DISCUSSION PAPER SERIES E

SHIGA UNIVERSITY

Discussion Paper No. E-9

Bitcoin or Gold? Risk Assessment Based on Continuous Wavelet Transform

Lei Xu

March 2021

The Institute for Economic and Business Research
Faculty of Economics
SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,
SHIGA 522-8522, JAPAN

Bitcoin or Gold? Risk Assessment Based on Continuous Wavelet Transform

Lei Xu

Graduate School of Economics, Kobe University, 2-1, Rokkodai, Nada-Ku, Kobe 657-8501, Japan.

E-mail: joker0813@hotmail.com

Abstract: In this paper, the risk reduction and downside risk reduction of Value at Risk and Expected shortfall are calculated using variance and covariance derived from wavelet analysis. By comparing bitcoin to currency (stock) portfolios and gold to currency (stock) portfolios in time-frequency domain, we find strong evidence that the portfolios containing bitcoin effectively reduce investment risk than the portfolios containing gold after 2019, and especially after 2020.

1. Introduction

The cryptocurrency is a medium of exchange that uses cryptographic principles to secure transactions and control the creation of exchange units. Cryptocurrency is a type of digital currency (or virtual currency). Among the many cryptocurrencies currently on the market, the most pioneering and well-known is Bitcoin, which became the first decentralized cryptocurrency in 2009.

As more and more economists are interested in Bitcoin, whether Bitcoin can be used as a safe-haven financial asset is always considered. Ji et al. (2018) demonstrate that the integration between Bitcoin and other financial assets is a continuous process and will change over time. Selmi et al. (2018) apply the conditional Value-at-Risk approach to risk management and demonstrates that bitcoin and gold can diversify opportunities and reduce downside risk in an extended oil portfolio. Pal and Mitra (2019) compute optimal hedge ratios between bitcoin and other financial assets by using conditional volatility estimates of different GARCH models and find Gold provides a better hedge against bitcoin. The results of Urquhart and Zhang (2019) indicate that, Bitcoin does act as an intraday hedge, diversifier and safe haven for certain currencies.

On the other hand, as a traditional safe-haven asset, gold is often used for hedging risk research. Wang et al. (2011) investigate the benefit of using gold to hedge against inflation, and Reboredo

(2013) study the gold's hedge and safe-haven status with respect to oil price changes. Between the gold and stock market movements, there are also very extensive researches (e.g. Baur and McDermott, 2010; Baur and Lucey, 2010; Miyazaki et al., 2012; Beckmann et al., 2015). Among the various types of studies, the most common ones are still about gold, bitcoin, and oil. Jin et al. (2019) find the dynamic correlations between gold and crude oil markets are almost positive, while those between Bitcoin and gold, and those between Bitcoin and oil markets are nearly negative. Their results indicate that gold market dominates crude oil and Bitcoin markets and contributes more explanatory power in price movements of the hedging assets system. Al-Yahyaee et al. (2019) examine the diversification and hedging properties of Bitcoin and gold assets for oil and S&P GSCI investors with GARCH model. Their results denote the importance of Bitcoin and gold in oil and S&P GSCI portfolio management in hedging effectiveness and downside risk reductions.

Additionally, as an analytical tool capable of providing a time-frequency dimension, multi-perspective analysis of bitcoin or gold is also becoming possible through wavelet analysis. Kang et al. (2019) employ both the DCC-GARCH and wavelet coherence methods to investigate the volatility persistence, causality, and phase differences between Bitcoin and gold futures prices. Berdin et al. (2015) find that gold acts as a hedge for a variety of international equity and debt markets for horizons of up to one year by applying the continuous wavelet transform. Considering the conclusions of the above papers all point out that the obtained analysis results are different at different times and frequencies. Therefore, to perform a comprehensive analysis in terms of the time-frequency domain, we applied the wavelet analysis.

Our contribution to the literature is three-fold. First, Bitcoin, Gold, Swiss Franc, Japan Yen, and European euro show larger variance in daily returns over the short to medium term in the early 2020s. Besides, of the daily returns of equities, S&P500 and Nikkei index has a large variance in the short and medium-term in the first half of the 2020s and the long term with significantly larger variance. Therefore, we can speculate that these fluctuations in the first half of 2020 are likely related to COVID-19. Second, the correlation between the daily returns of gold and the three currencies is higher in the short to medium term, while the correlation between the daily returns of bitcoin and the three stocks is higher in the medium to long term in 2020. And differences in correlations in the time-frequency domain can affect the assessment of investment risk. Third, by comparing the risk reduction spectrum of each portfolio and comparing the average of the three metrics (the risk reduction, the VaR downside risk reduction, and the ES downside risk reduction) for each year from 2017 to 2020, we conclude that the bitcoin and 6 asset portfolios can reduce the risk greater than the gold and 6 asset portfolios do after 2019, and especially after 2020.

The remainder of this article is organized as follows. Section 2 explains the methodology and describes the data. Section 3 presents the empirical results. Finally, Section 4 concludes.

2. Methodology and Data

2.1 Wavelet transform

Morlet et al. (1982) introduced the wavelet transform and provided a mathematical tool for seismic wave analysis, which has proven to be a powerful signal processing tool that can explain the joint behavior of time series in terms of both frequency and time. We can define the parent wavelet function, $\varphi(.) \in L^2(\mathbb{R})$, as

$$\varphi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\varphi\left(\frac{t-\tau}{s}\right), s\tau \in \mathbb{R}, s \neq 0, \tag{1}$$

where s is the scaling factor or frequency, which measures the degree of compression, and τ defines the translation parameter, which determines the time location of the wavelet. The term $\frac{1}{\sqrt{s}}$ denotes the normalization factor to ensure the unit variance of the wavelet and $\left\|\varphi_{r,s}\right\|^2 = 1$. In this paper, we apply the Morlet wavelet introduced by Goupillaud et al. (1984), which is an appropriate method for identifying oscillatory components of a signal. According to Addison (2002), another advantage of the Morlet wavelet is that it is a complex or analytic wavelet within a Gaussian envelope with good time-frequency localization. Formally, the Morlet wavelet is

$$\varphi^M(t) = \frac{1}{\pi^{1/4}}e^{i\omega_0 t}e^{-t^2/2}, \tag{2}$$

where t denotes the time and, ω_0 is the central frequency of the wavelet. ω_0 usually equals 6, which enables the Morlet wavelet to balance the localization of time and frequency.

2.2 Wavelet power spectrum and cross-wavelet transform

The main purpose of using wavelet analysis is to calculate the variance and covariance in the time-frequency domain. Thus, we use the continuous wavelet transform and the cross-wavelet transform as our main analytical tools.

The continuous wavelet transform (CWT) of a time series $y(t) \in L^2(\mathbb{R})$ is given by

$$W_{y;\varphi}(\tau,s) = \int_{-\infty}^{\infty} y(t)\frac{1}{\sqrt{|s|}}\varphi^*(\frac{t-\tau}{s})dt, \tag{3}$$

where $*$ denotes the complex conjugate form. By representing the original time series in the functions of τ and s, the wavelet transform can simultaneously provide information in the time-frequency space. We can obtain the time series by taking the inverse of the CWT function:

$$y(t) = \frac{1}{C_\varphi}\int_0^{\infty}\left[\int_{-\infty}^{\infty} W_{y;\varphi}(\tau,s)\varphi_{\tau,s}(t)d\tau\right]\frac{ds}{s^2}, s > 0, \tag{4}$$

Thus, we can state the variance of the power spectrum as

$$\|y\|^2 = \frac{1}{C_\varphi}\int_0^{\infty}[\int_{-\infty}^{\infty}\left|W_{y;\varphi}(\tau,s)\right|^2 d\tau]\frac{ds}{s^2}, s > 0, \tag{5}$$

where $\left|W_{y;\varphi}(\tau,s)\right|^2$ is the wavelet power spectrum, which can interpret the degree of the local variance of $y(t)$ frequency by frequency.

Following Grinsted et al. (2004), we can assess the statistical significance against the null hypothesis that the time series generating process is given by an AR(1) stationary process with a certain background power spectrum (P_f).[1] Torrence and Compo (1998) compute the white noise and red noise wavelet power spectra based on Monte Carlo simulations and derive the following corresponding distribution for the local wavelet power spectrum under the null, where

$$D\left(\frac{|W_{y,t}(s)|^2}{\sigma_y^2} < p\right) = \frac{1}{2} P_f y_v^2, \tag{6}$$

at time, t, and frequency, s. P_f is the mean spectrum at the Fourier frequency, f, that corresponds to the wavelet frequencies s ($f \approx 1/s$). v is equal to 1 or 2 for real or complex wavelets, respectively. Therefore, a high power spectrum denotes a large variance of the series at time, t, and frequency, s.

Next, we can define the cross-wavelet transform for two time series, $x(t)$ and $y(t)$, as

$$W_{x,y}(\tau, s) = W_x(\tau, s) W_y^*(\tau, s), \tag{7}$$

where W_y^* is the complex conjugate of W_y. Then, the cross-wavelet power (XWP) is

$$(XWP)_{xy} = \left|W_{xy}\right|, \tag{8}$$

which denotes the local covariance in the pairs of assets' daily returns at each time and frequency and shows the area in the time-frequency domain with high common power. In addition, Torrence and Compo (1998) derive the theoretical distribution of the cross-wavelet power of two time series with background power spectra P_f^x and P_f^y as

[1] The Fourier power spectrum of an autoregressive process with lag-1 autocorrelation α is $P_f = \frac{1-\alpha^2}{|1-\alpha e^{-2i\pi k}|^2}$.

$$D\left(\frac{|W_{x,t}(s)W_{y,t}^{*}(s)|}{\sigma_x\sigma_y} < p\right) = \frac{Z_v(p)}{v}\sqrt{P_f^x P_f^y}, \tag{9}$$

where $Z_v(p)$ is the confidence level associated with the probability p for a probability density function defined by the square root of the product of two χ^2 distributions.[2] Thus, the cross-wavelet power indicates the degree of the local covariance of the pairs of assets' daily returns at time, t, and frequency, s.

2.3 Wavelet coherence

In this section, we employ a wavelet coherence analysis, which can identify information about the dependencies and correlations between two signals. We can write the wavelet coherence as

$$R_{xy}^2 = \frac{|S(W_{xy})|^2}{S(|W_x|^2)S(|W_y|^2)}, \tag{10}$$

where S is a smoothing operator in both time and frequency, and R_{xy}^2 takes a value between 0 and 1. Values of R_{xy}^2 closer to 1 indicate a stronger correlation, while values closer to 0 indicate a weaker correlation between the pairs of assets' daily returns. Thus, the wavelet coherence analysis generates insight on the conditional correlations.

2.4 The risk management

[2] For example, in our analysis, we calculate the 5% significance level using $Z_2(0.95) = 3.999$.

Since our ultimate goal is to evaluate the impact of bitcoin and gold on investment risk, we need to calculate Value at Risk (VaR), Expected shortfall (ES), the risk reduction (RR), and the downside risk reduction (DRR) in addition to applying wavelet analysis. After we calculate the variance and covariance which obtained from wavelet analysis, we use them to compute the weight of asset X in the X-Y portfolio. Following by Kroner and Ng (1998), the weight is defined as

$$w_{s,t}^{X} = \frac{var_{s,t}^{Y} - cov(r^{X}, r^{Y})_{s,t}}{var_{s,t}^{X} - 2cov(r^{X}, r^{Y})_{s,t} + var_{s,t}^{Y}}, \tag{11}$$

where s denotes the frequency, r denotes the daily return, $w_{s,t}^{X} = 0$ when $w_{s,t}^{X} < 0$, and $w_{s,t}^{X} = 1$ when $w_{s,t}^{X} > 1$. Therefore, $w_{s,t}^{Y} = 1 - w_{s,t}^{X}$. The portfolio's mean and variance is given by

$$r_{s,t}^{portfolio} = w_{s,t}^{X} r_{t}^{X} + w_{s,t}^{Y} r_{t}^{Y}, \tag{12}$$

$$var_{s,t}^{portfolio} = w_{s,t}^{X} var_{s,t}^{X} + w_{s,t}^{Y} var_{s,t}^{Y} + w_{s,t}^{X} w_{s,t}^{Y} cov(r^{X}, r^{Y})_{s,t}. \tag{13}$$

Then, according to Reboredo and Rivera-Castro (2014) the risk reduction, defined as the percentage reduction in the X-Y portfolio variance with respect to the Y portfolio:

$$RR_{s,t} = 1 - \frac{var_{s,t}^{portfolio}}{var_{s,t}^{Y}}, \tag{14}$$

a higher value of $RR_{s,t}$ means that the greater X-Y optimal weight portfolio reduced risk better.

Next, we also measure the downside risk reduction (DRR) by calculating the ratio between the X-Y portfolio VaR and ES with respect to those of the Y portfolio. The VaR at the $(1 - 0.05)\%$ confidence level of a portfolio is defined as

$$VaR_{s,t}^{portfolio} = V_0 \emptyset^{-1}(1-0.05) var_{s,t}^{portfolio}, \tag{15}$$

where V_0 is the value of the initial investment and $\emptyset(\cdot)$ is the cumulative normal distribution. Therefore, the ES can be given by

$$ES_{s,t}^{portfolio} = E[r_{s,t}^{portfolio} | r_{s,t}^{portfolio} \leq VaR_{s,t}]. \tag{16}$$

In the end, we calculate the downside risk reduction as follows:

$$DRR_{s.t}^{VaR} = 1 - \frac{VaR_{s,t}^{portfolio}}{VaR_{s,t}^{Y}}, \tag{17}$$

$$DRR_{s.t}^{ES} = 1 - \frac{VaR_{s,t}^{ES}}{VaR_{s,t}^{Y}}. \tag{18}$$

Similar to the risk reduction Eq. (14), a higher value of $DRR_{s,t}$ implies that X can diversify the downside risk better. Moreover, values of $DRR_{s,t}$ varying over different frequencies also imply an evolving risk reduction at different horizons.

2.5 Data

First, we source all the US dollar-based price of Bitcoin, Gold, Swiss Franc, Japan Yen, European euro, S&P500, Nikkei index, and CSI 300 index from Bloomberg. The sample of series covers the period from Jan. 3, 2017, to Jan. 15, 2021. Second, we calculate the returns of Bitcoin (BTC), Gold (GOLD), Swiss Franc (CHF), Japan Yen (JPY), European euro (EURO), S&P500 (SP500), Nikkei index (NIKKEI), and CSI 300 index (SHSZ300) with the log-difference of the price series after we removed the missing values which denote the market holidays. Thus, the date of the asset returns covers spans from Jan. 5, 2017, to Jan. 15, 2021. Finally, we apply the continuous wavelet analysis to compute the variance, the covariance, and the correlation of the asset returns, which the former two are used to calculate the RR, the VaR, and the ES.

Table 1 presents the statistics of the asset returns. All of the series are skewed with excess kurtosis, and the Jarque-Bera statistics indicate that the distribution of the series is non-normal. According to the Augmented Dickey-Fuller (ADF) unit-root test, all of the asset returns series are stationary.

Table 1. The statistics of the assets' returns.

	BTC	GOLD	CHF	JPY	EURO	SP500	NIKKEI	SHSZ300
Observation	877	877	877	877	877	877	877	877
Mean	0.40	0.05	-0.02	-0.01	0.02	0.06	0.04	0.06
Median	0.35	0.07	0.00	-0.01	0.01	0.09	0.06	0.06
Max	29.26	5.42	1.81	3.15	1.56	8.97	7.73	7.43
Min	-31.73	-6.27	-1.58	-2.92	-2.06	-12.77	-6.27	-8.21
Standard deviation	5.32	0.93	0.44	0.51	0.46	1.38	1.23	1.28
Skewness	-0.15	-0.40	0.09	0.11	-0.11	-1.33	0.03	-0.31
Kurtosis	4.74	6.80	1.12	5.41	1.20	19.84	5.65	4.74
Jarque-Bera statistic	829.78***	1727.20***	48.03***	1079.30***	55.38***	14717***	1174.90***	843.29***
ADF unit-root test	-8.29***	-10.15***	-9.84***	-10.95***	-9.66***	-8.65***	-9.03***	-9.12***

Note: *** Denotes the 1% significance level.

Table 2 shows the Pearson correlations of the pairwise for all pairs of the asset returns series in our sample. Here, we note that both bitcoin and gold have negative correlation coefficients for CHF and JPY. Besides, gold also shows a negative correlation with NIKKEI. Comparing the absolute values, we can see that the correlation between gold and CHF, JPY, and EURO is stronger than the correlation between bitcoin and these three. When compared to currencies, the correlation between stocks and bitcoin or gold is not strong. In addition, the correlation is less than 0.4 for all groups except for the stronger correlation between the three currencies and between NIKKEI and SHSZ300.

In the following analysis, we use the normalized daily returns, which are derived by centering and scaling the series.[3]

Table 2. The Pearson correlations for the pairs of the asset returns.

	BTC	GOLD	CHF	JPY	EURO	SP500	NIKKEI	SHSZ300
BTC	1							
GOLD	0.12	1						
CHF	-0.04	-0.47	1					
JPY	-0.01	-0.48	0.58	1				
EURO	0.06	0.42	-0.79	-0.44	1			
SP500	0.18	0.01	0.08	0.34	0.03	1		
NIKKEI	0.06	-0.03	-0.01	0.21	0.08	0.29	1	
SHSZ300	0.02	0.08	-0.04	0.10	0.06	0.27	0.42	1

[3] To center the series, we subtract the average value from all the values of the series. To scale the series, we divide each value of the series by its standard deviation.

3. Empirical Results

In our paper, we decompose the data series into 8 levels covering short term (from one day to one week), medium term (from more than one week to less than two months), and long term (from more than two months to one year).

3.1 Wavelet power spectrum

Figure 1 illustrates the continuous wavelet power spectrum for the returns of bitcoin, gold, three selected currencies, and three selected stocks, which also denotes the variance of the assets returns. In the wavelet power spectrum, the black contour shows the 5% significance level which estimated by applying the Monte Carlo simulations, using phase randomized surrogate series. The power levels range from blue (low power) to red (high power), and the cone of influence indicated by the solid curve indicates the area affected by edge effects. The red area at the bottom (top) of the graph indicates strong variations at low (high) frequencies of the sample period. In contrast, the red area on the left (right) side of the graph indicates significant variations at the beginning (end) of the sample period.

In Figure 1, the return of bitcoin price has a large variance on the short and medium-term from mid-2017 to mid-2018, followed by a variance of less than 0.3 on the long term from mid-2018 to mid-2019, and again shows a large variance in the early part of the year. The conclusion of the variance of this return is factually consistent considering that the bitcoin price started to rise significantly in late 2017 and in 2020. In contrast, the variance of the gold price return is relatively small before 2020, while there is a large variance on the short and medium-term in early and late 2020, which is perhaps brought about by the effect of COVID-19. Similarly, the variance of CHF, JPY, and EURO also show large values in the short and medium-term in the first half of 2020, while the SP500 and NIKKEI, although they also have a large variance in the short and medium-term in the first half of 2020, but have statistically significant variance in the medium and long term, despite the variance being only about 0.3. In slight contrast to the variance of the same stock returns, SHSZ300 also has a large variance in the short to medium term in the first half of 2020 but does not have a statistically significant large variance in the long term.

Furthermore, in Figure 1, we can visually compare the assets' prices and the returns and the continuous wavelet power spectrum of the returns. If we look at the post half of 2020, we can clearly see that, in the top part, the successively higher asset prices do not bring statistically meaningful high variance in the continuous wavelet spectrum. On the other side, the common feature of the returns of these assets is high volatility around February or March 2020, and this is also reflected in the continuous wavelet spectrum (large red areas enclosed by black lines).

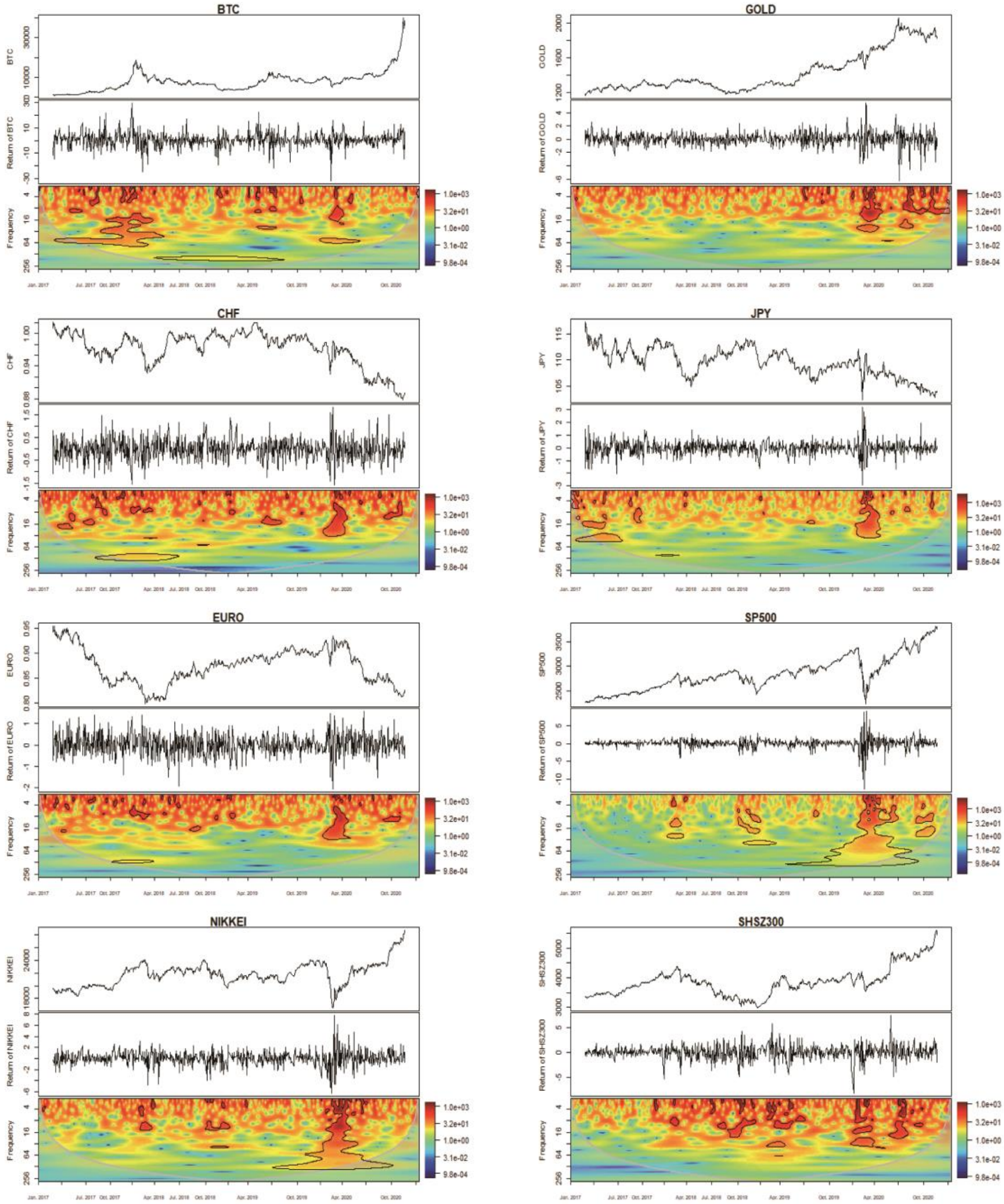

Figure 1. The US dollar-based price of assets (the top part), and their returns (the medium part), and the continuous wavelet power spectrum of the assets price returns (the bottom part).

3.2 Cross-wavelet transform

We present the covariance of the pair of the bitcoin price return and the currency returns and the stock returns with cross-wavelet transform in Figure 2a and Figure 2b.

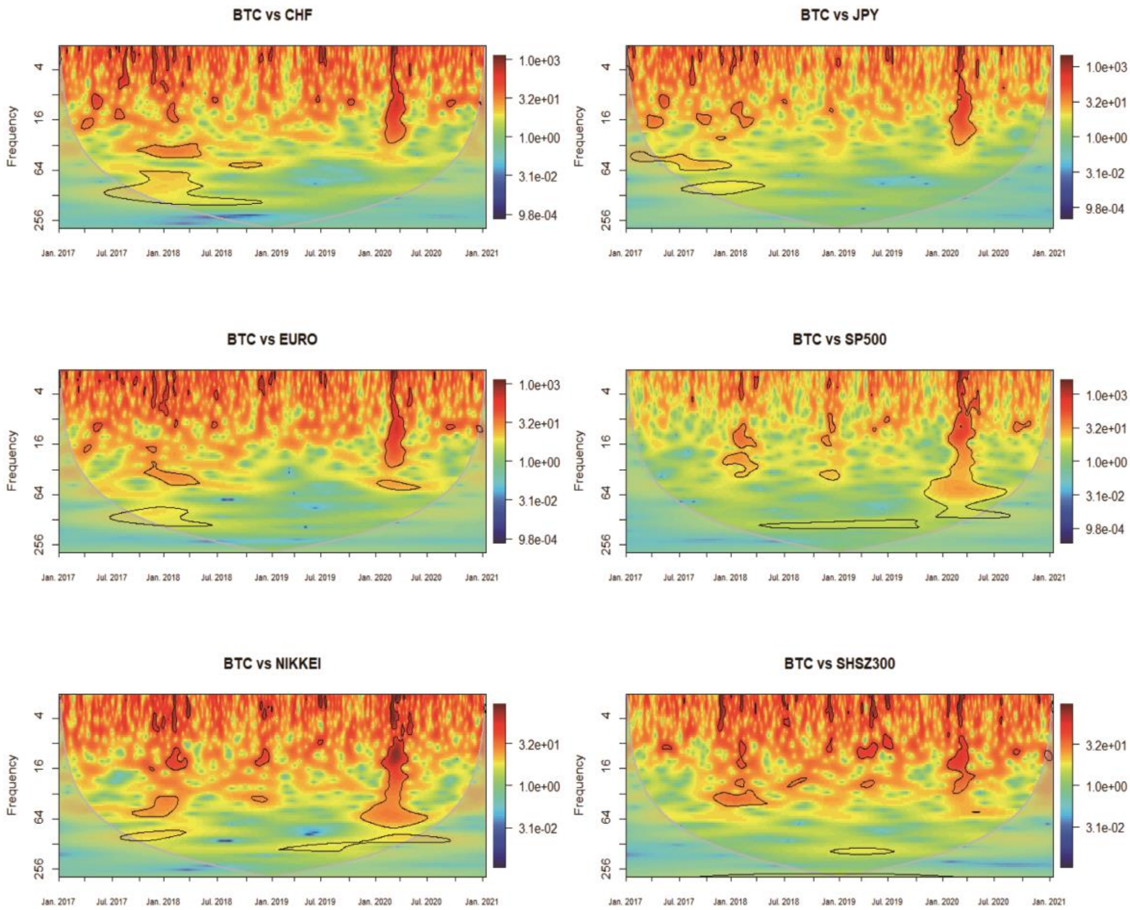

Figure 2a. The cross-wavelet transform between BTC and the selected 6 assets daily returns.

In Figure 2a, the covariance in the pair of BTC and three selected currency returns have a statistically significant covariance of less than 0.3 in the medium to long term from late 2017 to mid-2018, in addition to a few areas of significance in the short term. On the short- and medium-term in the early 2020s, we also observe relatively large red areas, which represent a large covariance in the three pairs on this time-frequency domain. In the pairs of BTC and three stock returns, in addition to a statistically significant covariance of less than 0.3 in the long term, there is a very significant large covariance on the short and medium-term in 2018 and the first half of

2020, especially the short term in 2020 presents the largest value of covariance (the darkest red area).

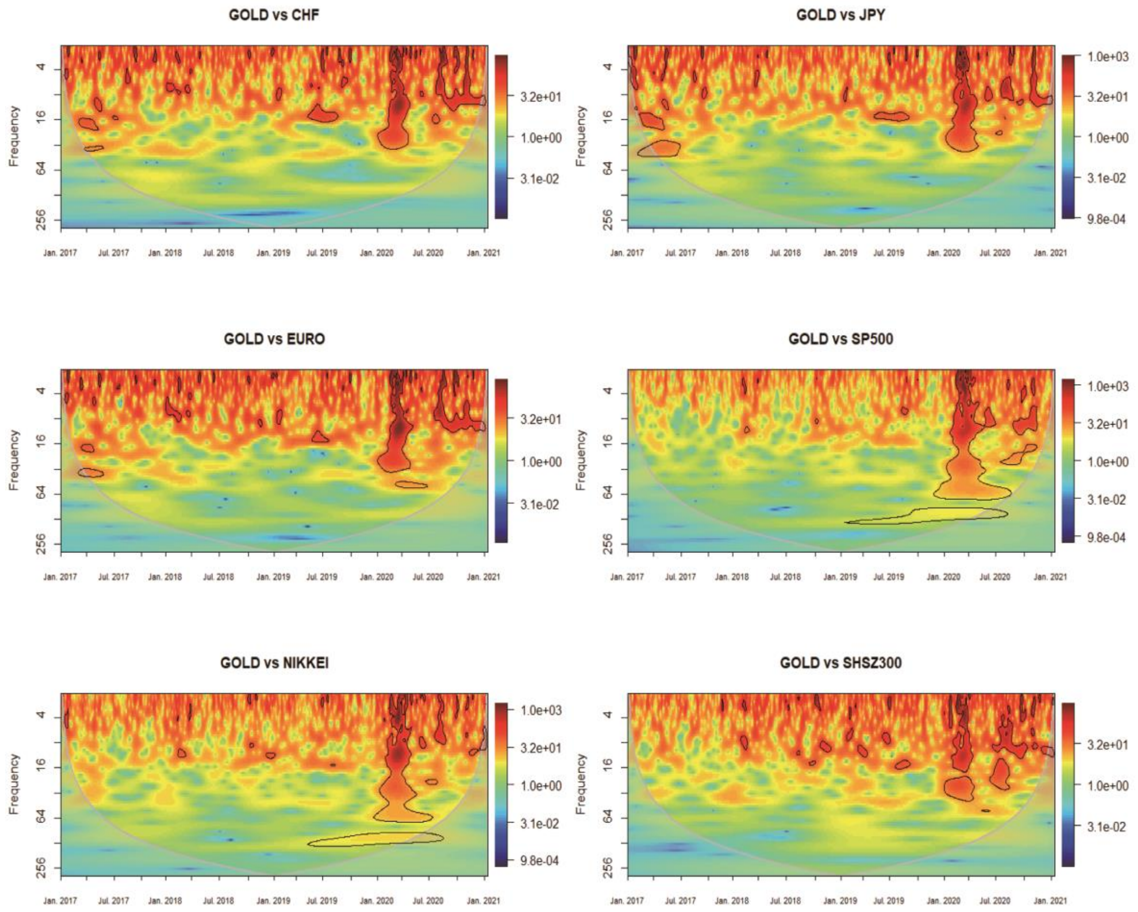

Figure 2b. The cross-wavelet transform between GOLD and the selected 6 assets daily returns.

Similar to Figure 2a, Figure 2b shows the cross-wavelet transform between gold and three currency returns and three stock returns. In Figure 2b, the biggest difference from Figure 2a is that the meaningful dark red area in the early 2020s is larger than the area at other times. While there are still several statistically significant areas present in the short term, it is undoubtedly the early 2020s, especially in the short to medium term, that is the most pronounced. Another difference from Figure 2a is that in late 2020, significantly large covariance continues to be observed at short- and medium-term frequencies.

3.3 Wavelet coherence

While we present the Pearson correlations in Table 2, we also prefer to present a spectrum of correlations in the time-frequency domain. Therefore, we present the wavelet coherence between the bitcoin and six selected assets daily returns in Figure 3a. A statistically significant correlation above 0.8 can be seen in the pair of daily returns for Bitcoin and the three currencies at a frequency of 8-16 days in late 2020. The few yellow-red areas which indicate correlations between 0.6 and 0.8 can also be observed on the other time-frequencies. On the other hand, in the pairs of bitcoin and SP500 and NIKKEI daily returns, on the medium and long term frequencies from mid-2019 to mid-2020, we can observe large red areas representing high correlations. And in the pairs of bitcoin and NIKKEI daily returns, if we look only at the long term, a high correlation can be observed across almost all timelines, except for the time period from early to mid-2019. As for the remaining pair of bitcoin and SHSZ300 daily return. However, there are a few statistically significant areas of high correlation; the number of areas and the magnitude of the correlation represented are not as large as the other two pairs of bitcoin and stock returns.

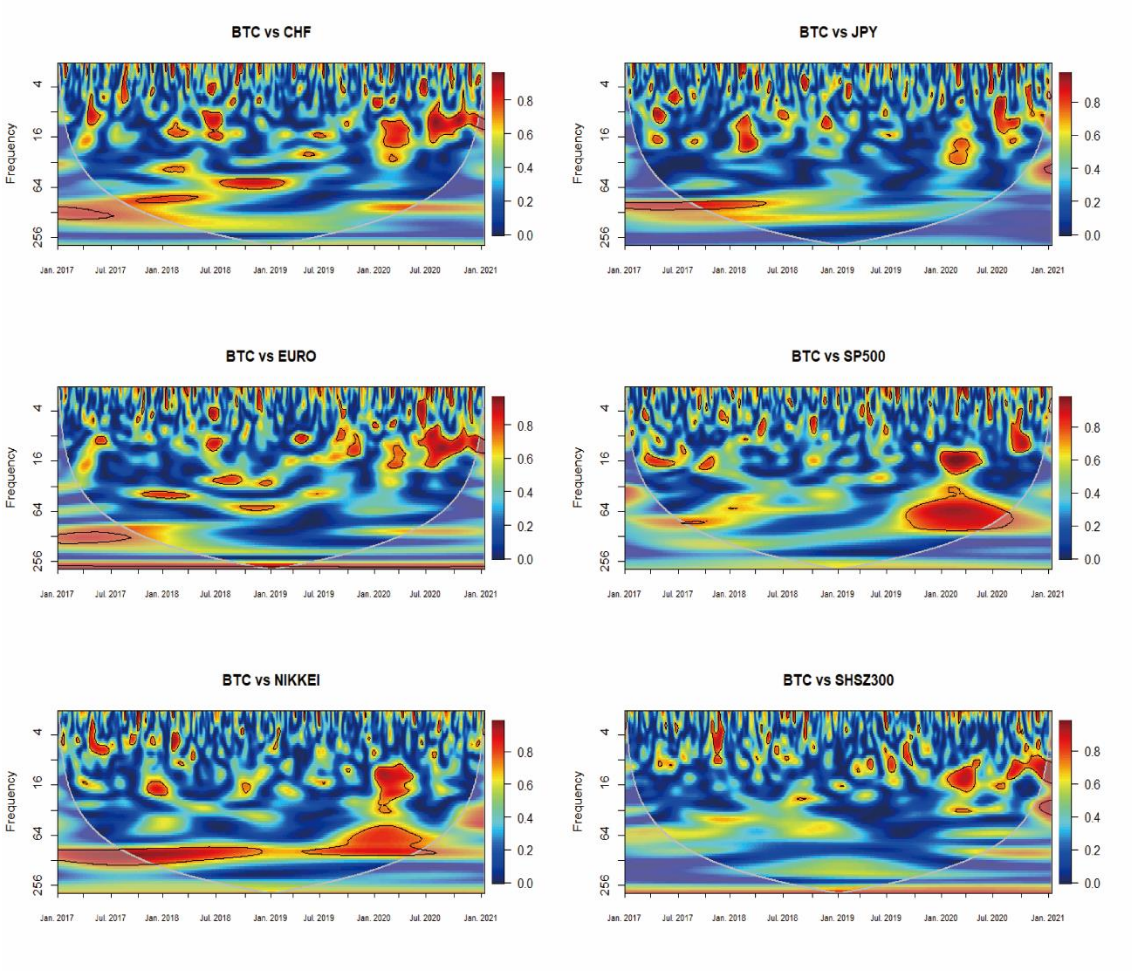

Figure 3a. The wavelet coherence between BTC and the selected 6 assets price returns.

Fig. 3b presents the wavelet coherence between gold and the selected 6 assets price returns. When compared to the daily return pairs of bitcoin and the three currencies in Figure 3a, the correlation between gold and the three currencies daily returns is stronger in the short to medium term and across almost all timelines, which is consistent with the prior studies. On the other hand, in the pairs of gold with SP500 and NIKKEI daily returns, the red area of the values of statistically significant correlations in the short term is not large, and correlations in the medium term frequency in the early 2020s are more pronounced. And the correlation between gold and

SHSZ300 daily returns is only more pronounced at frequencies of about 8 days in 2017 and early 2020.

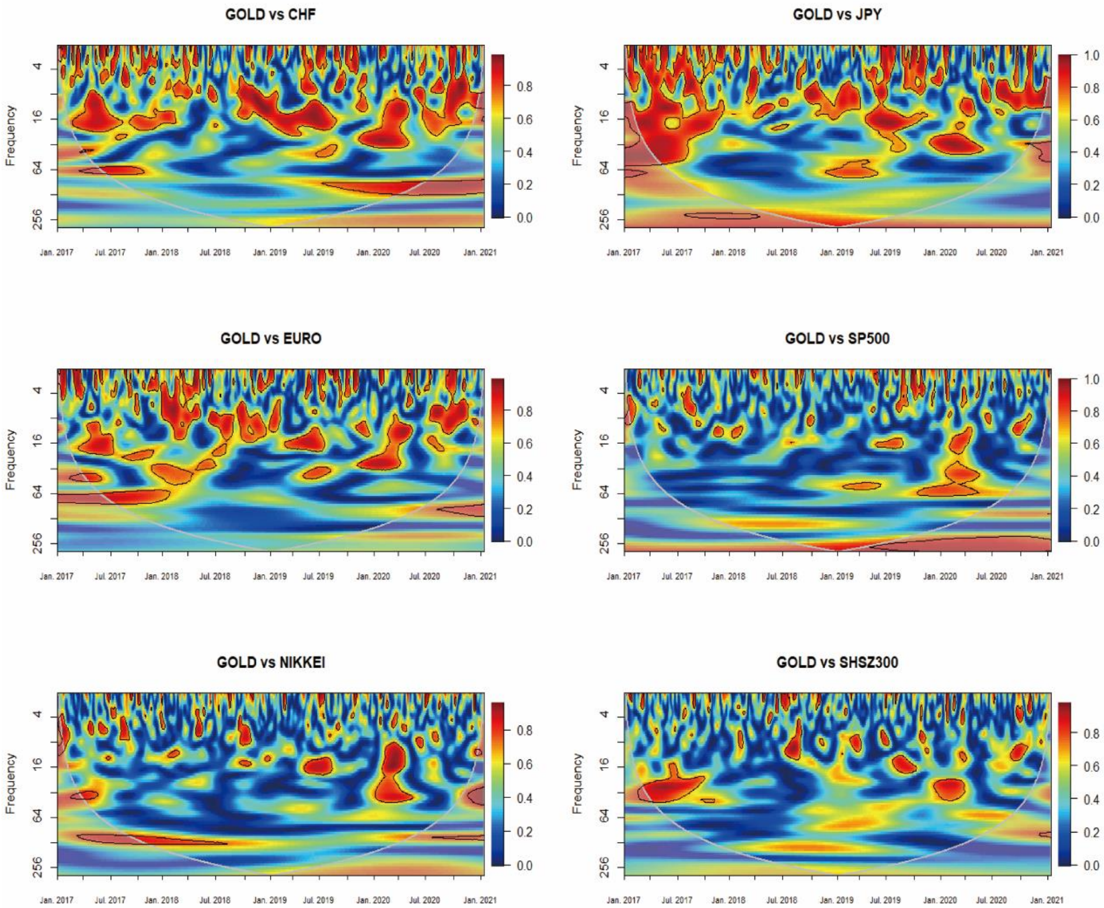

Figure 3b. The wavelet coherence between GOLD and the selected 6 assets price returns.

3.4 Risk management

Following Kroner and Ng (1998) and Reboredo and Rivera-Castro (2014), we compute a time-frequency domain RR, and a higher value of RR means that the greater BTC (gold)-currency (stock) optimal weight portfolio reduced risk better. Moreover, values of RR varied over time and different frequencies imply an evolving risk reduction at different horizons, which is convenient for international short and long-term investors who are more interested in short and long-run risk reduction.

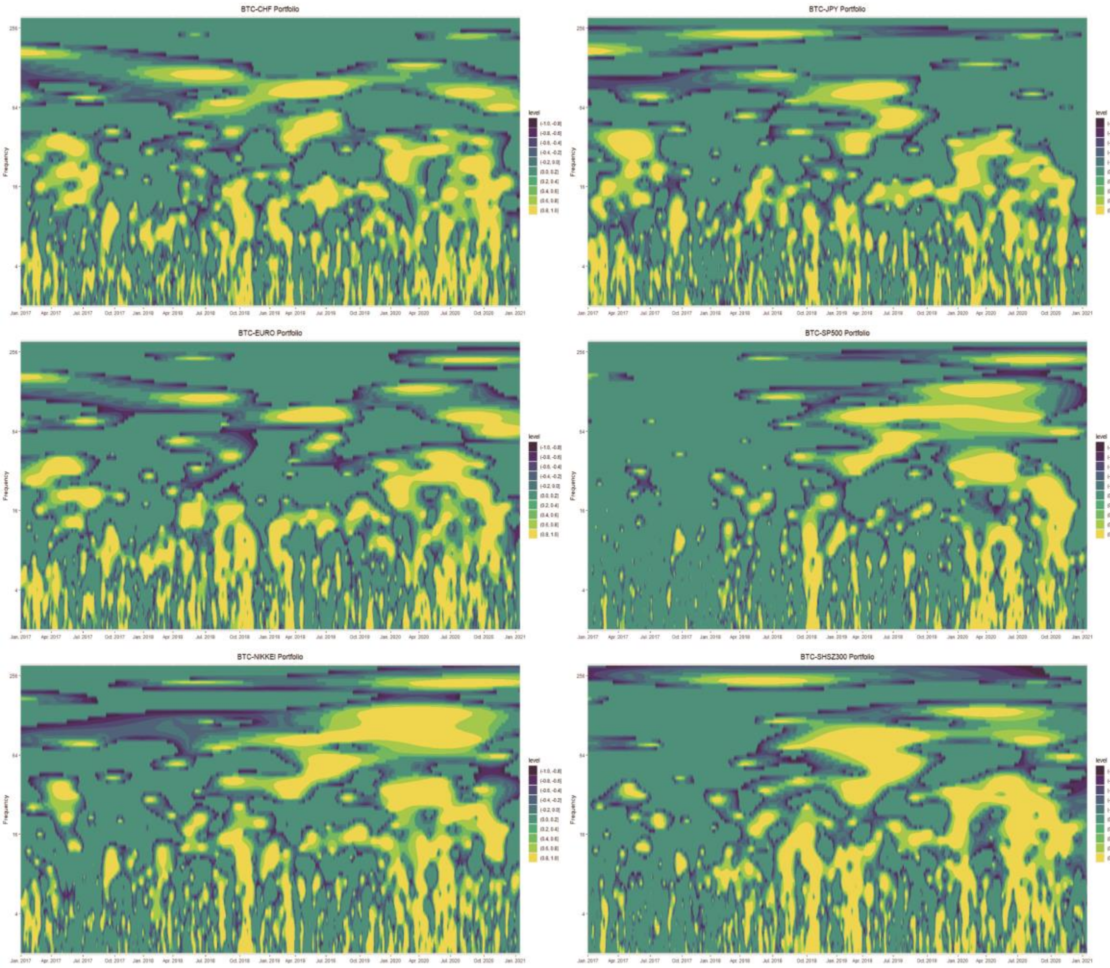

Figure 4a. The risk reduction in the BTC-currency (stock) portfolio variance. Notes: Unlike the graph of wavelet analysis, the Y-axis of this graph represents a gradual increase in frequency from top to bottom, and the dark blue to yellow gradient represents the change in the value of risk reduction from -1 to 1.

In Figure 4a, we present the RR in the bitcoin-currency (stock) portfolio variance. In the portfolio of bitcoin and the three currencies, in the short and medium-term (bottom of the figure), there are many uniformly distributed yellow areas (although discontinuous) and almost no dark blue areas observed on the entire timeline, which means that the three portfolios are effective in reducing the risk of investment at high frequencies. In the medium to long term (top of the figure), although there are yellow areas representing values close to 1 for the risk reduction, large dark blue areas can also be observed, which means that the risk of investment at this time is rather high. Therefore, great caution is needed when holding these three portfolios over the long term. Furthermore, large areas of dark blue and jungle green can be observed in the portfolio of bitcoin and stocks until 2019. And on the medium and long term after 2019, and especially after 2020, there are continuous and large yellow areas, while there are significantly more yellow areas in the short and medium-term than before 2019. These results mean that these three bitcoin and stock portfolios can effectively reduce investment risk after 2019, and especially after 2020. And compared to the bitcoin-currency portfolios, the bitcoin-stock portfolios are more worthy of selection.

Similarly, in Figure 4b, we present the RR in the gold-currency (stock) portfolio variance. There is a big difference compared to Figure 4a, wherein the 3 portfolios of gold and currency, large areas of dark blue and jungle green can be observed after 2019 (especially after 2020) compared to the large yellow areas evident before 2019, and it is not just a short term reduction in the yellow areas, but a much larger difference on the long term. This means that the 3 portfolios of gold and currencies are already very unsuitable to hold after 2019, and especially after 2020.

Perhaps foreign exchange market investors should exclude gold as an option when considering risk aversion. Moreover, the yellow areas in the portfolios for gold and SP500 are very sparse in the short term, with large areas of yellow concentrated in the medium term from 2018 to 2019

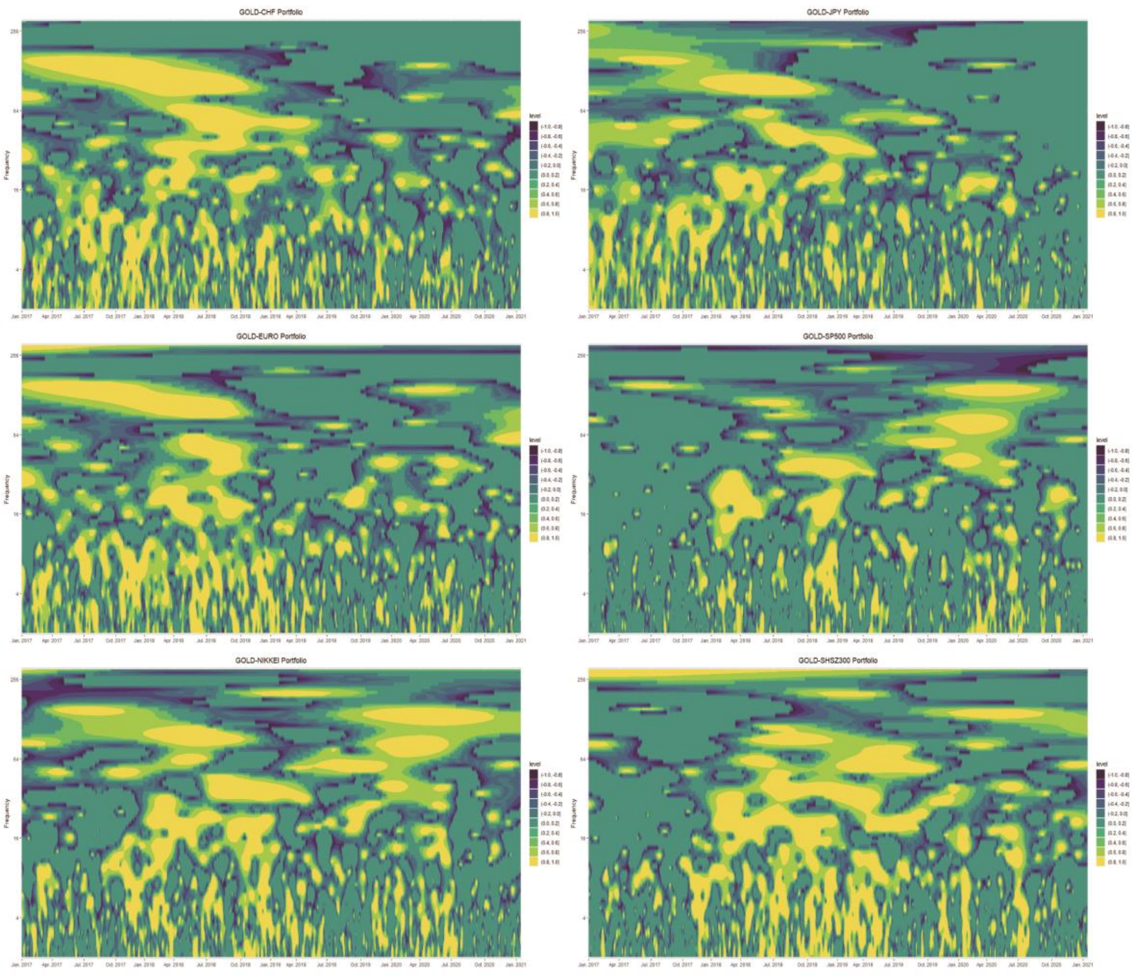

Figure 4b. The risk reduction in the GOLD-currency (stock) portfolio variance. Notes: Unlike the graph of wavelet analysis, the Y-axis of this graph represents a gradual increase in frequency from top to bottom, and the dark blue to yellow gradient represents the change in the value of risk reduction from -1 to 1.

and the long term from 2019 to 2020. And close to 256 days in frequency, large dark blue areas can be observed, implying that investors should be careful and risky in their choice of holding time when holding this portfolio. The remaining yellow areas in the portfolios of Gold and

NIKKEI and SHSZ300 are more evenly distributed and concentrated between 2018 and 2019. And while the yellow areas are observable in the long term after 2020, the blue and green areas (negative or less than 0.5) are also very visible. This result continues to imply the need for great caution in choosing the portfolios of gold and equities.

As a summary of Figure 4a and Figure 4b, we can visually compare the two figures' yellow areas. From this, we can see that the yellow area on the left side of each graph in Figure 4a is less than each graph in Figure 4b, while when we look at the right side, Figure 4a has more yellow areas than Figure 4b. In particular, when we compare the high frequencies after 2020 (the lower part of the y-axis), there are more yellow areas in Figure 4a than in Figure 4b, in other words, we can interpret that the risk reduction effect of Bitcoin on selected six assets is stronger than gold at the high frequencies after 2020.

We also measure the downside risk reduction (DRR) by calculating the ratio between the bitcoin (gold)-currency (stock) portfolio VaR and ES with respect to those of the currency (stock) portfolio. To make the results clearer, we averaged the RR and DRR according to year and frequency, and the results are shown in Table 3.

In Table 3, the common feature across all frequencies is that the RR and DRR of the portfolios containing gold and 6 assets are larger in 2017 and 2018 than the RR and DRR of the portfolios containing bitcoin at the same frequency in the same years. However, in 2019 and 2020, the RR and DRR of portfolios containing bitcoin and 6 assets are larger instead. While there are instances where the DRR of portfolios containing gold is larger than the DRR of portfolios containing bitcoin at some frequencies after 2019 (e.g., DDR.ES for Gold-CHF portfolio at the 32-64 day frequency in 2019), considering all portfolios and all frequencies and metrics, after 2019, portfolios containing bitcoin coin are undoubtedly more able to reduce the risk of investments than the portfolios containing gold.

Table 3. The downside risk reduction effectiveness of BTC (GOLD)-currency (stock) portfolio.

			BTC						GOLD					
Frequencies	Year		CHF	JPY	EURO	SP500	NIKKEI	SHSZ300	CHF	JPY	EURO	SP500	NIKKEI	SHSZ300
1-2	2017	RR	0.21	0.24	0.24	0.04	0.04	0.09	0.33	0.32	0.30	-0.01	0.06	0.05
		DRR.VaR	0.17	0.18	0.19	0.03	0.04	0.07	0.25	0.24	0.22	0.01	0.07	0.04
		DRR.ES	0.17	0.19	0.19	0.03	0.04	0.07	0.25	0.24	0.22	0.01	0.07	0.05
	2018	RR	0.22	0.15	0.20	0.15	0.16	0.30	0.31	0.25	0.34	0.21	0.35	0.45
		DRR.VaR	0.17	0.13	0.18	0.14	0.15	0.24	0.24	0.19	0.27	0.16	0.26	0.34
		DRR.ES	0.18	0.13	0.18	0.14	0.16	0.24	0.24	0.19	0.27	0.16	0.27	0.34
	2019	RR	0.24	0.14	0.21	0.15	0.21	0.30	0.15	0.09	0.16	0.08	0.10	0.24
		DRR.VaR	0.19	0.12	0.17	0.13	0.18	0.24	0.13	0.08	0.13	0.08	0.08	0.20
		DRR.ES	0.19	0.12	0.18	0.13	0.18	0.24	0.13	0.08	0.13	0.08	0.09	0.20
	2020	RR	0.25	0.23	0.27	0.38	0.35	0.23	0.06	0.13	0.12	0.29	0.25	0.19
		DRR.VaR	0.19	0.18	0.20	0.30	0.27	0.18	0.06	0.11	0.11	0.22	0.19	0.15
		DRR.ES	0.19	0.18	0.20	0.30	0.27	0.19	0.06	0.11	0.11	0.22	0.19	0.15
2-4	2017	RR	0.25	0.29	0.31	0.04	0.13	0.06	0.33	0.30	0.31	-0.01	0.11	0.06
		DRR.VaR	0.21	0.23	0.24	0.04	0.11	0.05	0.24	0.22	0.24	0.01	0.10	0.06
		DRR.ES	0.21	0.23	0.24	0.04	0.11	0.04	0.24	0.21	0.24	0.01	0.10	0.06
	2018	RR	0.24	0.19	0.29	0.14	0.20	0.28	0.26	0.19	0.35	0.16	0.25	0.32
		DRR.VaR	0.19	0.15	0.23	0.11	0.17	0.23	0.20	0.15	0.27	0.12	0.19	0.25
		DRR.ES	0.19	0.16	0.24	0.12	0.17	0.23	0.20	0.15	0.27	0.13	0.19	0.25
	2019	RR	0.25	0.20	0.21	0.09	0.22	0.27	0.18	0.16	0.15	0.11	0.23	0.32
		DRR.VaR	0.20	0.16	0.16	0.07	0.18	0.22	0.16	0.12	0.13	0.08	0.13	0.25
		DRR.ES	0.20	0.17	0.17	0.08	0.18	0.22	0.15	0.12	0.14	0.09	0.17	0.26
	2020	RR	0.37	0.25	0.34	0.38	0.36	0.34	0.11	0.06	0.08	0.16	0.17	0.15
		DRR.VaR	0.28	-0.06	0.26	0.30	0.30	0.26	0.10	0.06	0.08	0.13	0.14	0.13
		DRR.ES	0.29	0.19	0.29	0.30	0.29	0.27	0.10	0.06	0.08	0.13	0.14	0.13
4-8	2017	RR	0.14	0.17	0.22	0.00	0.08	0.01	0.37	0.41	0.37	0.03	0.22	0.19
		DRR.VaR	0.12	0.15	0.18	0.01	0.07	0.02	0.28	0.30	0.30	-0.14	0.18	0.15
		DRR.ES	0.12	0.15	0.17	0.01	0.07	0.03	0.28	0.28	0.30	0.00	0.17	0.16
	2018	RR	0.26	0.14	0.39	0.15	0.20	0.32	0.32	0.15	0.52	0.23	0.27	0.42
		DRR.VaR	0.20	0.11	0.29	0.11	0.16	0.25	0.23	0.14	0.38	0.19	0.21	0.32
		DRR.ES	0.21	0.11	0.30	0.12	0.16	0.25	0.24	0.13	0.38	0.18	0.21	0.33
	2019	RR	0.23	0.17	0.14	0.09	0.11	0.24	0.15	0.06	0.08	0.04	0.13	0.29
		DRR.VaR	0.18	0.10	0.13	0.08	0.10	0.20	0.12	-0.19	0.08	0.05	0.09	0.22
		DRR.ES	0.18	0.14	0.11	0.09	0.11	0.20	0.12	0.04	0.07	0.10	0.11	0.22
	2020	RR	0.35	0.35	0.35	0.35	0.42	0.38	0.04	0.04	0.05	0.13	0.13	0.13
		DRR.VaR	0.26	0.27	0.26	0.28	0.32	0.30	0.03	0.05	0.06	0.10	0.10	0.12
		DRR.ES	0.26	0.27	0.26	0.28	0.32	0.30	0.04	0.04	0.06	0.10	0.11	0.12
8-16	2017	RR	0.18	0.19	0.09	0.02	0.07	0.04	0.26	0.32	0.15	0.00	0.17	0.07
		DRR.VaR	0.15	0.17	-0.54	0.09	0.05	0.03	0.18	0.26	0.11	-1.19	0.12	0.08
		DRR.ES	0.14	0.16	0.07	0.01	0.06	0.04	0.21	0.21	0.11	-0.01	0.11	0.06
	2018	RR	0.14	0.19	0.28	0.13	0.25	0.30	0.22	0.27	0.27	0.22	0.41	0.44
		DRR.VaR	0.08	0.14	0.22	0.10	0.20	0.29	0.65	0.17	0.11	0.17	0.30	0.32
		DRR.ES	0.12	0.14	0.22	0.10	0.20	0.22	0.12	0.20	0.19	0.15	0.36	0.32
	2019	RR	0.34	0.31	0.33	0.05	0.24	0.31	0.12	0.02	0.06	0.05	0.01	0.14
		DRR.VaR	0.26	0.24	0.25	-0.01	0.17	0.24	0.08	0.02	0.08	-0.01	0.02	0.13
		DRR.ES	0.26	0.24	0.25	0.05	0.18	0.23	0.09	0.03	0.08	0.05	0.03	0.12
	2020	RR	0.36	0.31	0.31	0.33	0.38	0.38	-0.04	0.05	-0.03	0.02	0.10	0.14
		DRR.VaR	0.26	0.23	0.23	0.25	0.27	0.28	0.00	0.09	0.00	0.03	0.08	0.11
		DRR.ES	0.26	0.23	0.23	0.24	0.30	0.30	-0.01	0.05	0.01	0.04	0.08	0.12

Table 3 (continued)

			BTC						GOLD					
Frequencies	Year		CHF	JPY	EURO	SP500	NIKKEI	SHSZ300	CHF	JPY	EURO	SP500	NIKKEI	SHSZ300
16-32	2017	RR	0.30	0.22	0.30	-0.03	0.09	0.01	0.10	-0.02	0.14	-0.02	-0.06	-0.03
		DRR.VaR	0.22	0.17	0.21	-0.02	0.07	-0.03	0.05	0.06	0.09	-0.02	-0.05	0.37
		DRR.ES	0.22	0.19	0.22	0.02	0.08	0.01	0.08	0.01	0.10	-0.01	-0.02	0.04
	2018	RR	0.00	0.02	0.00	0.06	0.07	0.14	0.45	0.35	0.46	0.45	0.60	0.65
		DRR.VaR	0.00	0.02	0.03	0.06	0.06	0.11	0.33	0.26	0.38	0.35	0.46	0.50
		DRR.ES	0.01	0.05	0.02	0.07	0.06	0.11	0.30	0.26	0.36	0.36	0.45	0.50
	2019	RR	0.07	0.05	0.02	0.01	0.04	0.16	0.11	0.10	0.10	0.13	0.21	0.42
		DRR.VaR	0.05	0.03	0.03	0.01	0.03	0.11	0.01	0.14	0.05	0.08	0.11	0.30
		DRR.ES	0.05	0.05	0.02	0.01	0.04	0.13	0.05	0.04	0.08	0.09	0.14	0.31
	2020	RR	0.44	0.45	0.46	0.38	0.48	0.55	0.04	0.04	0.09	0.09	0.07	0.10
		DRR.VaR	0.31	0.76	0.33	0.31	0.25	0.42	0.03	0.03	0.06	0.07	0.06	0.08
		DRR.ES	0.33	0.31	0.34	0.31	0.36	0.42	0.04	0.04	0.07	0.08	0.06	0.09
32-64	2017	RR	0.06	0.14	0.07	0.00	0.01	0.02	-0.05	0.36	0.01	-0.01	0.02	-0.07
		DRR.VaR	0.06	0.10	0.09	-0.05	0.02	-0.21	-0.23	0.06	-0.03	-0.01	-0.14	-0.85
		DRR.ES	0.03	0.11	0.05	0.04	0.02	0.01	-0.37	-0.38	0.12	-0.02	-0.02	-0.65
	2018	RR	0.08	-0.04	-0.05	0.02	0.06	-0.03	0.61	0.32	0.42	0.19	0.35	0.54
		DRR.VaR	0.04	-0.01	-0.98	0.02	0.01	-0.14	0.35	0.19	0.67	0.11	0.22	0.37
		DRR.ES	0.06	-0.03	-0.04	0.01	0.04	-0.01	0.41	0.25	0.39	0.22	0.29	0.37
	2019	RR	0.24	0.25	0.00	0.31	0.39	0.45	0.16	0.19	0.01	0.27	0.38	0.44
		DRR.VaR	0.17	0.15	-0.34	0.24	0.30	0.37	0.07	0.00	0.12	0.34	0.30	0.36
		DRR.ES	0.15	0.17	-0.26	0.23	0.26	0.35	2.00	0.18	0.02	0.19	0.23	0.32
	2020	RR	0.13	0.06	0.18	0.28	0.13	0.13	0.00	-0.01	0.12	0.26	0.17	0.05
		DRR.VaR	0.08	0.10	0.14	0.20	0.08	0.12	-0.06	-0.01	0.09	0.18	0.07	-0.01
		DRR.ES	0.08	0.04	0.14	0.19	-0.13	0.11	-0.01	0.00	0.09	0.12	-0.53	0.05
64-128	2017	RR	-0.13	-0.19	-0.15	0.00	-0.21	-0.04	0.35	0.16	0.22	-0.03	0.28	-0.01
		DRR.VaR	-0.29	-0.09	0.22	0.02	-0.06	-0.29	0.25	0.44	0.17	0.06	0.23	-0.01
		DRR.ES	-0.24	-0.12	0.00	0.07	-0.04	0.02	0.51	1.02	0.18	-0.01	0.18	-0.01
	2018	RR	0.31	0.12	0.06	-0.01	-0.11	0.16	0.62	0.41	0.38	0.06	0.41	0.30
		DRR.VaR	-0.80	0.13	0.09	0.01	-0.18	-0.05	-0.15	0.47	0.30	-0.01	0.32	0.10
		DRR.ES	0.21	0.16	0.00	0.02	-0.17	0.18	0.47	0.24	2.33	0.37	0.27	0.32
	2019	RR	0.28	0.23	0.14	0.43	0.49	0.43	-0.17	-0.03	-0.06	0.08	0.08	0.18
		DRR.VaR	0.33	0.13	0.08	0.34	0.32	0.75	0.24	-0.03	-0.02	-0.15	0.19	0.13
		DRR.ES	0.26	0.13	0.08	0.30	0.33	0.32	0.20	0.16	-0.01	0.01	0.05	0.12
	2020	RR	0.22	-0.01	0.22	0.45	0.62	0.27	-0.02	-0.01	0.06	0.36	0.58	0.19
		DRR.VaR	0.31	0.01	0.06	0.32	0.41	0.18	7.56	0.09	0.08	0.24	0.39	0.19
		DRR.ES	0.09	-0.02	0.11	0.31	0.40	0.05	-0.10	0.07	0.02	0.25	0.41	0.08
128-256	2017	RR	0.04	0.03	0.02	-0.01	-0.09	-0.04	0.31	0.51	0.26	0.05	-0.06	0.01
		DRR.VaR	0.06	0.11	0.02	0.03	-0.13	-0.11	0.22	0.53	1.04	0.00	0.58	-0.15
		DRR.ES	-0.02	0.01	0.00	-0.01	-0.07	-0.23	0.16	0.27	0.19	0.07	0.17	0.02
	2018	RR	-0.06	0.13	-0.04	-0.03	-0.11	0.17	0.00	0.02	-0.06	-0.08	-0.09	0.02
		DRR.VaR	-0.01	0.11	0.08	-0.12	1.02	0.15	-0.20	0.02	0.81	0.01	-1.77	-0.14
		DRR.ES	-0.49	0.14	0.14	0.02	-0.27	0.12	-0.04	0.00	-0.07	-0.11	0.67	0.58
	2019	RR	-0.01	-0.11	-0.01	-0.12	-0.05	-0.07	-0.07	-0.08	-0.03	-0.11	0.17	0.06
		DRR.VaR	0.26	-0.19	-0.11	-0.07	-0.07	0.80	0.48	-0.28	0.13	-0.03	0.07	0.47
		DRR.ES	0.00	0.22	-0.04	-0.16	-0.23	-0.03	-0.06	0.72	-0.37	-0.21	0.10	0.12
	2020	RR	0.01	-0.01	0.03	0.14	0.33	-0.01	0.03	0.01	0.05	-0.11	0.13	0.15
		DRR.VaR	-1.23	-0.27	-0.11	0.10	-0.17	-0.02	-1.51	1.11	0.10	-0.11	-0.12	-5.84
		DRR.ES	0.21	-0.08	-0.04	0.16	0.06	1.79	0.01	0.03	0.00	-0.05	0.03	0.09

4 Conclusions

In this paper, we apply the wavelet analysis to compute the variance and covariance in the time-frequency domain, to provide a new perspective for portfolio diversification allocation and risk management analysis. More specifically, we analyze the risk assessment in the time-frequency domain for bitcoin-to-currency (CHF, JPY, and EURO), bitcoin-to-stock (SP500, NIKKEI, SHSZ300), and gold-to-currency and gold-to-stock, for a total of 12 portfolios.

We can summarize the major findings of this study as follows. First, bitcoin, gold, and 3 currencies, all 5 assets' daily returns show large variance in the short to medium term in the early 2020s, except for bitcoin, which also has a large variance in the long to medium term from 2017 to 2018. And of the 3 stocks' daily returns, SP500 and NIKKEI have large variance not only in the short and medium-term in the first half of 2020 but also have a significantly large variance in the long term. The larger variance of the series in 2020 could be related to COVID-19 which has a huge impact on the economy in 2020 in the world.

Second, the correlation between the daily returns of gold and the three currencies is higher in the short to medium term, while the correlation between the daily returns of bitcoin and the three stocks is higher in the medium to long term in 2020 than the correlation between the daily returns of gold and the three stocks.

Third, by comparing the risk reduction spectrum of each portfolio and comparing the average of the three metrics (the risk reduction, the VaR downside risk reduction, and the ES downside risk reduction) for each year from 2017 to 2020, we conclude that the bitcoin and 6 asset portfolios can reduce the risk greater than the gold and 6 asset portfolios do after 2019, and especially after 2020.

Taken together, our findings suggest that gold, as a safe-haven asset, is being challenged by bitcoin after 2019, and especially after 2020, both for short-term and long-term investments. But notably, whatever asset portfolio is held should be carefully chosen for its holding duration.

In addition, wavelet analysis is widely known as a means of noise reduction. For further analysis, we can use wavelet noise reduction for each series, and then the new data obtained will be analyzed further.

References

Addison, P., 2002. *The Illustrated Wavelet Transform Handbook*. The Institute of Physics, London.

Al-Yahyaee, K.H., Mensi, W., Al-Jarrah, I.M.W., Hamdi, A., Kang, S.H., Volatility forecasting, downside risk, and diversification benefits of Bitcoin and oil and international commodity markets: A comparative analysis with yellow metal. *The North American Journal of Economics and Finance* 49, 104-120.

Baur, D., Lucey, B., 2010. Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *The Financial Review* 5 (3), 217–229.

Baur, D., McDermott, T., 2010. Is gold a safe haven? International evidence. *Journal of Banking and Finance* 34 (8), 1886–1898.

Beckmann, J., Berger, T., Czudaj, R., 2015. Does gold act as a hedge or a safe haven for stocks? A smooth transition approach. *Finance Research Letters* 31, 171-178.

Bredin, D., Conlon, T., Potì, V., 2015. Does gold glitter in the long-run? Gold as a hedge and safe haven across time and investment horizon. *International Review of Financial Analysis* 41, 320-328.

Goupillaud, P., Grossman, A., Morlet, J., 1984. Cycle-octave and related transforms in seismic signal analysis. *Geoexploration* 23, 85-102.

Grinsted, A., Moore, J.C., Jevrejeva, S., 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Process Geophysics* 11, 561-566.

Ji, Q., Bouri, E., Gupta, R., Roubaud, D., 2018. Network causality structures among Bitcoin and other financial assets: A directed acyclic graph approach. *The Quarterly Review of Economics and Finance* 70, 203-213.

Jin, J.Y., Yu, J., Hu, Y., Shang, Y., 2019. Which one is more informative in determining price movements of hedging assets? Evidence from Bitcoin, gold and crude oil markets. *Physica A: Statistical Mechanics and its Applications* 527, 121121.

Kang, S.H., McIver, R.P., Hernandez, J.A., 2019. Co-movements between Bitcoin and Gold: A wavelet coherence analysis. *Physica A: Statistical Mechanics and its Applications* 536, 120888.

Kroner, K.F., and Ng, V.K., 1998. Modeling Asymmetric Comovements of Asset Returns. *Review of Financial Studies* 11, 817-844.

Miyazaki, T., Toyoshima, Y., Hamori, S., 2012. Exploring the dynamic interdependence between gold and other financial markets. *Economics Bulletin* 32 (1), 37–50.

Morlet, J., Arens, G., Fourgeau, E., Giard, D., 1982. Wave Propagation and Sampling Theory, Parts I and II. *Geophysics* 47, 203-236.

Pal, D., Mitra, S.K., 2019. Hedging bitcoin with other financial assets. *Finance Research Letters* 30, 30-36.

Reboredo, J. C., 2013. Is gold a hedge or safe haven against oil price movements? *Resources Policy* 38 (2), 130–137.

Reboredo, J.C., Rivera-Castro, M.A., 2014. Gold and exchange rates: Downside risk and hedging at different investment horizons. *International Review of Economics and Finance* 34, 267-279.

Selmi, R., Mensi, W., Hammoudeh, S., Bouoiyour, J., 2018. Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold. *Energy Economics* 74, 787-801,

Torrence, C., Compo, G.P., 1998. A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society* 79, 61-78.

Urquhart, A., and Zhang, H.X., 2019. Is Bitcoin a hedge or safe haven for currencies? An intraday analysis. *International Review of Financial Analysis* 63, 49-57.

Wang, K., Lee, Y., Thi, T., 2011. Time and place where gold acts as an inflation hedge: An application of long-run and short-run threshold model. *Economic Modelling* 28, 806–819.