$$\dot{B} = rB - \left[\tau_C C + \tau_I(wL + rW) - gY\right].$$

It holds from the definition of total asset (1), aggregate output (14), and the first order conditions of the firm (15) (16) that

$$\begin{aligned} \dot{B} &= (1-\tau_I)rB - \left(\tau_C C + \tau_I(wL + rK) - gAKL^{1-\alpha}\right) \\ &= (1-\tau_I)\alpha AL^{1-\alpha}B \\ &\quad - \left[\tau_C C + \tau_I(1-\alpha)AKL^{1-\alpha} + \tau\alpha AL^{1-\alpha}K - gAKL^{1-\alpha}\right]. \end{aligned}$$

Thus,

$$\dot{B} = (1-\tau_I)\alpha AL^{1-\alpha}B - \left[\tau_C C + (\tau_I - g)AKL^{1-\alpha}\right]. \tag{24}$$

Substituting the first and the second equations of (23) into (24) yields

$$\dot{B} = (1-\tau_I)\alpha AL^{1-\alpha}B - \left[\tau_C C(0) + (\tau_I - g)AL^{1-\alpha}K(0)\right]e^{\theta t}.$$

Using equation (22), which shows the initial level of consumption, obtains

$$\begin{aligned} \dot{B} &= (1-\tau_I)\alpha AL^{1-\alpha}B \\ &\quad - \left[\tau_C\left\{(1-g)AL^{1-\alpha} - \theta\right\}K(0) + (\tau_I - g)AL^{1-\alpha}K(0)\right]e^{\theta t} \\ &= (1-\tau_I)\alpha AL^{1-\alpha}B - HK(0)e^{\theta t}, \end{aligned} \tag{25}$$

where H is a constant defined by

$$H \equiv \tau_C \left[(1-g) AL^{1-\alpha} - \theta \right] + (\tau_I - g) AL^{1-\alpha}. \tag{26}$$

Solving the differential equation (25), obtains

$$B(t) = e^{(1-\tau_I)\alpha AL^{1-\alpha} t} \left[B(0) - HK(0) \int_0^t e^{\left[\theta - (1-\tau_I)\alpha AL^{1-\alpha}\right] s} ds \right].$$

Thus,

$$\begin{aligned} B &= e^{(1-\tau_I)\alpha AL^{1-\alpha} t} \left[B(0) + \frac{H}{\theta - (1-\tau_I)\alpha AL^{1-\alpha}} K(0) \right] \\ & \quad - \frac{H}{\theta - (1-\tau_I)\alpha AL^{1-\alpha}} K(0) e^{\theta t}. \end{aligned}$$

From the definition of θ (19),

$$B = e^{(1-\tau_I)\alpha AL^{1-\alpha} t} \left[B(0) - \frac{H}{\rho} K(0) \right] + \frac{H}{\rho} K(0) e^{\theta t}. \tag{27}$$

Thus, it holds that

$$B(t) = B(0) e^{\theta t} \tag{28}$$

if and only if

$$B(0) = \frac{H}{\rho} K(0) = \frac{\tau_C \left[(1-g) AL^{1-\alpha} - \theta \right] + (\tau_I - g) AL^{1-\alpha}}{\rho} K(0). \tag{29}$$

The equation system (23) and (28) gives the BGP condition in the present economy.

The following assumptions are now made.

Assumption 2. $\tau_I - g > 0$.

Assumption 2, together with Assumption 1, guarantees that the constant variable H is positive (see (26)). Further, it implies that an initial level of public debt that is compatible with the BGP is positive (see (29)), i.e., the government is a borrower. This is reasonable given the high level of accumulated public debt in many advanced countries in the real world.

The public debt-to-GDP ratio on the BGP is worth investigating. It can be derived from (23), (28) and (29) as

$$\frac{B}{Y} = \frac{H}{AL^{1-\alpha}\rho} = \frac{\tau_C\left[(1-g)\,AL^{1-\alpha} - \theta\right] + (\tau_I - g)\,AL^{1-\alpha}}{AL^{1-\alpha}\rho} \tag{30}$$

The next proposition is established.

Proposition 1 *(Balanced Growth Path)*

In the present economy, a BGP is given by (23) *and* (28)*, with the initial level of the public debt* (29) *given* $K(0) > 0$*. The parameter* θ *is defined in* (19)*. The public debt-to-GDP ratio on the BGP is given by* (30)*.*

4 Main Analysis

This section investigates an initial level of public debt-to-GDP ratio that is compatible with a BGP. That can be interpreted as a sustainable public debt-to-GDP ratio, which is explained in Section 4.1. How this ratio depends on fiscal policies (τ_C, τ_I, g) is theoretically examined in Section 4.2.

4.1 Critical Initial Level of Public Debt-to-GDP Ratio

This subsection explicitly states the critical initial level of public debt-to-GDP ratio that is sustainable. It is clear from (27) that if $B(0) > \frac{H}{\rho}K(0)$, then the public debt-to-capital ratio diverges to $+\infty$ as time passes. It is equivalent in the present economy that the public debt-to-GDP ratio diverges to $+\infty$, which means that the public debt is not sustainable given the other parameters. Thus, the initial level of public debt must satisfy

$$B(0) \leq \frac{\tau_C\left[(1-g)\,AL^{1-\alpha} - \theta\right] + (\tau_I - g)\,AL^{1-\alpha}}{\rho}K(0)\,. \tag{31}$$

Using $Y(0) = AL^{1-\alpha}K(0)$, the critical level of public debt-to-GDP ratio that is sustainable must be

$$\frac{B(0)}{Y(0)} = \frac{\tau_C\left[(1-g)\,AL^{1-\alpha} - \theta\right] + (\tau_I - g)\,AL^{1-\alpha}}{AL^{1-\alpha}\rho}, \tag{32}$$

given fiscal policies (τ_C, τ_I, g) and other parameters. Substituting (19) into the right hand side of equation (32), gives a critical initial level of public debt-to-GDP ratio that is sustainable as follows:

$$\frac{B(0)}{Y(0)} = \varphi \equiv \frac{\tau_C \left[(1-g) AL^{1-\alpha} - \{(1-\tau_I)\alpha AL^{1-\alpha} - \rho\}\right] + (\tau_I - g) AL^{1-\alpha}}{AL^{1-\alpha}\rho}. \tag{33}$$

Under Assumption 1 and 2, it holds that $\varphi > 0$. If the parameter constellation satisfies equation (33), then the public debt is *sustainable*, or the fiscal policies (τ_C, τ_I, g) are *sustainable*.

A government should set policy parameters (τ_C, τ_I, g) to satisfy equation (33) given the initial levels of public debt $B(0)$, GDP $Y(0)$, labor endowment L, technology parameters A and α, and the subjective discount rate ρ. Inversely, this can be interpreted as follows: given the other parameters, equation (33) explicitly shows how the fiscal policies (τ_C, τ_I, g) affects the sustainability of public debt—the critical initial level of public debt-to-GDP ratio that is sustainable φ.

This result is summarized as a theorem.

Theorem 1 *(Critical Initial Level of Public Debt-to-GDP Ratio)*
The initial level of public debt-to-GDP ratio that is sustainable (or that is compatible with a BGP) is given by (33).

4.2 Fiscal Policy Interconnections

Equation (33) explicitly shows how the critical level of sustainable public debt-to-GDP ratio φ depends on fiscal policies (τ_C, τ_I, g). To investigate the interrelationships between fiscal policies and sustainable public debt, the paper derives the marginal effects of fiscal policies on φ. Elementary calculations from (33) yield

$$\frac{\partial \varphi}{\partial \tau_C} = \frac{(1-g) AL^{1-\alpha} - (1-\tau_I)\alpha AL^{1-\alpha} + \rho}{AL^{1-\alpha}\rho}, \tag{34}$$

$$\frac{\partial \varphi}{\partial \tau_I} = \frac{\alpha\tau_C + 1}{\rho} > 0, \tag{35}$$

$$\frac{\partial \varphi}{\partial g} = -\frac{\tau_C + 1}{\rho} < 0. \tag{36}$$

It is obvious that $\partial\varphi/\partial\tau_I > 0$ and $\partial\varphi/\partial g < 0$. Further, it is easily ascertained that under Assumption 1, the sign of $\partial\varphi/\partial\tau_C$ is positive. The signs of the derivatives $\partial\varphi/\partial\tau_C > 0$, $\partial\varphi/\partial\tau_I > 0$ and $\partial\varphi/\partial g < 0$ are plausible from an economics perspective. For example, $\partial\varphi/\partial\tau_I > 0$ implies that if the initial level of public debt-to-GDP ratio is high, then the government must set a high level of income tax rate to maintain the sustainability of public debt given the other parameters.

To study the interconnections between fiscal policies for the sustainability of public debt, the following relationships are derived by partially differentiating (34)-(36):

$$\frac{\partial^2\varphi}{\partial\tau_C\partial g} = -\frac{1}{\rho} < 0, \tag{37}$$

$$\frac{\partial^2\varphi}{\partial\tau_C\partial\tau_I} = \frac{\alpha}{\rho} > 0, \tag{38}$$

$$\frac{\partial^2\varphi}{\partial\tau_I\partial g} = 0. \tag{39}$$

Economic Implications.

What are the economic implications of equations (37)-(39)? First, for equation (37), consider a situation in which the government cuts public spending g. Equation (37) shows that in this situation there is an increase in the marginal effect of the consumption tax rate on the sustainability of public debt $\partial\varphi/\partial\tau_C$. That is, the marginal effects of raising the consumption tax rate to raise the critical level of the public debt-to-GDP ratio that is compatible with BGP increases as public spending decreases. Why does this phenomenon occur? The depression of public spending stimulates the consumption of households; this effect is captured by equation (22) (see Remark 2). As a result, the government revenue from the consumption taxation increases. Thus, the marginal effects of the consumption tax rate for the sustainability of public debt increases.

Second, consider equation (38). As noted in Remark 3, the income tax rate affects the consumption path. This drives an interaction between consumption tax and income tax for achieving the sustainability of public debt. Equation (38) captures this aspect: it shows that income tax always increases the marginal effects of the consumption tax rate for the sustainability of public debt. Contrastingly, equation (39) shows no such cross-effects between public spending and income tax in the present setting, that is, the cross derivative $\partial^2\varphi/\partial\tau_I\partial g$ is 0.

Third, consider (37) and (39). Equation (37) means that the marginal effect that reducing public spending has on the sustainability of public debt

$\partial\varphi/\partial g$ is a monotone decreasing function of the consumption tax rate τ_C. That is, an increase in a consumption tax rate enhances the marginal effect of reducing public spending to maintain the sustainable public debt-to-GDP ratio φ, while equation (39) shows that increasing the income tax rate has no such effect. Accordingly, increasing the consumption tax rate is a better strategy than increasing the income tax rate to improve a government's fiscal health.

Finally, consider (38) and (39). Equation (38) implies that the marginal effect that increasing the income tax rate has on the sustainability of public debt $\partial\varphi/\partial\tau_I$ is a monotone increasing function of the consumption tax rate τ_C. That is, an increase in a consumption tax rate enhances the marginal effect of increasing the income tax rate to maintain the sustainable public debt-to-GDP ratio φ, while equation (39) shows that reducing public spending has no such effect. Accordingly, increasing the consumption tax rate is a better strategy than decreasing public spending to improve a government's fiscal status.

These results are summarized as the following two theorems.

Theorem 2 *(Cross-Effects of Fiscal Policies)*
The marginal effect of the consumption tax rate on the sustainability of public debt increases as a government decreases public spending or as they increase the income tax rate. The marginal effect of income tax on the sustainability of public debt does not interconnect with government spending.

Theorem 3 *(Advantages of Consumption Taxation)*
If a government increases the consumption tax rate, then there is an increase in the marginal effects of raising the income tax rate, and of reducing public spending, to increase the critical level of the public debt-to-GDP ratio.

5 Concluding Remarks

This paper explicitly derives the upper bound of public debt that is compatible with a BGP in an AK-type growth model. Further, the paper examines how a government's various fiscal policies are interconnected with regard to the sustainability of public debt. The paper finds that as public spending decreases, or as the income tax rate increases, so there is an increase in the marginal contribution of the consumption tax rate on the sustainability of public debt. The paper also reveals that consumption taxation enhances the marginal effects of increasing the income tax rate and of decreasing public spending, which may be thought of as an advantage of consumption taxation. These results imply that a government should consider its public policies as an integrated module, rather than as stand-alone policy instruments, to effectively maintain a sustainable level of public debt.

The study results are relevant to economists and to policy makers in countries that are experiencing, or that are scheduling, fiscal reforms under situations with high levels of accumulated public debt. However, the model used is somewhat simple. For in-depth policy insights, the following extensions should be implemented in possible future studies.

This paper assumes that the population is constant over time. The analyzes were considerably simplified because of this assumption; however, it is desirable to take a population decline into account given the declining birth rates observed in many developed countries. How do the population movements influence the public debt-to-GDP ratio that is compatible with the sustainability of public debt? This presents an interesting research topic. Another direction for further research is taking unemployment and wage rigidity into account. How the tax rates and/or public spending co-work on the sustainability of public debt under situations in which public spending stimulates a firm's employment activities is an interesting research topic.

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