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Information Exchanges among Firms and Their Welfare Implications (Part Ⅱ): Alternative Duopoly Models with Different Types of Risks

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Information Exchanges among Firms and Their Welfare Implications (Part Ⅱ): Alternative Duopoly Models with Different Types of Risks

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Abstract. The purpose of this paper is to overview and evaluate the problem of information exchanges in oligopoly, an important topic in contemporary economics. It is intended as a synthesis of the two streams of economic theories, the economics of imperfect competition and the economics of risk and information.

This long series of papers consist of three parts. The previous paper, which dealt with Part I, discussed the dual relations between the Cournot and Bertrand duopoly models in the absence of risk.

This paper turns to Part II, focusing on many duopoly models in which a common risk is present. The starting point of discussion is the Cournot duopoly model with an industry-wide common demand risk. Many other duopoly models such as the Cournot duopoly with cost risk and the Bertrand duopoly with demand or cost risk are successively discussed. It will be seen that the existence of various risk factors and the informational exchanges between Cournot or Bertrand firms influence the welfare implications on consumers and the society in many complicated ways.

The next paper which deals with Part III will be concerned with more complicated problems such as private risks and/or oligopoly models.

Keywords Duopoly・Cournot・Bertrand・ common risk・information exchanges

1. Introduction

This paper explores the working and performance of a Cournot duopoly model when the firms face a common demand risk. It will serve as a starting point for our later discussions of all types of oligopoly models under conditions of various risks. William Shakespeare (1564-1616), a great English dramatist, once remarked: "All's well that ends well." There should be no objections against such a maxim. We wish to add that another maxim is also valid: "All's well that begins well."

In historical perspective, the problem of information exchanges in oligopoly was initiated with this simple type of duopoly by Basar and Ho (1974) and Ponnsard (1979a), and later developed by Novshek and Sonnenschein (1982), Clark (1983b) and Sakai (1984a) and many others. While they all assumed that goods are just homogeneous (namely, $\theta = 1$), Vives (1984) extended their results to cover the more general case of product differentiation (i.e., $-1 \leqq \theta \leqq 1$). Moreover, there is a growing body of works in the 1990s and even the 2000s. [1)]

This long series of papers consist of three parts. The last paper, namely Sakai (2016), which corresponded to Part I, dealt with alternative models of oligopoly in the absence of risks. We focused on the nice relations between the Cournot duopoly model with output strategies and the Bertrand duopoly model with price strategies. More specifically, the Cournot model where goods are substitutable (or complementary) and the Bertrand model where goods are complements (or substitutes) are really dual in the following sense: The welfare results obtained in one system may nicely be applicable to those in the other system. In that paper, we also made introductory remarks on alternative oligopoly models under demand or cost risks.

This paper turns to Part II, which systematically discusses many duopoly models in which a common risk is present. As a starting point of discussions, we pick up the Cournot duopoly model with a common demand risk. To this end, we newly introduce the two different effects, namely the variation and efficiency effects. Those effects are quite useful in analyzing the issue of information sharing in oligopoly theory. We are also concerned with other duopoly models such as the Cournot duopoly cost risk as well as the Bertrand duopoly with demand or cost risk .

Part III, which is the target of the next paper, will aim to first discuss many duopoly models with private risks, and later extend the welfare results obtained for those duopoly models to the very general case of oligopoly where there are any finite number of firms. Let us increase the number of firms from 2 to 3, 4, ..., n. Then it is expected that the welfare of "consumers as outsiders" may increase as the number of "firms as insiders" increases. Such a kind of "spill-over effect" will have very interesting reactions. Besides, some policy implications of information sharing among firms will also be our consistent concern.

1.1 Four Information Structures : Game-Theoretic Interpretations

There are two Cournot-type of firms —— firm 1 and firm 2. We assume that each firm is confronted with the common demand risk which is indicated by the value of the demand parameter α. It must determine the optimal level of output on an *ex ante* basis, namely on the basis of its estimate of α.

In line with Marschak and Radner (1972), we will find it useful to represent the information structure as a vector $\eta = [\eta_1, \eta_2]$ in the following manner: For each *i,*

η_i = 1 if firm i is informed of the realized value of α,
η_i = 0 if it is not so informed.

Note that η_i takes on either 1 or 0. Therefore there exist four information structures conceivable:

(i) $\eta^O = [0,0]$: Neither firm 1 nor firm 2 has information about α.
(ii) $\eta^{N1} = [1,0]$: Firm 1 is informed of α, but firm 2 remains to be ignorant.
(iii) $\eta^{N2} = [0,1]$: In contrast to (ii), only firm 2 is informed of α.
(iv) $\eta^S = [1,1]$: The two firms agree to share information about α, so that both of them are well informed of α.

As later discussions will show, it is convenient to treat $\eta^O = [0,0]$ as a reference point. We may order these four information structures by means of "fineness." Let us take a look at Fig. 1. For any two information structures, the notation "$\eta^A \Rightarrow \eta^B$" means that "$\eta^B$ is finer than η^A." As can easily be seen, $\eta^S = [1,1]$ is finer than $\eta^{N1} = [1,0]$ or $\eta^{N2} = [0,1]$, each of which in turn is finer than $\eta^O = [0,0]$. However , $\eta^{N1} = [1,0]$ and $\eta^{N2} = [0,1]$ are not comparable by fineness. In this paper, we are especially interested in comparing the two structures, $\eta^{N1} = [1,0]$ and $\eta^S = [1,1]$.

Regarding those four information structures aforementioned, we will find it quite convenient to give game-theoretic interpretations. More specifically, the extensive forms of games will be very instructive in understanding the similarities and

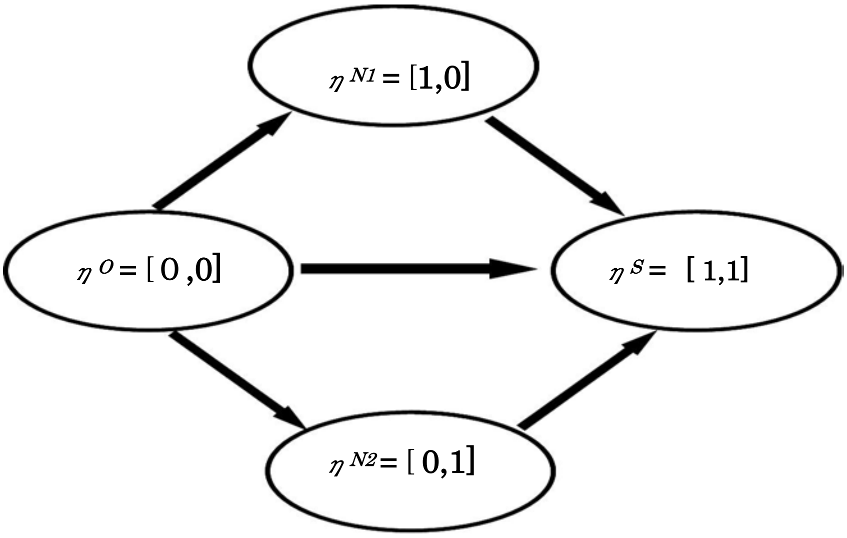
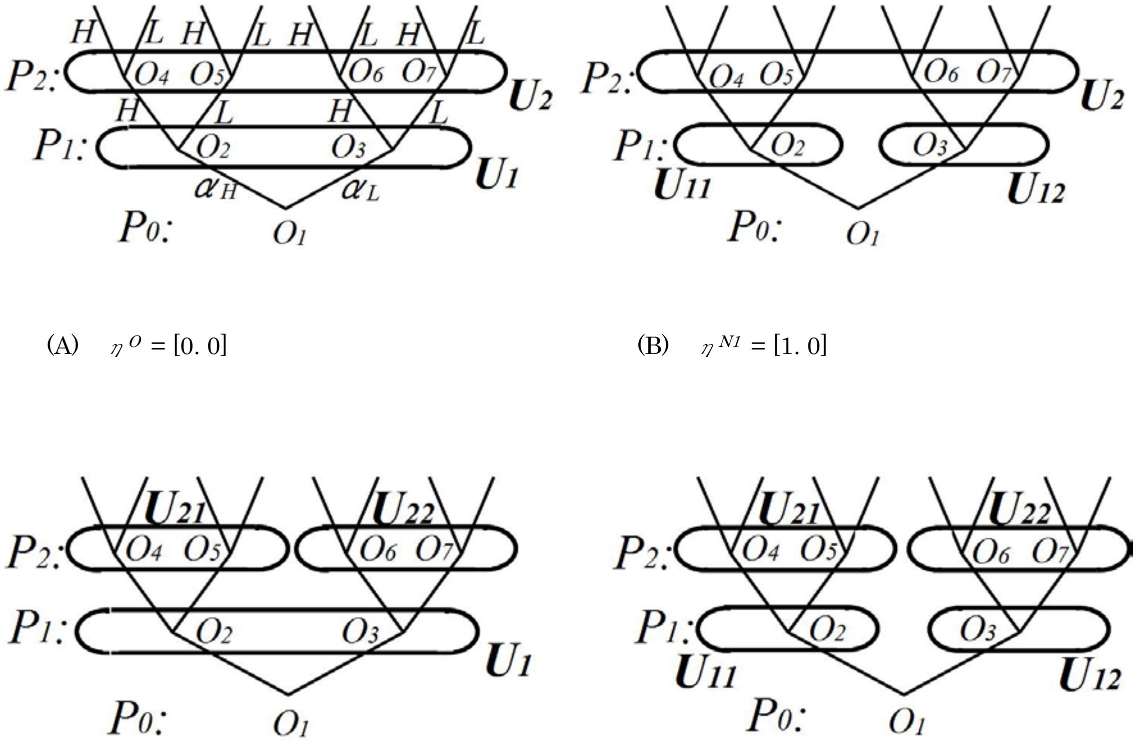


Fig. 1　The ordering of information structures by means of fineness

differences among the information systems.

In Fig. 2, Chart (A) illustrates the extensive form of the Cournot duopoly game with no information, η^{O}. The point P_O is regarded as the "Nature" who behaves like a person and selects, from the start point O_1, two alternatives (α_H or α_L) with certain combinations. A simple case for this situation would be the case in which they are evenly distributed: $Prob(\alpha_H) = Prob(\alpha_L) = 1/2$. The point P_1 indicates player 1 (namely, firm 1) and P_2 player 2 (namely, firm 2). Assume that the two players, P_1 and P_2 , must choose either a high (H) or low (L) level of output. Since P_1 does not know α in advance, it cannot distinguish between the points O_2 and O_3, whence these two points belong to the same information set U_1. In a similar fashion, since P_2 is not informed of α, the four points from O_4 through O_7 belong to the same information set U_2.

The Charts (B) and (C) respectively correspond to the Cournot duopoly games under non-symmetric information, η^{N1} and η^{N2}. The case in which only P_1 has information about α is depicted in Chart (B). Such a case is radically different from the previous case of no information, η^{O}. Since P_1 can now distinguish between O_2 and O_3, its information structure comprises the two information sets: $U_{11} = \{O_2\}$ and $U_{12} = \{O_3\}$. However, because no information is available to P_2, its information structure continues to contain only one information set: $U_2 = \{O_4, O_5, O_6, O_7\}$.



(A) $\eta^{O} = [0.\ 0]$ (B) $\eta^{N1} = [1.\ 0]$

(C) $\eta^{N2} = [0.\ 1]$ (D) $\eta^{S} = [1.\ 1]$

Fig. 2 Cournot Duopoly with a Common Demand Risk:
Extensive Game Presentations

In contrast to this, the case where only P_2 is informed of α is shown in Chart (C). Since P_1 is now ignorant, the two points O_2 and O_3 belong to the same information

structure U_1: $U_1 = \{O_2, O_3\}$.

Chart (D) represents the Cournot duopoly game under shared information, η^S . For the one hand, because P_1 is informed of α, it can distinguish the two points O_2 and O_3. As a result, there exist the two information sets like Chart (B): $U_{11} = \{O_2\}$ and $U_{12} = \{O_3\}$. For the other hand, since P_2 is informed of α, the number of information structures is also two like Panel (C): $U_{21} = \{O_4, O_5\}$ and $U_{22} = \{O_6, O_7\}$. It should be noted here that the duopoly game we are dealing with is a sort of simultaneous game in the sense that both players make their moves simultaneously. In other words, unlike a sequential game with the first and second movers being present, P_1 cannot still distinguish the points O_4 and O_5, nor P_2 the points O_6 and O_7 . It is also worthy of attention that the game under shared information is a refinement of the game under non-symmetric information. [2)]

In this paper, we are particularly eager to pick up and compare the two cases represented by Charts (B) and (D). Such a comparison enables us to systematically analyze how and to what extent the information transmission from one player (namely, one firm in the duopoly game) to the other player (i.e., the other firm) affects the welfare of each player, and the one of a third party such as consumers, as well as the welfare of the whole society. [3)]

1.2 The Cournot Equilibriums under Different Information Structures

The equilibrium concept we are going to employ throughout this paper is the Cournot equilibrium, which can be regarded as a predecessor of Nash equilibrium. There are essentially the two types of information structures: A pair of symmetrical cases, $\eta^0 = [0,0]$ and $\eta^S = [1,1]$, and another pair of non-symmetrical cases, $\eta^{N1} = [1,0]$. and $\eta^{N2} = [0,1]$. While the former cases are easier to handle, the latter cases require a special care for computation. [4)]

First of all, given $\eta^0 = [0,0]$, we say that the pair (x_1^0, x_2^0) of output strategies is an equilibrium pair under η^0 if the following equations hold:

$$x_1^0 = \text{Arg} \quad \text{Max}_{x_1} \; E_\alpha \left[\Pi_1 \left(x_1, x_2^0, \alpha \right) \right] ,$$
$$x_2^0 = \text{Arg} \quad \text{Max}_{x_2} \; E_\alpha \left[\Pi_2 \left(x_1^0, x_2, \alpha \right) \right] .$$

Therefore, when an equilibrium is reached, no firm has an incentive to deviate from it. In order to find the concrete values of $Ex_1 = x_1^0$ and $Ex_2 = x_2^0$, we first note that

$$\Pi_1 = (p_1 - \kappa_1)\, x_1,$$

where $p_1 = \alpha - \beta x_1 - \beta\theta x_2$. By maximizing

$$E\Pi_1 = E \left\{ \left[(\alpha - \kappa_1) - \beta x_1 - \beta\theta x_2 \right] x_1 \right\}$$

with respect to x_1, we find that

$$2\beta x_1^0 + \beta\theta x_2^0 = \mu - \kappa_1. \tag{1}$$

. In a similar fashion, if we maximize $E\Pi_2 = [(\alpha - \kappa_2) - \beta x_2 - \beta\theta x_1]\, x_2$ with respect to x_2, we obtain

$$2\beta x_2^0 + \beta\theta x_1^0 = \mu - \kappa_2. \tag{2}$$

The equations (1) and (2) can be combined in matrix notation:

$$\beta \begin{bmatrix} 2 & \theta \\ \theta & 2 \end{bmatrix} \begin{bmatrix} x_1^{\ 0} \\ x_2^{\ 0} \end{bmatrix} = \begin{bmatrix} \mu - \kappa_1 \\ \mu - \kappa_2 \end{bmatrix}. \tag{3}$$

Solving for x_1^0 and x_2^0, we obtain

$$x_1^0 = [\mu(2-\theta) - 2\kappa_1 + \theta\kappa_2] / \beta(4 - \theta^2), \tag{4}$$
$$x_2^0 = [\mu(2-\theta) - 2\kappa_2 + \theta\kappa_1] / \beta(4 - \theta^2). \tag{5}$$

Once a firm acquires information about α, its strategy becomes a contingent action, meaning that its output strategy now depends on the realized value of α. So given $\eta^{N1} = [1,0]$, the pair $(x_1^{N1}(\alpha), x_2^{N1})$ is called an equilibrium under η^{N1} if the following conditions are met:

$$x_1^{N1}(\alpha) = \text{Arg Max}_{x_1}\ \Pi_1(x_1, x_2^{N1}, \alpha) \quad \text{for any given } \alpha,$$
$$x_2^{N1} = \text{Arg Max}_{x_2}\ E_\alpha [\Pi_2(x_1^{N1}(\alpha), x_2, \alpha)].$$

This non-symmetrical case where only firm has information about α requires a special care for handling. Since we have $\Pi_1 = (p_1 - \kappa_1)\, x_1$, where $p_1 = \alpha - \beta x_1 - \beta\theta x_2$, maximization of Π_1 with respect to x_1 results in

$$2\beta x_1^{N1}(\alpha) + \beta\theta x_2^{N1} = \alpha - \kappa_1. \tag{6}$$

Remarkably, the term $x_1^{N1}(\alpha)$ in the above equation shows that the equilibrium value of x_1 depends on each α. In other words, firm 1 as an informed player must take a contingent action with the contingency related to the value of α.

In contrast to firm 1, firm 2 is an ignorant player whose action should not be contingent but rather routine. If we note that $E\Pi_2 = [(\alpha-\kappa_2) - \beta x_2 - \beta\theta x_1]x_2$, its maximization with respect to x_2 yields

$$2\beta x_2^{N1} + \beta\theta\ Ex_1^{N1}(\alpha) = \mu - \kappa_2. \tag{7}$$

Here again we note that the first variable, x_1, is a function of the demand parameter α under η^{N1}. We need to do a small trick to solve for $(x_1^{N1}(\alpha), x_2^{N1})$ from the two equations, (6) and (7). To this end, let us take expectations of both sides of (6). Then we have

$$2\beta Ex_1^{N1}(\alpha) + \beta\theta x_2^{N1} = \mu - \kappa_1. \tag{8}$$

We can easily combine (8) with (7) in matrix notation:

$$\beta\begin{bmatrix} 2 & \theta \\ \theta & 2 \end{bmatrix}\begin{bmatrix} Ex_1^{N1}(\alpha) \\ x_2^{N1} \end{bmatrix} = \begin{bmatrix} \mu-\kappa_1 \\ \mu-\kappa_2 \end{bmatrix}. \tag{9}$$

Comparison of (9) and (3) enables us to obtain $Ex_1^{N1}(\alpha) = x_1^0$ and $x_2^{N1} = x_2^0$. If we take care of (6) and (8), then it is not a difficult job to derive

$$2\beta\ x_1^{N1}(\alpha) - 2\beta Ex_1^{N1}(\alpha) = \alpha - \mu.$$

It immediately follows from this equation that

$$\begin{aligned} x_1^N(\alpha) &= Ex_1^{N1}(\alpha) + (\alpha-\mu)/2\beta \\ &= x_1^0 + (\alpha-\mu)/2\beta. \end{aligned} \tag{10}$$

When firm 1 decides to reveal its information to firm 2, the latter firm 's strategy becomes a contingent action as well. Therefore, given $\eta^S = [1,1]$, the pair $(x_1^S(\alpha), x_2^S(\alpha))$ is named an equilibrium under η^s if the following conditions are satisfied:

$$x_1^S(\alpha) = \text{Arg Max}_{x_1}\ \Pi_1\ (x_1, x_2^S(\alpha), \alpha) \qquad \text{for any given } \alpha,$$
$$x_2^S(\alpha) = \text{Arg Max}_{x_2}\ \Pi_2\ (x_1^S(\alpha), x_2, \alpha) \qquad \text{for any given } \alpha.$$

In this symmetric case, η^S, we can derive the equilibrium values of x_1 and x_2

Table 1 Equilibrium Output Strategies under η^O, η^{N1} and η^S: the Cournot Duopoly with a Common Demand Risk:

Information structures	Equilibrium output strategies	
	x_1	x_2
$\eta^0=[0,0]$	$x_1{}^0$	$x_2{}^0$
$\eta^{N1}=[1,0]$	$x_1{}^0+\dfrac{\alpha-\mu}{2\beta}$	$x_2{}^0$
$\eta^{NS}=[1,1]$	$x_1{}^0+\dfrac{\alpha-\mu}{\beta(2+\theta)}$	$x_2{}^0+\dfrac{\alpha-\mu}{\beta(2+\theta)}$

Remark :

$$x_1{}^0=\frac{\mu(2-\theta)-2\kappa_1+\theta\ \kappa_2}{\beta(4-\theta^2)},\quad x_2{}^0=\frac{\mu(2-\theta)-2\kappa_2+\theta\ \kappa_1}{\beta(4-\theta^2)},$$

where μ denotes the expected value of α, namely $\mu=E\alpha$.

in a way similar to another symmetric case, η^0. What we have to do is to simply replace μ by α. So we now have the following set of equations in matrix form:

$$\beta\begin{bmatrix}2 & \theta\\ \theta & 2\end{bmatrix}\begin{bmatrix}x_1{}^S(\alpha)\\ x_2{}^S(\alpha)\end{bmatrix}=\begin{bmatrix}\alpha-\kappa_1\\ \alpha-\kappa_2\end{bmatrix}\qquad . \tag{11}$$

Solving for $x_1{}^S(\alpha)$ and $x_2{}^S(\alpha)$, we find that

$$x_1{}^S(\alpha)=[\mu(2-\theta)-2\kappa_1+\theta\kappa_2]\,/\,\beta(4-\theta^2)\ =\ x_1{}^0\ +\ (\alpha-\mu)/\beta(2+\theta)\ , \tag{12}$$

$$x_2{}^S(\alpha) = [\mu(2-\theta)-2\kappa_2 + \theta\kappa_1] / \beta(4-\theta^2) = x_2{}^0 + (\alpha-\mu)/\beta(2+\theta) . \quad (13)$$

All the computational results derived above can systematically be summarized in Table 1. As may naturally be expected, the output pair ($x_1{}^0$, $x_2{}^0$) serve as a reference point for all the Cournot equilibriums: indeed, the whole analytical structure have been built on the base of no information, η^0. All's well that begins well!

1.3 Welfare Formulas

The purpose of this subsection is to compute and compare the following set of equilibrium values under alternative information structures.

Π_i : firm i's expected profit ($i = 1, 2$),
EPS: expected producer surplus,
ECS: expected consumer surplus,
ETS: expected total surplus.

In order to carry out such a hard task, it is quite useful to make use of a group of welfare formulas. To this end, we have to newly invent a set of small parts, the combinations of which will later lead to a big architecture.

More specifically, we are interested in comparing the equilibrium values under non-shared information, η^{N1}, and those under shared information, η^S. It is expected that such comparison is quite helpful in analyzing the welfare effects of an information transmission agreement on an *ex ante* basis if the timing structures of the two firms are to be carried out in the following four stages.

(i) At the first stage, both firms have the opportunity to make a certain *ex ante* agreement concerning the transmission of demand information from one firm to the other. Such an agreement can be made either by a binding contract or through a third independent agency such that a trade association.

(ii) At the second stage, firm 1 observes the realized value of a random demand parameter, α, whereas firm 2 remains to be ignorant.

(iii) Then at the third stage, firm 1 transmits its information to firm 2 according to the *ex ante* agreement made at the first stage. Garbling or cheating on the part of the informed firm (i.e., firm 1) is not permitted. In other words, both firms are supposed to be honest players in the information exchange agreement: implementation of the agreement must be done correctly and thoroughly.

(iv) At the fourth and final stage, each firm makes its production decision, thus selecting the optical level of its own output.

Now, let us try to express all the relevant welfare quantities in statistical terms, more exactly, in terms of variances and co-variances relative to strategic variables and stochastic parameters. Recalling that $\Pi_i = (p_i - \kappa_i)\, x_i$ by definition, it is not difficult to show that for $i = 1,2$, the equilibrium value of firm i' s expected profit is provided by

$$
\begin{aligned}
E\Pi_i &= E[(p_i - \kappa_i)x_i] \\
&= E(p_i - \kappa_i)\, E(x_i) + Cov(p_i - \kappa_i\, , x_i) \\
&= E\Pi_i^O + Cov(p_i\, , x_i)\, , \qquad (14)
\end{aligned}
$$

where $E\Pi_i^O = (E(p_i) - \kappa_i)\ E(x_i)$. Because expected total surplus is the sum of expected profits across firms, it is given by

$$EPS = EPS^O + \Sigma_i\ \mathrm{Cov}(p_i\, , x_i)\, , \qquad (15)$$

where $EPS^O = \Sigma_i\, E\Pi_i^O$.

In the case of a common demand risk (α), consumer surplus is simply given by

$$CS = (1/2)\, \Sigma_i\, (\alpha - p_i)\, x_i\ .$$

If we take the expectation of both sides of this equation, we obtain

$$
\begin{aligned}
ECS &= (1/2)\ \Sigma_i\, E(\alpha\, x_i) - (1/2)\ \Sigma_i\, E(p_i\, x_i) \\
&= (1/2)\ \Sigma_i \{E(\alpha)\ E(x_i) + Cov(\alpha\, , x_i)\} \\
&\quad - (1/2)\ \Sigma_i \{ E(p_i)E(x_i) + Cov(p_i\, , x_i)\} \\
&= ECS^O - (1/2)\ \Sigma_i\, Cov(p_i\, , x_i) + (1/2)\, \Sigma_i\, Cov(\alpha\, , x_i)\, , \qquad (16)
\end{aligned}
$$

where $ECS^O = (1/2)\ \Sigma_i\ \{E(\alpha) - E(p_i)\}\ E(x_i)$.

The welfare level of the whole society can be measured by expected total surplus. Since it is the sum of expected producer and consumer surpluses, it is provided by

$$
\begin{aligned}
ETS &= EPS + ECS \\
&= EPS^O + \Sigma_i\, Cov(p_i\, , x_i) \\
&\quad + ECS^O - (1/2)\, \Sigma_i\, Cov(p_i\, , x_i) + (1/2)\, \Sigma_i\, Cov(\alpha\, , x_i)
\end{aligned}
$$

$$= \quad ETS^O + \ (1/2)\,\Sigma_i\, Cov(p_i\,,\,x_i) + (1/2)\,\Sigma_i\, Cov(\alpha\,,\,x_i) \tag{17}$$

Now, let us break up the term $(Cov(p_i\,,\,x_i))$ into several parts. Since $p_i = \alpha - \beta x_i - \beta\theta x_j$ $(i, j = 1,2;\ i \neq j)$, it follows that

$$\begin{aligned} Cov(p_i\,,\,x_i) &= Cov(\alpha - \beta x_i - \beta\theta x_j,\quad x_i\ ) \\ &= -\beta\, Var(x_i) - \beta\theta\, Cov(x_1, x_2) + Cov(\alpha, x_i) \qquad (18) \\ &\qquad (i, j = 1,2;\ i \neq j) \end{aligned}$$

Consequently, by inserting (18) into (14), (15), (16) and (17), we can obtain the following set of welfare formulas:

$$E\Pi_i \ = \ E\Pi_i{}^O + \ -\beta\, Var(x_i) - \beta\theta\, Cov(x_1, x_2) + Cov(\alpha, x_i) \tag{19}$$

$$\begin{aligned} EPS &= EPS^O + \ \Sigma_i\ \{-\beta\, Var(x_i) - \beta\theta\, Cov(x_1\,, x_2) + Cov(\alpha, x_i)\} \\ &= EPS^O \ -\beta\,\Sigma_i\, Var(x_i) - 2\beta\theta\, Cov(x_1, x_2) + \ \Sigma_i\, Cov(\alpha, x_i), \end{aligned} \tag{20}$$

$$\begin{aligned} ECS &= ECS^O - (1/2)\ \ \Sigma_i\{-\beta\, Var(x_i) - \beta\theta\, Cov(x_1\,, x_2) + Cov(\alpha, x_i)\} \\ &\quad + (1/2)\,\Sigma_i\, Cov(\alpha\,,\,x_i) \\ &= ECS^O + (\beta/2)\ \ \Sigma_i\, Var(x_i) + \beta\theta\, Cov(x_1\,, x_2)\,, \end{aligned} \tag{21}$$

$$ETS \ = ETS^O - \ \ (\beta/2)\ \ \Sigma_i\, Var(x_i) - \beta\theta\, Cov(x_1\,, x_2) + \Sigma_i\, Cov(\alpha, x_i). \tag{22}$$

These formulas teach us that the relative strength of the following four component parts play a critical role in evaluating the welfare of producers, consumers, and the whole society: (i) $Var(x_i)$, (ii) $Cov(x_1, x_2)$, (iii) $Cov(\alpha\,,\,x_i)$, and (iv) θ.

If we compare (18) and (19), then we immediately see that an increase in the variance of each output affects the welfare of producers and the one of consumers in opposite directions: increased variability of each output, *cetris paribus*, makes producers worse off but consumers better off. This is due to the fact that firm 1's profit is a concave function of x_i ,and producer surplus a concave function of x_1 and x_2, but that consumer surplus a convex function of x_1 and x_2.

The sign and value of θ is very important, and plays a critical part in understanding of the welfare implications of our oligopoly models with risks. For the one hand, it measures the degree of technical substitutability or complementary relationship between the two goods, x_1 and x_2. For the other hand, it also demonstrates how the demands for these two goods are stochastically correlated.

If x_1 and x_2 are substitutes (or complements) then firms' reaction curves are negatively (or positively) sloping, so that the value of $Cov(x_1, x_2)$ must be negative (or positive). Therefore, the quantity $(-\theta\ Cov\ (x_1,\ x_2\ ))$ can measure the degree of combined interaction between x_1 and x_2, taking account of both physical and stochastic interaction. As can naturally be expected, the greater the value of this quantity, the more advantageous is the position of "producers as insiders" ,and the more disadvantages is the position of "consumers as outsiders." [5)]

So far, we have intensively discussed how the variability of each firm's strategic variable or the interaction between the two strategic variables influences the welfare of producers, consumers, and the whole society. This effect may be called the *variation effect*.

There is another sort of effect, however. Such a new effect is represented by the value of $Cov(\alpha, x_i)$, which shows how and to what extent the value of stochastic parameter α and the value of each strategic variable x_i are correlated. The better the correspondence between these values, the greater is the welfare of producers. This effect can be named the *efficiency effect*. It is noted that consumers are not directly affected by this effect although they could indirectly be affected via corresponding changes in x_1 and x_2.

These two effects — the variation and efficiency effects — might appear to be somehow interlocked, but must be separated for an exact and detailed investigation. Only after reasonable separations of things at an early stage, a full unification at a later stage will be feasible and truly effective! [6)]

1-4 The Impact of Informational Transmission on Various Welfare Components

We are now in a position to compare the non-shared information equilibrium (with only firm 1 being informed) and the shared information equilibrium on an *ex ante* basis. Suppose that the two firms make an arrangement of information transfer from firm 1 to firm 2 before the market demand is realized. The question of interest is how much and in what direction such an arrangement contributes to the welfare of producers, consumers, and the whole society. We assume here that each firm truthfully reveals its information by a binding contract or an unwritten rule, thus ignoring the problem of possible garbing and information manipulation. [7)]

As was shown above, there are several component parts which enter into formulas for each firm's expected profit, expected producer surplus, expected consumer surplus, and expected total surplus. So it would be a very good idea to separately analyze the

Table 2 The Equilibrium Values of Variation and Efficiency Components:
The Cournot Duopoly with a Common Demand Risk (α)

η	Own Variation			Cross Variation	Efficiency		
	V1	V2	V1+V2	CV	E1	E2	E1+E2
η^{N1}	$\frac{\sigma^2}{4\beta^2}$	0	$\frac{\sigma^2}{4\beta^2}$	0	$\frac{\sigma^2}{2\beta^2}$	0	$\frac{\sigma^2}{2\beta^2}$
η^{S}	$\frac{\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{2\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\theta\,\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\sigma^2}{\beta(2+\theta)}$	$\frac{\sigma^2}{\beta(2+\theta)}$	$\frac{2\sigma^2}{\beta(2+\theta)}$
$\eta^{S}-\eta^{N1}$	$\frac{-\theta(4+\theta)\sigma^2}{4\beta^2(2+\theta)^2}$	$\frac{\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\sigma^2\phi(\theta)}{4\beta^2(2+\theta)^2}$	$\frac{\theta\,\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{-\theta\,\sigma^2}{2\beta(2+\theta)}$	$\frac{\sigma^2}{\beta(2+\theta)}$	$\frac{\sigma^2(2-\theta)}{2\beta(2+\theta)}$

Remark. V1 = $Var(x_1)$, V2 = $Var(x_2)$, CV = $\theta\, Cov(x_1, x_2)$, E1 = $Cov(\alpha, x_1)$,
E2 = $Cov(\alpha, x_2)$,; $\phi(\theta) = 4 - 4\theta - \theta^2$. .

impact of information transmission on each of the components and then unite them together rather than to merely gloss over such impact on the whole entity. Taking advantage of Table 1, we can easily make such computations. Table 2 demonstrates the results obtained for the two information structures, η^{N1} and η^{S}.

With regard to Table 2, it is noted that the following notations are employed for the sake of simplification:

V1 = $Var(x_1)$ = the variance of x_1 ,

V2 = $Var(x_2)$ = the variance of x_2, ,

CV = $\theta\, Cov(x_1, x_2)$ = the product of the substitution coefficient θ and the covariance of x_1 and x_2 ,

E1 = $Cov(\alpha, x_1)$ = the covariance of α and x_1 , ,

E2 = $Cov(\alpha, x_2)$ = the covariance of α and x_2 .

Let us pay attention to the values in the last row starting with the difference term ($\eta^S - \eta^{N1}$). They clearly indicate exactly how the information transmission from firm 1 to firm 2 affects each welfare component.

(i) Such transmission decreases (or increases) the variability of x_1 if goods are substitutes (or complements), whereas it does increase the variability of x_2 regardless of the degree of technical substitutability between x_1 and x_2.

(ii) As can naturally be expected, it tends to reinforce the degree of interaction between the output strategies of the two firms, which is represented by the difference ($Cov(x_1^S, x_2^S) - Cov(x_1^{N1}, x_2^{N1})$).

(iii) Whereas it decreases (or increases) the covariance of α and x_1 whenever goods are substitutes (or complements), it always increases the covariance of α and x_2. As can be expected, this result has a nice correspondence to (i) above..

1.4. Visual Explanations by Means of Diagrams

We are concerned with the effects of information transmission on various welfare components. The situations we are facing appear rather complicated. As the saying goes, seeing is believing! It is certain that visual explanations by means of diagrams would be a great help. Let us take a close look at Fig. 3.

For simplicity, assume that the common demand intercept (α) can be two equally likely values —a high value (H) or a low value (L). The reaction functions, or the best response functions, are depicted in Fig. 3. If goods are substitutes (or complements) then the reaction lines are negatively (or positively) sloping. Suppose that both firms get information about α. When the demand is high (namely, $\alpha = H$), firm 1's reaction curve for its rival's choice x_2 is shown as R_1^H. It is actually linear since we assume linear demand and constant unit cost. When the demand is low (i.e., $\alpha = L$), firm 1's reaction curve for x^2 is drawn as R_1^L, which lies lower than R_1^H due to a fall in demand. A dotted line R_1^O denotes the average of these two reaction curves for firm 1. In a similar fashion, we can draw the two reaction curves R_2^H and R_2^L together with their average R_2^O for firm 2.

In Fig. 3, we are able to find the Cournot-Nash equilibriums under various information structures. When both firms are ignorant of α, Q^O represents an equilibrium point, with (x_1^O, x_2^O) being the pair of equilibrium output strategies.

When only firm 1 knows α, the equilibrium will be represented by the pair of the two points, Q^{HO} and Q^{LO}, with $(x_1^{HO}\ x_1^{LO};\ x_2^{O})$ being the vector of equilibrium output

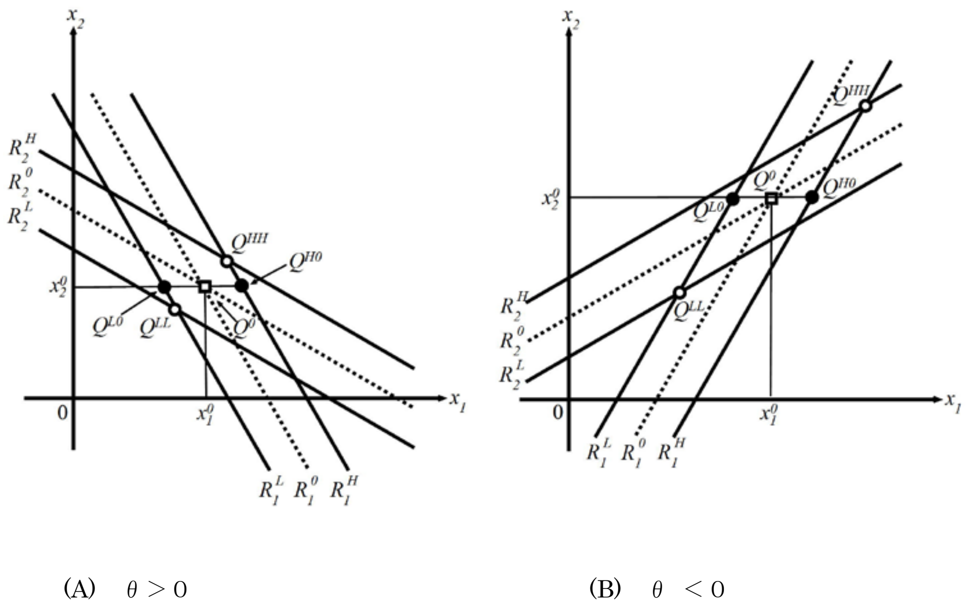


Fig. 3 Graphical illustrations of the Cournot duopoly equilibriums under η^{0}, η^{N1} and η^{S}: The case of a common demand risk

strategies. This is because x_1^{HO} and x_1^{LO} respectively represent firm1's best responses to x_2^{O} for the demands H and L while x_2^{O} remains firm2's best response to the average of these two demand values. In case both firms can know α, the equilibrium will be shown by the pair of the two points, Q^{HH} and Q^{LL}. In this symmetric case, it is quite clear that the vector $(x_1^{HH},\ x_1^{LL};\ x_2^{HH},\ x_2^{LL})$ stands for the equilibrium output strategies of the two firms. 8)

We are ready to see diagrammatically how the information transmission from firm 1 to firm 2 influences various welfare components. First, take a look at Chart (A). In the case of substitutable goods ($\theta > 0$), it is readily seen that Q^{HH} lies west of Q^{HO} and Q^{LL} east of Q^{LO}. As a result, the information transmission makes both $Var(x_1)$ and $Cov(\alpha, x_1)$ smaller. Next, see Chart (B). In the case of complementary goods ($\theta < 0$),

Q^{HH} lies east of Q^{HO} and Q^{LL} west of Q^{LO}, so that the information transmission makes both $Var(x_1)$ and $Cov(\alpha, x_1)$ larger. Although such a visual approach is quite useful, we must bear in mind its inescapable limitations as well. For instance, by merely looking at Chart (A) only, we cannot determine the sign of the sum term $\Sigma_i\ Var(x_i)$, which comprises one of the key components in the welfare formulas (19)–(22) aforementioned.

1.5 Comparisons between the equilibrium values under non-symmetric information and those under symmetric information

Let us make a sequence of comparisons between the equilibrium values of each firm's profit, producer surplus, consumer surplus, and total surplus under the two information structures: (i) non-symmetric information, η^{N1}, and symmetric information, η^{S}.

To this end, for any arbitrary variable Z, let us denote by ΔZ the difference between the equilibrium value under η^{S} and the one under η^{N1}. Then in the light of (19) − (22), it is a straightforward job to derive the following set of equations:

$$\begin{aligned}
\Delta E\Pi_i &= E\Pi_i{}^{S} - E\Pi_i{}^{N1} \\
&= E\Pi_i{}^{O} + -\beta\, Var(x_i{}^{S}) - \beta\ \theta\ Cov(x_i{}^{S}, x_j{}^{S}) + Cov(\alpha, x_i{}^{S}) \\
&\quad - [\, E\Pi_i{}^{O} + -\beta\, Var(x_i{}^{N1}) - \beta\ \theta\ Cov(x_i{}^{N1}, x_j{}^{N1}) + Cov(\alpha, x_i{}^{N1})\,] \\
&= -\beta\, \Delta\, Var(x_i) - \beta\ \theta\ \Delta\, Cov(x_1, x_2) + \Delta\, Cov(\alpha, x_i),
\end{aligned} \tag{23}$$

$$\begin{aligned}
\Delta EPS &= EPS^{S} - EPS^{N1} \\
&= -\beta\, \Sigma_i \Delta\, Var(x_i) - 2\beta\ \theta\ \Delta\, Cov(x_1, x_2) + \Sigma_i\ \Delta\, Cov(\alpha, x_i),
\end{aligned} \tag{24}$$

$$\begin{aligned}
\Delta ECS &= ECS^{S} - ECS^{N1} \\
&= (\beta/2) \Sigma_i \Delta\, Var(x_i) + \beta\ \theta\ \Delta\, Cov(x_1, x_2),
\end{aligned} \tag{25}$$

$$\begin{aligned}
\Delta ETS &= ETS^{S} - ETS^{N1} \\
&= -(\beta/2) \Sigma_i \Delta\, Var(x_i) - \beta\ \theta\ \Delta\, Cov(x_1, x_2) + \Sigma_i\ \Delta\, Cov(\alpha, x_i)
\end{aligned} \tag{26}$$

The welfare effects of information transmission through variation and efficiency channels are carefully summarized in Table 3. For example, the information transmission from firm 1 to firm 2 leads to a decrease or an increase in $\Sigma_i\ Var(x_i)$ according to whether θ is larger or smaller than θ^*, where θ^* is a larger root x_i of the quadratic equation:

Table 3　The Welfare Impact of Information Transmission through Variation and Efficiency Channels:　The Cournot Duopoly with a Common Demand Risk (α)

The Welfare Impact	Own Variation			Cross Variation	Efficiency			Total
	V1	V2	V1+V2	CV	E1	E2	E1+E2	
	$-(\theta>0)$ $+(\theta<0)$	+	$-(\theta>\theta^*)$ $+(\theta<\theta^*)$	$+(\theta>0)$ $-(\theta<0)$	$-(\theta>0)$ $+(\theta<0)$	+	+	
$\Delta E\Pi_1$	$+(\theta>0)$ $-(\theta<0)$	0		$-(\theta>0)$ $+(\theta<0)$	$-(\theta>0)$ $+(\theta<0)$	0		$-(\theta>0)$ $+(\theta<0)$
$\Delta E\Pi_2$	0	−		$-(\theta>0)$ $+(\theta<0)$	0	+		+
ΔEPS			$+(\theta>\theta^*)$ $-(\theta<\theta^*)$	$-(\theta>0)$ $+(\theta<0)$			+	$-(\theta>\theta^*)$ $+(\theta<\theta^*)$
ΔECS			$-(\theta>\theta^*)$ $+(\theta<\theta^*)$	$+(\theta>0)$ $-(\theta<0)$			0	$+(\theta>-\theta^*)$ $-(\theta<-\theta^*)$
ΔETS			$+(\theta>\theta^*)$ $-(\theta<\theta^*)$	$-(\theta>0)$ $+(\theta<0)$			+	+

Remark.　$\theta^* = 2(\sqrt{2}-1)$ ≒ 0.8284

$$4-4\theta-\theta^2=0,$$

from which we find $\theta^* = 2(\sqrt{2}-1)$ ≒0.8284.

If we observe a mosaic-type diagram enchased with many plus and minus signs in Table 3, we would immediately see that it is no easy job to systematically analyze the welfare effects of the information transmission from firm 1 to firm 2.　First of all, there are various (own and cross) variation and efficiency channels through which such information transmission influences expected profits, expected producer surplus, expected consumer surplus, and expected total surplus.　Besides, in most of these channels, the direction of influence (a positive or negative sign) cannot uniquely be determined, depending on the value of θ.　One of few exceptions for this is the efficiency impact on *EPS* and *ECS*:　Whereas the information transmission contributes positively to *EPS* through the efficiency channel, regardless of the value of θ, there is no efficiency effect present on the part of *ECS.*

Table 4 The Degree of Technical Substitution and the Welfare Impact of Information Transmission: The Cournot Duopoly with a Common Demand Risk (α)

θ	- -	$-\theta^*$	-	0	+	θ^*	+ +
$\Delta E\Pi_1$	+	+	+	0	-	-	-
$\Delta E\Pi_2$	+	+	+	+	+	+	+
ΔEPS	+	+	+	+	+	0	-
ΔECS	-	0	+	+	+	+	+
ΔETS	+	+	+	+	+	+	+

Remark. $\theta^* \fallingdotseq 0.8284$

The last column of Table 4 indicates the *total* welfare impact of information transmission combining variation and efficiency effects: $\Delta ETS = \Delta EPS + \Delta ECS$. There are three critical values of θ for the determination of the total impact: $\theta = -\theta^*, 0, \theta^*$. It is recalled here that θ^* is a larger root of the quadratic equation: $4 - 4\theta - \theta^2 = 0$, so that $\theta^* = 2(\sqrt{2} - 1) \fallingdotseq 0.8284$.

The relationship between the degree of technical substitution and the information transmission from firm 1 to firm 2 may systematically be shown in Table 4.

If we carefully observe Table 4, then we are able to obtain the following welfare results.

(i) If goods are substitutes (namely, $\theta > 0$), then we find $E\Pi_1 < 0$, so that firm 1 does not wish to reveal information. [9] In particular, when goods are strong substitutes and nearly homogeneous (i.e., $\theta > \theta^*$), the loss of firm 1 from the information agreement overpowers the benefit of firm 2, with the result that expected producer surplus must decline: Therefore, $\Delta EPS = EPS^S - EPS^{N1} < 0$. It should also be noted that in this case of strong substitutes , the information transmission surely *increases* *ETS* although it decreases *EPS*. Therefore, when implementing industrial policies for information flows, the government authority should be encouraged to somehow mix them with other supplementary measures.

(ii) In case goods are weak complements (i.e., $-\theta^* < \theta < 0$), we find $\Delta E\Pi_1$, $\Delta E\Pi_2$, and ΔECS are all positive. For this case, the revealing case is Pareto-superior to the non-revealing case. This possibility is clearly indicated by a solid enclosure in Table 4.

(iii) In a wide range of intermediate (either substitutable or complementary) cases in which $-\theta^* < \theta < \theta^*$, we find ΔEPS, ΔECS, and ΔETS all positive ,although $\Delta E\Pi_1$ may be negative for substitutable goods. So if a side payment from firm 2 to firm 1 is permitted and really carried out, then the information agreement between the two firms can increase the welfare of all the parties concerned. The possibility of such a happy "win-win-win situation" is shown by a dotted enclosure in Table 4.

(iv) If goods are strong complements (namely, $\theta < -\theta^*$), then we find *EPS* increasing but *ECS* decreasing, showing that a conflict between producers' and consumers' interests regarding the information transmission. This is the case in which we must find a way to promote supplementary measures for consumers.

(v) Regardless of the value of θ, the information flow from firm 1 to firm 2 increases firm 2's expected profit. Consequently, firm 2 always wishes to acquire the information as it should do. Besides, in spite of the value of θ, ETS must go up by the information transmission. So information is always good for the society as a whole.

It is needless to say that those welfare results aforementioned are subject to some limitations because of the specifications of equations in the model. We do think, however, that they are fundamentally robust, thus being applicable for more general models than we have adopted here.

2. Other Duopoly Models with a Common Risk

We can generally expect that the welfare implications of the information transmission would be different if some of the following factors are subject to change:

(i) strategic variables (prices instead of quantities),
(ii) the source of risk (cost instead of demand),
(iii) the type of risk (idiosyncratic risks instead of a common risk),
(iv) the number of firms (oligopoly instead of duopoly).

At this point, we are content to limit our attention to duopoly models, leaving more general oligopoly models to another piece of paper. Even within such a duopoly framework, however, we have to consider the following three different models :

(i) the Cournot duopoly with a common *cost* risk,
(ii) the Bertrand duopoly with a common *demand* risk,
(iii) the Bertrand duopoly with a common *cost* risk.

Each of these models will successively be discussed in the following subsections. We need to take a step-to-step approach to reach the peak of the "academic mountain" from which we may enjoy getting an overall view of the whole landscape.

2.1 The Cournot Duopoly with a Common Cost Risk

Having discussed so far the Cournot duopoly model with a common *demand* risk, we would have no difficulty to analyze the same type of duopoly with a common *cost* risk. What matters in Cournot models is risk about the *net demand* intercept which is nothing but the difference between the demand intercept (α) and the constant unit cost (κ).

Suppose that the common cost parameter (κ) instead of the common demand parameter (α) is a random variable. Then we can define and compute Nash-Cournot equilibriums under various information structures exactly in the same fashion as we did for the case of a common demand risk. All we have to do now is to replace $Cov(\alpha, x_i)$ with ($-$ $Cov(\kappa, x_i)$). In order to see the relationship between the degree of technical substitution and the welfare impact, we may apply Table 4 again to the present case of cost risk. Only reinterpretation of the same framework would do a necessary trick!

2.2. The Bertrand Duopoly with a Common Demand Risk

We now turn to the situation under which firms act as Bertrand competitors rather than Cournot competitors. While there have been many papers dealing with the Cournot duopoly with a common demand risk, there are still a very few papers available for the Bertrand duopoly with the same kind of risk. [10)]

Assume that risk is about the demand side. There is a nice dual relationship between the Bertrand and Cournot equilibriums: Bertrand equilibrium with substitutable (or complementary) output is the dual of Cournot equilibrium with complements (or substitutes). However, as was carefully discussed in Part I , such nice duality argument applies only to the part of producers, but not to the part of consumers: the argument may be effective, but should not be almighty.

It is quite useful to employ the following set of formulas:

$$\begin{aligned}\Delta E\Pi_i &= E\Pi_i^S - E\Pi_i^{N1} \\ &= -b\ \Delta Var(p_i) + b\theta\,\Delta Cov(p_1,p_2) + \Delta Cov(a, p_i), \end{aligned} \tag{27}$$

$$\begin{aligned}\Delta EPS &= EPS^S - EPS^{N1} \\ &= -b\Sigma_i \Delta Var(p_i) + 2b\theta\,\Delta Cov(p_1,p_2) + \Sigma_i\ \Delta Cov(a, p_i), \end{aligned} \tag{28}$$

$$\begin{aligned}\Delta ECS &= ECS^S - ECS^{N1} \\ &= (b/2)\Sigma_i \Delta Var(p_i) - b\theta\,\Delta Cov(p_1,p_2) - \Sigma_i\ \Delta Cov(a,p_i), \end{aligned} \tag{29}$$

$$\begin{aligned}\Delta ETS &= ETS^S - ETS^{N1} \\ &= -(b/2)\Sigma_i \Delta Var(p_i) + b\theta\,\Delta Cov(p_1,p_2) \end{aligned} \tag{30}$$

Let us compare the Bertrand system (27)-(30) with the Cournot system (23)-(26). Then we immediately see that it is possible to automatically derive the first two equations (23) - (24) from (27) - (28) by simply replacing x_i by p_i, α by a, β by b, and θ by $(-\theta)$; which conforms a duality on the part of producers between the Bertrand and Cournot equilibriums.

However, such a replacement work is not feasible between the last two equations (29) - (30) and (25) - (26). In fact, compared with the Cournot system, there exists now an efficiency effect term represented by the term $(-\ \Sigma_i\ \Delta Cov(a, p_i))$ in the Bertrand system. Therefore, an information agreement affects the welfare of consumers not only through variation channels but also through efficiency channels; which

Table 5 The Bertrand Duopoly with a Common Demand Risk (α):
Various Degrees of Technical Substitution

θ	- -	- θ^*	-	0	+	θ^*	+ +
$\Delta E\Pi_1$	-	-	-	0	+	+	+
$\Delta E\Pi_2$	+	+	+	+	+	+	+
ΔEPS	-	0	+	+	+	+	+
ΔECS	-	-	-	-	-	-	-
ΔETS	-	-	-	-	-	0	+

Remark. $\theta^* \fallingdotseq 0.8284$

demonstrates a striking feature of the Bertrand model with demand risk. Moreover, the welfare loss of consumers through efficiency channels is just counterbalanced by the welfare gain of producers through the same channels, so that no efficiency effects are working for welfare of the whole society. [11)]

A more intriguing question would be how and to what extent the total welfare impact of the information transmission between the Bertrand firms is dependent on the degree of technical substitution between x_1 and x_2. An answer to this question may be shown in Table 5. Comparison of Table 5 with Table 4 enables us to enumerate the following features.

(i) As far as $\Delta E\Pi_1$, $\Delta E\Pi_2$ and ΔEPS are concerned, the sign pattern in Table 5 is dual to the one in Table 4. When we move from left to right in one table, we only have to move from right to left in the other table because a positive (or negative) θ in the Bertrand system corresponds to a negative (or positive) θ in the Cournot system.

(ii) No matter what the value of θ may be, the information transmission leads to a decline in *ECS*. When Bertrand competitors are subject to a common demand risk, the information revelation by one firm to the other is always against the interest of consumers. This is because the efficiency effects are now working strongly against *ECS*.

(iii) Unless goods are strong substitutes, the "welfare pie" gets smaller by the information transmission. To put it differently, information is good for the whole society only when x_1 and x_2 are nearly homogeneous (i.e., $\theta > \theta^*$).

(iv) In the case of strong complements (viz., $\theta < -\theta^*$), we observe *EPS*, *ECS* and *ETS* all decreasing. Therefore, the information transmission is harmful to the welfare of producers and of consumers. See a double-dotted enclosure in the lower left corner in Table 5. Clearly, this is the worst situation we could imagine regarding the information transmission. It is quite unfortunate that such possibility has drawn little attention in the existing literature on oligopoly and information.

2.3 The Bertrand Duopoly with a Common Cost Risk

Let us assume that Bertrand competitors face a common cost risk such that the common unit cost (κ) is a stochastic variable. By introducing cost risk into the Bertrand duopoly, as was noted above, a completely new situation would come out and the simple duality argument could no longer be applicable. The combination of Bertrand and cost risk would turn out to be very alarming! [12)]

A set of welfare formulas we are going to use for the Bertrand duopoly with a common cost risk are as follows:

$$\Delta E\Pi_i = -b\,\Delta Var(p_i) + b\theta\,\Delta Cov(p_1,p_2) + b\,\Delta Cov(\kappa,p_i) - b\theta\,\Delta Cov(\kappa,p_j) \qquad (i \neq j), \tag{31}$$

$$\Delta EPS = -b\Sigma_i \Delta Var(p_i) + 2b\theta\,\Delta Cov(p_1,p_2) + \mathrm{b}(1-\theta)\Sigma_i\,\Delta Cov(\kappa,p_i), \tag{32}$$

$$\Delta ECS = (b/2)\Sigma_i \Delta Var(p_i) - b\theta\,\Delta Cov(p_1,p_2), \tag{33}$$

$$\Delta ETS = -(b/2)\Sigma_i \Delta Var(p_i) + b\theta\,\Delta Cov(p_1,p_2) + b(1-\theta)\Sigma_i\,\Delta Cov(\kappa,p_i), \tag{34}$$

Table 6 The Bertrand Duopoly with a Common Cost Risk (κ):

Various Degrees of Technical Substitution

θ	- -	- θ**	-	0	+	θ*	+ +
$\Delta E\Pi_1$	+	+	+	0	-	-	-
$\Delta E\Pi_2$	-	0	+	+	+	+	+
ΔEPS	+	+	+	+	+	+	+
ΔECS	+	+	+	+	+	0	-
ΔETS	+	+	+	+	+	+	+

Remark. $\theta^* \fallingdotseq 0.8284$, $-\theta^{**} \fallingdotseq -0.8393$

As is seen in (31), regarding the welfare impact of firm i 's expected profit, there is a cross efficiency term associating κ with p_j ($j \neq i$). For example, the information transmission from firm 1 to firm 2 changes not only the value of $Cov(\kappa, p_1)$ but also the value of $Cov(\kappa, p_2)$. This is a completely new situation we have never seen for other duopoly cases.

The sensitivity of the welfare impact to the value of θ is well represented by Table 6. It is noted here that there emerges a new critical value of θ, denoted by $-\theta^{**} \fallingdotseq -0.8393$, which is the only real root of the following cubic equation:

$$2 - 2\theta^2 + \theta^3 = 0.$$

It is noted that the value of ($-\theta^{**}$) is slightly less than the value of ($-\theta^{*}$). This is because, as stated above, $-\theta^{*} = -2(\sqrt{2}-1) \fallingdotseq -0.8284$.

By taking a close look at Table 6, we are able to derive the following welfare implications.

(i) Concerning the sign pattern of $\Delta E\Pi_1$, Table 6 resembles Table 4 although there is now a cross efficiency effect working behind the scene. When goods are complements (or substitutes), firm 1 wishes (or does not wish) to reveal the information to firm 2. In contrast to the previous cases, however, there emerges the new possibility that the value of receiving information is amazingly negative. Indeed, when goods are strong complements (viz., $\theta < -\theta^{**}$), the welfare of firm 2 must go down by acquiring the information from its rival firm: namely, $\Delta E\Pi_2 < 0$. As the saying goes, ignorance may sometimes be bliss!

(ii) Independently of the value of θ, the information transmission increases *EPS*. If a side payment is feasible between the firms, the transmission may make both firms better-off. Concerning the impact on *ECS*, the sign pattern in Table 6 is just the opposite of the one in Table 4. Unless goods are strong substitutes, the information revelation is beneficial to consumers as outsiders.

(iii) If goods are weak complements in the sense that $-\theta^{**} < \theta < 0$, then we find $E\Pi_1$,$E\Pi_2$, *ECS* and *ETC* all increasing. For such a case, an information transmission agreement represents a Pareto improvement as is indicated by a solid enclosure in Table 6..

(iv) If goods are not strong substitutes in the sense that $\theta < \theta^{*}$, then an information agreement followed by a side payment would result in the improvement of the welfare of all the parties. as is shown by a dotted enclosure there.

Finally, let us attempt to compare Table 6 with Table 5. Then we readily see a remarkable difference between these two tables regarding the appearance of plus and minus signs. For the welfare analysis of Bertrand competitors, it is very critical whether the information one firm reveals to the other is about the cost side or the the demand side. This is in sharp contrast to the Cournot case in which the two cases of cost and demand information result in the same welfare implications. The fundamental difference between the Bertrand and Cournot systems regarding this matter cannot be overemphasized. Indeed, Bertrand is really something new, something different from Cournot !

5. Concluding Remarks

In the above, we have systematically discussed alternative duopoly models with different types of risks. The starting point of our discussion is clearly the Cournot duopoly model with a common demand risk. This clearly demonstrates how great A.A. Cournot (1801-77) has been as a pioneer of modern oligopoly theory. In this paper, we have exerted all our energy to extend the Cournot theory to the world with various risks. Cournot is so great because he seems still alive after 140 years of his death!

Although these analyses are very useful and yield many interesting implications, there is no need to say that they are subject to some limitations. First of all, the situation where only one risk is present must be of limited interest. The case of private risks in which each firm faces its own demand or cost risk are more realistic and more intriguing. Next, the number of firm in an industry should not be limited to two: In other words. it may be any finite number; two, three, four, ... , fifty, and more. How and to what we can extend our duopoly analysis to the general case of oligopoly is surely a very important question, and will therefore be explored in the next paper dealing with Part III.

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Endnotes

1) For instance, see Kühn & Vives (1994), Sakai (1990), Vives (1990, 1992, 1999, 2008).

2) For a detailed discussion on simultaneous and sequential games, see Suzuki (1999).

3) If such transformation transmission benefits all the parties concerned, we are in the "win-win-win situation" in the sense that it is good for the information transmitter, good for the information receiver, and also good for the whole society. Needless to say, it should be an ideal world that can hardly be attainable in the real world.

4) The extension of Nash equilibrium (1951) to the situation of imperfect information was successfully done by Harsanyi (1967-68), Selten (1975, 78), and others.

5) For the properties of reaction curves in the case of differentiated products, see Gal-Or (1985) and Sakai (1984, 87, 1990).

6) The term "the variation and efficient effects" were first introduced and intensively discussed by Sakai & Yamato (1989, 90).

7) For the problem of garbling and information manipulation, see Marschack & Radner (1972), Crawford & Sobel (1982), Okuno-Fujiwara, Postlewaite & Suzumura (1986) and others.

8) For a diagrammatic representation of Cournot-Nash equilibriums under η^{N1} and η^{S}, see Okuno-Fujiwara, Postlewaite & Suzumura (1986).

9) The significance of this point was first emphasized by Ponssard (1979) and Clarke (1983) for the special case of perfect substitutes (namely, $\theta = 1$). However, these results are no longer valid if goods are complements (i.e., $\theta < 0$).

10) While there have been many papers dealing with the Cournot duopoly with a common demand risk, there still exist a very few articles for the Bertrand duopoly with the same kind of risk. Vives (1984) is an excellent piece of work in the latter area, but he has failed to divide the welfare impact into various and efficiency channels.

11) In order to save the space, detailed tables showing the welfare effects through variation and efficiency channels for the present and following cases are omitted in this paper. See Sakai (1989).

12) It seems to be a rather common misunderstanding that when we move from the world of a common *demand* risk into the world of a common *cost* risk, the Cournot and Bertrand models continue to have nice dual relations. This is perhaps the reason

why so few papers on the Bertrand duopoly with a common cost risk have been published so far. Filling in such a gap is really the goal of this paper.