The Hicks-Morishima Approach Reconsidered

Another Look at the Interdependence of Several Markets

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I Michio Morishima and John R. Hicks: the Two Superstars of Economics

There are the two economics superstars who have greatly influenced my academic career. They are Michio Morishima, Osaka University and London School of Economics, and John R. Hicks, London School of Economics and Oxford University. To tell the truth, both of them already passed away. Academically, however, they seem to be still alive: indeed, their outstanding accomplishments will never be forgotten in economic science. The main aim of this paper is to reconsider and shed new light on a topic named the Hicks-Morishima approach to the interdependence of several markets. Although such approach has been rather neglected for some time, I am sure that even now, it is worthy of serious reconsideration, presumably giving a new guide toward an integrated grand theory of social science in the twenty-first century. No doubt, this should be the direction of future research to be strongly desired by both Morishima and Hicks in their late years.

When I was a graduate student at the University of Rochester, I was fortunate to meet with Mr. Masayoshi Hirota, a "very serious man with highly samurai spirits," who so fondly argued night and day with me about life philosophy, world matters, and of course

economic science. According to Hirota, we should not have any fears about the seemingly high levels of American universities. Osaka University seemed to be at least on a par with, and even better than, University of Rochester. Hirota proposed me to do a joint work on general equilibrium theory with the final aim of publishing academic papers in professional journals of high prestige. We worked very hard by utilizing every space of the Rochester campus, namely the university library, class rooms, student dining rooms, and the graduate student center. After some time of our joint research, there came our final product, Hirota and Sakai (1969), which aimed to open a new horizon in theoretical studies in production theory. Although our joint paper had a humble yet attractive title "On Substitution and Scale Effects in Production Theory," it was quite unfortunate that because of some minor technical reasons, it missed a publication opportunity.

In hindsight, I am sure that my joint research experience with Mr. Hirota, or "Osaka samurai scholar," has constantly given us a great stimulus for the execution of our research project on economic theory. Indeed, having been encouraged a great deal by the Hirota-Sakai cooperation, I myself continued to learn many advanced mathematical tools which were applicable not only to production theory, but also to consumption theory as well. The final result of my continuous research effort was the completion of my doctoral dissertation (1972) *Axiomatic Foundations of Consumption and Production Theories* submitted to The University of Rochester, with my main thesis advisor being Professor Lionel W. McKenzie. After finishing my Ph. D. work and publishing many theoretical papers in international journals, I could spend a happy life at Pittsburgh, in which Professor Asatoshi Maeshiro and his wife Kazuko kindly helped me in many ways. All those nice things happened to my rather long American life in the period 1968-1976.[1)]

I can vividly recall that at the very end of 1973, my important meeting with Michio Morishima took place in New York City. More exactly, he was invited to deliver a special lecture at Econometric Society North American Meeting at New York Hilton Hotel, with Professor Lawrence W. Klein of Pennsylvania serving as a respectable chairman. The audience filled the hall to overflowing. Professor Akira Takayama, a noted international economist, sat in the very front seat, taking so many pictures. All the people seemed to have a very happy time by enjoying the presence of the memorable trio, namely Morishima, Klein and Takayama. Then, Morishima stood up and delivered a historical speech: "Kark Marx is so great! I have no doubt that he is academically still alive after one hundred years of his death!" In an instant, the whole audience stood up and applauded his powerful speech.[2)]

1) In September 1972, I submitted my Ph.D. dissertation to the University of Rochester, with Professors Lionel W. McKenzie and James Friedman being thesis advisers. As we seen in Sakai (1972), the thesis consisted of the two parts. The first part carefully discussed axiomatic approaches to consumption theory, the second part mathematical foundations of production theory. I would like to point out that Chapter 3 in Part 2, titled *An Axiomatic Approach to Input Demand Theory*, was based on my joint work with Dr. Masayoshi Hirota with further refinements. Very fortunately, a still further revision of this Chapter was published in *International Economic Review*, which was first published by Michio Morishima of Osaka University and Lawrence Klein of the University of Pennsylvania.

2) More exactly speaking, Professor Morishima's invited lecture was titled *Marx in the Light of Modern Economic Theory*. It was nothing but a Walras memorial lecture at the North American Meeting of Econometric Society. Although it was first read at Hilton Hotel, New York, in 28 December 1973, it was later published as a leading article of *Econometrica* (see Morishima, 1974). It was really lucky that I myself was right there as an attentive audience. This constitutes one of never-to-be-forgotten memories in my long academic life.

In what follows, I would like to just pick up Hicks' great book *Value and Capital* (1939, 1946), carefully discussing how closely Hicks and Morishima have been connected by this masterpiece. A famous writer Naoki Komuro (2004) once remarked:

> During the Second World War, Morishima served as a naval cadet whose duty wasto break a secret code of the enemy. When he left home toward a battlefront, he carried an important book with himself. That book was nothing but a newly published text *Value and Capital* by Hicks, then a rising star economist of the United Kingdom, one of Japan's hostile countries. Nevertheless, Morishima, who faithfully conducted his secret mission of code breaking, dared to exert all his energy to fully understand the Hicks new book, and safely could return home
>
> (Komuro, 2004)

Later in his life, Morishima (1999) got a sudden surge of nostalgia for his youth:

> When I [Morishima] was a Kyoto University student just after the last war, I was fondly reading *Value and Capital* by J.R. Hicks. He discovered the following law of critical importance: "Let the excess demand for one good increase. Then, not only will its price will rise, but also the prices of other substitutive goods will rise, whereas the prices of complementary goods will decline." I noticed, however, that he carelessly neglected the undeniable truth that any economic activity including market trading had to operate at some distance in a space. My question which would naturally arise in my mind was how Hicks' market laws aforementioned would had to change by the introduction of distance and space. In my student days, I was literally swamped with such a very difficult problem.
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> (Morishima, 1999)

It was quite fair to say that in those days, Morishima stood high and alone in the Japanese academia: in fact, the research level of Morishima went far beyond the one of any ordinary Japanese economist. He was a man of courage and determination. The major aim of this paper is to reconsider Hicks' law of market exchange in a new modern fashion, thus further extending the Hicks-Morishima approach with my unique flavor attached to it.

I have met with Hicks several times. My most memorable moment took place when I attended the International Conference in Honor of J.R. Hicks, Bologna, Italy, in 1991. It gave me a great and unforgettable memory.

The outline of this paper is as follows. The second section will deal with s simple model of general equilibrium — the excess demand approach a la Hicks and Morishima. The third section will discuss many non-normal cases. The issues of instability, multiple equilibrium and non-existence will be successively taken up. In the fourth section, we will turn to the problem of comparative statics. The impact of parameter changes on equilibrium values will carefully be investigated. Final remarks will be made in the fifth section, with reference to Morishima's life-long dream of a grand synthetic approach to a so-called "symphonic economics."

II A Simple Model of General Equilibrium: The Excess Function Approach a la Hicks and Morishima

Saying is one thing. But doing is another. Now let us discuss the working and performance of a simple model of general equilibrium. To this end, we will adopt the excess demand approach a la Hicks and Morishima. The Hicks-Morishima approach is a very effective one to shed light on interrelation of excess demand curves in the price space for an in investigation of the working and performance of general equilibrium. Although we focus on a simple two-good model in this paper, its generalization to any several good model would require no difficult task.

Honestly speaking, in spite of its rather simple and ambitious framework, it is quite unfortunate that the Hicks-Morishima approach has been long neglected in the academic circle. I suppose that there are several reasons for this. First of all, the traditional general equilibrium approach, developed by Lionel W. McKenjie, Gerald Debreu and Kenneth J. Arrow, exclusively works with the goods space rather than the price space. In contrast, the Hicks-Morishima approach exclusively operates on the price space, thus against the current main stream of economic theory. Next, the majority of economics readers are usually familiar with the straightforward notion of demand and supply curves, but not with the twisted concept of excess demand curves. And of course, many people prefer simplicity to complexity.

I suppose that Morishima would apparently agree with those reasons. In order for the excess demand approach to become very familiar to the academic circle, we would have to do a sort of extraordinary work for publicity and sales effort. Morishima had special courage to write a new style of economics text (1984) *The Economics of Industrial Society*, in which he adopted the Hicks-Morishima approach again. Unfortunately, history repeated itself : it again received a rather dull response. Since I regard myself as an "academic son" of Morishima and also as an "academic grandson" of Hicks, I have to be strong in the neglected stream of academia, strongly arguing usefulness and applicability of the Hicks-Morishima Approach.

2-1 Are A Pair of Two Goods Substitutive or Complementary ?

In reality, there seem to be many diversified markets which are more or less interlocked with each other. If we take a simple example from a female clothes market, it is true that skirt and jeans markets are mutually interrelated. Their relations, however, are much more complicated that a man in the street would usually imagine. The demand for female cloths may be influenced by so many possible factors. First of all, a young stylish girl may want to be dressed in the height of fashion: indeed, being behind the fashion would be the most terrible thing to do. If wearing slim jeans, but not wide skirt, becomes more fashionable than before, then there would be so many young girls who want to change their fashion style from skirt to jeans. Besides, if skirt becomes more expensive due to an increased cost of fabric, then there would be a tendency that the sale of jeans as a

good substitute of skirt will rise. Moreover, the overall demand for cloths, jeans or skirt, is determined by the purchasing power of buyers. When the economic conditions become improved (or worsened), people want to spend more (or less) money for clothing, so that the sales of skirt and jeans will overall tend to go up (or down).

This may not be all of the story. Let us recall that we live in the world of risk and uncertainty. If we consider people's forecasts and expectations, the demand and supply of a good should get much more complicated.

For instance, even if the price of a skirt has risen "this week", no one can exactly tell what will happen to that price "next week." If a young girl expects it to rise higher next week, she has a good incentive to buy it this week. Besides, even if her income stays sluggish this month, yet she expects it to go up significantly the next month, she might feel rich and affluent even now, possibly showing positive buying behavior. Similar reasoning will be applicable to the supply side, not only the demand side. In general, if the trader expects the sale to be sluggish this year, but to be increasing next year, he is likely to positively respond to his optimistic forecast.

To sum up, human beings are complicated, and so are markets. In fact, a skirt market and a jeans market are so intricate that they cannot simply be analyzed. It is the Hicks-Morishima approach that can shed a nice analytical light on the entangled interdependence of several markets. In historical perspective, that approach was originated in Chapter 5 of Hicks (1939, 1946), and later highly developed in Morishima (1984) in many possible ways. In our regret, however, time seems to pass too slowly to recognize a masterpiece: indeed, the magnificent Hicks-Morishima approach has been long undervalued or neglected. As I have said repeatedly, it is my sincere desire to revive the approach once more and hopefully attach some new findings and flavors to it.

2-2 A Simple Two-Good Model : The Starting Point

At the starting point of discussion, let us begin our market model analysis with a simple case of two goods, x_1 and x_2. More specifically, let us suppose that x_1 stands for tea and x_2 coffee. Further assume that the price of x_1 and the one of x_2 are respectively denoted by p_1 and p_2

It is natural to assume that the amount of demand for x 1 is dependent not only on p_1, but also on p_2. Since tea and coffee are presumably in competitive relations, they are formally regarded as "substitutive" in microeconomic theory. Therefore the demand function, D_1, for good 1 may be formulated as follows:

$$D_1 : x_1 = -p_1 + 0.5\, p_2 + 3 \qquad (1)$$

It is supposed here that there are two different routes by which the amount of demand of coffee, x_1, rises. The first route is the direct one by means of a fall in p_1, the price of coffee per se. This may be called the "own effect" of a price change. The second route is the indirect one in the sense that the price rise of tea as a substitute of coffee causes the change of demand from tea to coffee. This may be named the "cross effect" of a price change. As is clearly seen in Eq. (1), it is supposed that the first own effect overpowers the second cross effect.

Concerning the supply side, we assume that its direction of change is exactly the oppo-

site of the demand side. More specifically, let us consider the following linear supply function:

$$S_1 : x_1 = p_1 - 0.5\, p_2 - 1 \qquad (2)$$

Let the price of coffee rise. Then, we obtain the following two effects. First, the distributor intends to increase the supply of coffee in the market. This is the "own effect" of a more expensive coffee. Second, when the price of tea falls, the distributor not only wishes to sell tea less, but also he tends to sell coffee more. This is the "cross effect" or the "spill over" effect over the two markets. Besides, it is supposed here that the first own effect overwhelms the second cross one.

We are now in a position to introduce the concept of an excess demand function. Note that excess demand means "demand minus supply": namely, $E_1 = D_1 - S_1$.

Thus, in the light of Eqs. (1) and (2), we can easily derive the excess demand function for x_1 as follows:

$$E_1 = D_1 - S_1 : x_1 = 2p_1 + p_2 + 4 \qquad (3)$$

This demonstrates that the amount of excess demand for good 1 is *inversely* proportional to its price (the own effect), whereas the former amount is in direct proportion to the price of good 2 (the cross effect). Besides, understandably, the own effect overpowers the cross one.

Market equilibrium of good 1 is realized when its demand and supply are just equal, namely its excess demand completely vanishes. It is at this equilibrium point that the price of good 1 ceases to change. On the one hand, if the demand for good 1 exceeds its supply, then the excess demand occurs, implying that the price of good 1 must go up. On the other, if the demand is less than the supply, then the excess supply occurs, so that the price has to fall. This is a famous adjustment process a la Leon Walras, which may be summarized as follows:

$E_1 > 0 : D_1 > S_1$(disequilibrium)$\rightarrow p_1$ goes up.
$E_1 = 0 : D_1 = S_1$(equilibrium)$\rightarrow p_1$ does not change.
$E_1 < 0 : D_1 = S_1$(disequilibrium)$\rightarrow p_1$ goes down.

Let us take a look at Fig. 1. In the left chart (A), the straight line on the price plane represents the excess demand function: $-2\,p_1 + p_2 + 4 = 0$. (Note that this function can also be rewritten as $p_2 = 2p_1 - 4$.) Since the two goods are substitutes, the line is positively sloped. Moreover, reflecting the fact that the own effect overpowers the cross effect, the slope of the straight line for p_1-axis should be steeper than the one for p_2-axis , so that the former slope must exceed 45 degree.

Suppose that a pair of prices (p_1, p_2) lies right on the straight line E_1. Then, p_1 does not change at all, and maintains the same value as before. Clearly, the excess demand line $E_1 = 0$ divides the whole price plane into two areas. In the *left* area where $E_1 > 0$, p_1 rises in a direction indicated by the right arrow ($\rightarrow$). In contrast, in the *right* area where $E_1 < 0$, p_1 falls in an opposite direction indicated by the left arrow ($\leftarrow$).

A similar argument will apply for good 2, or coffee. So, we can formulate the demand , the supply and the excess demand functions in the following way:

$$D_2 : x_2 = 0.5\,p_1 - 2p_2 + 4 \quad (4)$$
$$S_2 : x_2 = -0.5\,p_1 + p_2 - 2 \quad (5)$$
$$E_2 = D_2 - S_2 : x_2 = p_1 - 3p_2 + 6 \quad (6)$$

This teaches us that the amount of excess demand for good 2 is inversely proportional to its price p_2 (the own effect), whereas the former amount is in direct proportion to p_1, the price of good 1 (the cross effect). Besides, as before, the own effect overpowers the cross one.

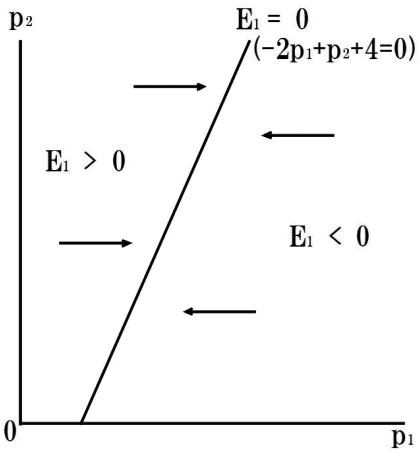
The Walras-type adjustment process for good 2 may be formulated analogously:

$E_2 > 0 : D_2 > S_2$(disequilibrium)$\rightarrow p_2$ rises.
$E_2 = 0 : D_2 = S_2$(equilibrium)$\rightarrow p_2$ does not change.
$E_2 < 0 : D_2 < S_2$(disequilibrium)$\rightarrow p_2$ falls.

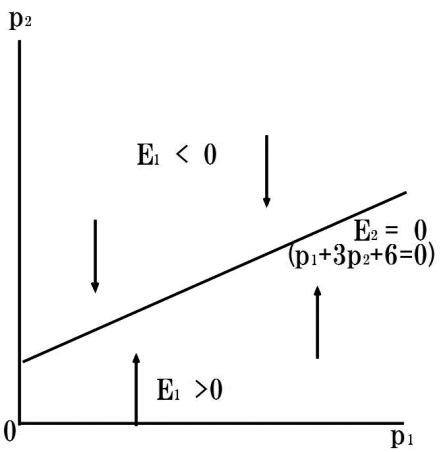
Let us take a look at Fig. 1 again. In the right chart diagram (B), the straight line $E_2 = 0$, or $p_1 - 3p_2 + 6 = 0$, represents the excess demand for good 2. Note that this line may be rewritten as $p_1 = 3p_2 - 6$. This is the case in which the two goods are substitutes and each own effect is dominating, so that the excess demand line is positively sloped and its "slope for p_2-axis" must exceed 45 degree. The directions of price adjustment p_2 are indicated by arrows in the figure.: namely, p_2 rises in the lower area, but falls in the upper area.

2-3 General Equilibrium of Two Markets: The Simplest Case

We are now ready to get into the world of "general equilibrium" with the two goods, in which the demand and supply of each good are just equal and the changes of prices stop moving. In other words, general equilibrium looks like a stationary state.



(A) Excess demand for x_1 :changes in p_1 and p_2



(B) Excess demand for x_2 : changes in p_1 and p_2

Fig. 1 The Relation between the Two Markets

Now, let us carefully investigate what happens if the two chart diagrams (A) and (B) of Fig. 1 as well as the corresponding sets of arrows are superimposed. Then, we will immediately see that a very charming float diagram like Fig. 2 appears. At the intersection point Q of the two excess demand lines, $E_1 = 0$ and $E_2 = 0$, the following two equations must be satisfied :

$$-2p_1 + p_2 + 4 = 0 \quad (7)$$
$$p_1 - 3p_2 + 6 = 0 \quad (8)$$

Solving Eqs. (7) and (8) for p_1 and p_2 simultaneously, we can see that a pair of coordinates of the intersection point Q is shown by $Q = (18/5, 16/5)$. Therefore, the equilibrium prices of p_1 and p_2 are respectively 18/5 (= 3.6) and 16/5 (= 3.2).[3)]

To sum up, the intersection point Q indicates the point of "general equilibrium" of the two markets. Remarkably, it should be a stable point as well. Note that any price movements may be shown by the arrows of the pair (p_1, p_2). In whatever direction the price pair may move in Fig. 2, it is eventually destined to converge at the equilibrium point Q.

III Many Non-Normal Cases: Instability, Multiple-Equilibrium, and Nonexistence

3-1 The Saddle-Point Equilibrium: Substitutable Goods yet Dominating Cross Effects

As the saying goes, saying is one thing but doing is really another. All the foregoing analyses have just dealt with the ideal, utopian two-

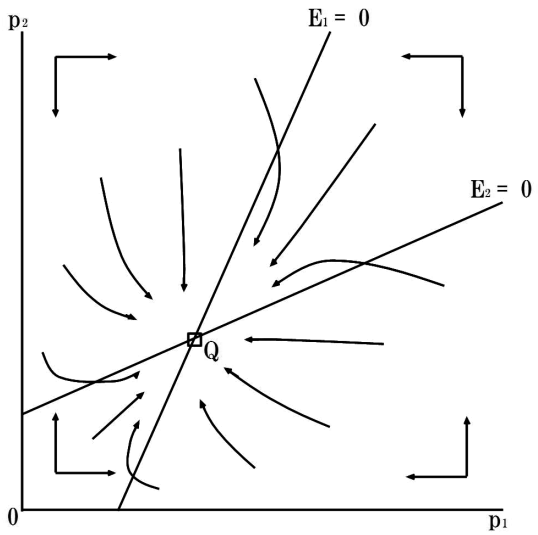


Fig. 2 The existence and stability of the two-good markets: An ideal state

3) A group of chart diagrams like Fig. 2 were first introduced by Hicks (1939). Morishima who admired Hicks so much intended to make the chart diagrams revived, with the aim of establishing his own "new economics." To our regret, however, such revival of the Hicks-Morishima approach has not been very successful so far. It is our real intention that we give the approach one more chance.

good world in which the only one point of general equilibrium uniquely exists and its stability is also guaranteed. One might wonder if such utopian world really exists. When we are getting into the real world as it is, it is far from perfect, with a lot of irregularities. For instance, general equilibrium might not exist at all. There would be more than one equilibrium. Besides, many other "irregular situations" including unstable points and/or saddle points might occur.

Suppose that there is a market equilibrium with substitutable goods. Then, the most important question to ask is whether and to what extent the stability of the equilibrium is guaranteed. Interesting enough, the equilibrium is not always stable. In fact, we will see that the cross effects of price changes act as disturbing factors of instability. In fact, if the cross effects are strong enough to outweigh the own effects, then the equilibrium point will become a non-stable saddle point. To clarify such an interesting point, let us focus on the case that the excess demand functions are of the following forms:

$$E_1 : E_1 = -p_1 + 2p_2 - 4 \quad (9)$$
$$E_2 : E_2 = 3p_1 - p_2 - 6 \quad (10)$$

The excess demand lines of good 1 and good 2 are depicted in Fig. 3. Since we are dealing with substitutable goods, it follows that both lines should be positively sloped. There is an important point we should not miss, however. Note, the cross effects are strong enough to outweigh the own effects. Indeed, in Eq. (9) above, the absolute value 2 of the coefficient of p_2 is greater than the one 1 of the coefficient of p_1 , whereas in Eq. (10), the absolute value 3 of

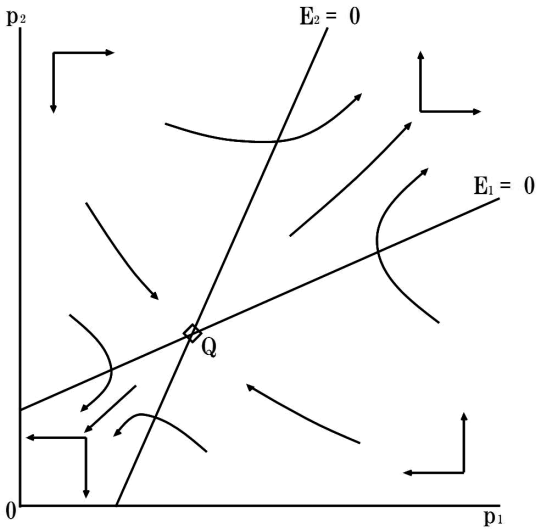


Fig. 3 The equilibrium point is an unstable saddle point: The cross effect is so strong

the coefficient of p_1 exceed the one 1 of the coefficient of p_2.

As a result, in sharp contrast to the idealistic case discussed above, the slope of E_1- line is steeper for p_2-axis, whereas the one of E_2- line is steeper for p_1- axis.

Let compare Fig. 2 and Fig. 3. Then, we clearly understand that the positional relationship of the lines $E_1 = 0$ and $E_2 = 0$ has been reversed. Consequently, such reversal of locations would critically change the adjustment processes of prices.

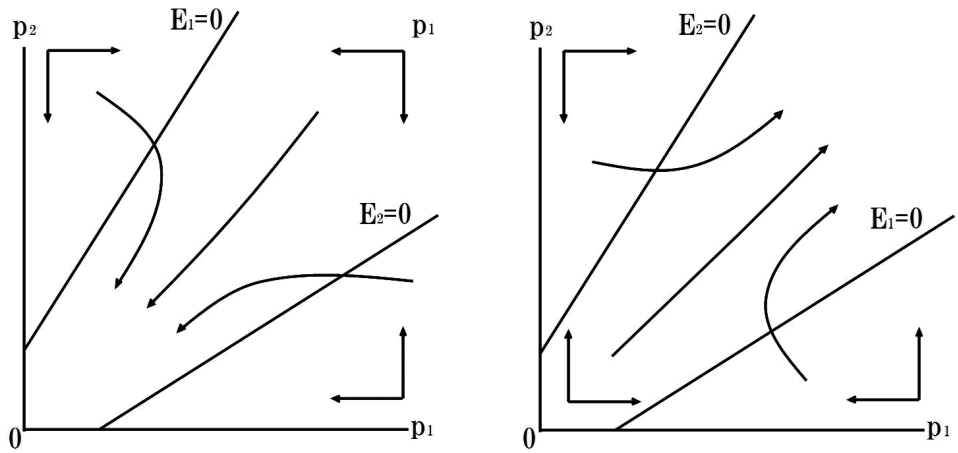
The coordinates of equilibrium point is $Q = (16/5, 18/5)$. Apart from the previous discussion, however, we can no longer say that this point is a stable point in its true sense. In fact, it should be a saddle point that is unstable on almost all paths. As is seen in Fig. 3, except only for the two stable paths, one path going down from the North-West to Q, and another going up from the South-East to Q, all other possible paths are not stable at all. This means that the saddle point Q should substantially be unstable.

3-2 Non-Existence of the Two-Market Equilibrium

What we should consider next is the emergence of "non-normal situations." Since all of them are of vital importance, they respectively require for very careful investigations. As the first example, let us examine the case in which the excess demand functions are written as follows:

$$E_1 : E_1 = -2p_1 + p_2 - 2 \quad (11)$$
$$E_2 : E_2 = p_1 - 2p_2 - 2 \quad (12)$$

Let us take a look at Fig. 4. As is seen in Chart (A), the two excess demand lines, $E_1 = 0$



(A) The prices are declining to zero

(B) The prices are expanding forever

Fig. 4 Non-existence of equilibriums

and E_2= 0, never intersect at all in the positive coordinates of the price space. If we dared to solve for p_1 and p_2 in Eqs. (11) and (12), we would obtain a pair of NEGATIVE prices (p_1, p_2) = (–2, –2), which should not economically be meaningful. Eventually, as is shown in Chart (A), both prices, p_1 and p_2 , tend to eventually decline toward zero along the directions of arrows.

If we interchange the positions of the two lines, $E_1 = 0$ and $E_2 = 0$ in the left chart (A), then we immediately obtain those lines in the right chart (B). Then, we have to deal with the following set of equations:

$$E_1 : E_1 = p_1 + 2p_2 + 4 \quad (11)$$
$$E_2 : E_2 = 3p_1 - p_2 + 6 \quad (12)$$

In Chart (B), there exist no intersections of the two lines in the positive price quadrant. If we dared to simultaneously solve $E_1 = 0$ and $E_2 = 0$, then we would find (p_1, p_2) = (–16/5, –18/5), a pair of negative and economically meaningless prices. As a result, there are no general equilibrium solutions, and the prices eventually tend to rise forever as indicated by arrows.

3-3 The Case of Complementary Goods : A Completely New Situation

In the above, we have limited our investigation to the case of competitive or substitutable goods. Looking at the real world, however, we can eyewitness many other cases in which some goods may not be mutually competitive but rather move in the same direction. For instance, while tea and coffee are substitutes, tea and sugar are complementary, and so are coffee and sugar.

As the first example of complementary goods, let us discuss the following excess demand functions of the two goods:

$$E_1 : E_1 = 2p_1 - p_2 + 4 \quad (13)$$
$$E_2 : E_2 = - p_1 - 3p_2 + 6 \quad (14)$$

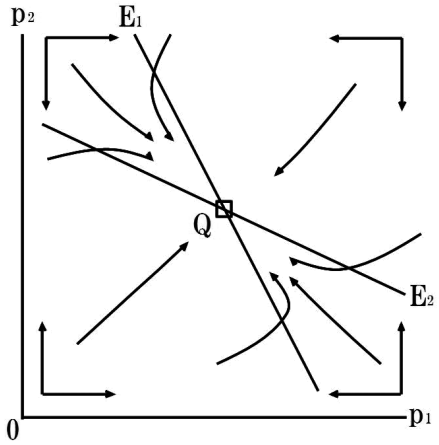
Let take a close look at Eq. (13) and (14). The, we will see that in the right-hand sides of those equations, the values of coefficients of p_1 – term and p_2 – term are both negative. This means that both the excess demands for x_1 and x_2 have to increase when p_1 falls, or when p_2 falls. This implies that the two goods are complementary, not substitutes. Besides, we assume here that the own effects overpowers the cross effects. Therefore, we would rightly conjecture that the equilibrium point Q , if it exists, must be stable. Surely, such conjecture will turn out be right by looking at the left chart (A) of Fig. 5.

Concerning Fig. 5, there is one more point to watch. We must make sure that the two excess demand lines, E_1 and E_2, are now NEGATIVELY sloped. Since the own effects dominate the cross effects in Chart (A), it follows that both the slope of E_1 for p_1 – axis and the slope of E_2 for p_2 – axis should be steeper.

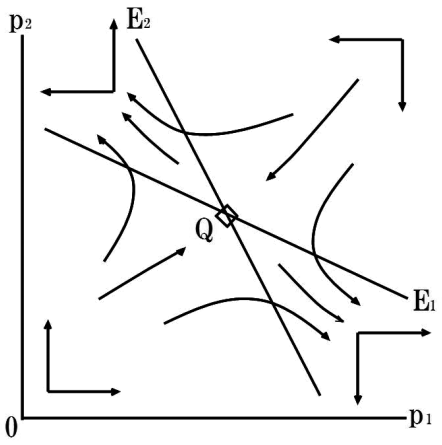
We will turn to the second case which is represented by the following pair of excess demand functions:

$$E_1 : E_1 - p_1 - 3p_2 + 6 \quad (15)$$
$$E_2 : E_2 - 2p_1 - p_2 + 4 \quad (16)$$

Now, the cross effects are now more powerful than the own effects. As can easily be



(A) Point Q is stable since the own effects are dominating



(B) Point Q is unstable since the cross effects are dominating

Fig. 5 The two goods are complementary

seen in Chart (B), the slope of E_1 for p_2 – axis (not for p_1 – axis) is steeper while the the one of E_2 for p_1 – axis (not for p_2 – axis) is steeper. Therefore, the equilibrium point Q is an almost unstable saddle point.

In conclusion, the power relation between the own effects and the cross effects plays a vital role in the determination of the stability of an equilibrium point. Whether the goods are substitutes or complementary, the dominance of the own effects over the cross effects clearly guarantees the stability of an equilibrium. On the contrary, if the cross effects dominate the own effects, then an equilibrium point becomes an almost unstable saddle point.

IV Comparative Statics: The Impact of Parameter Changes on Equilibriums

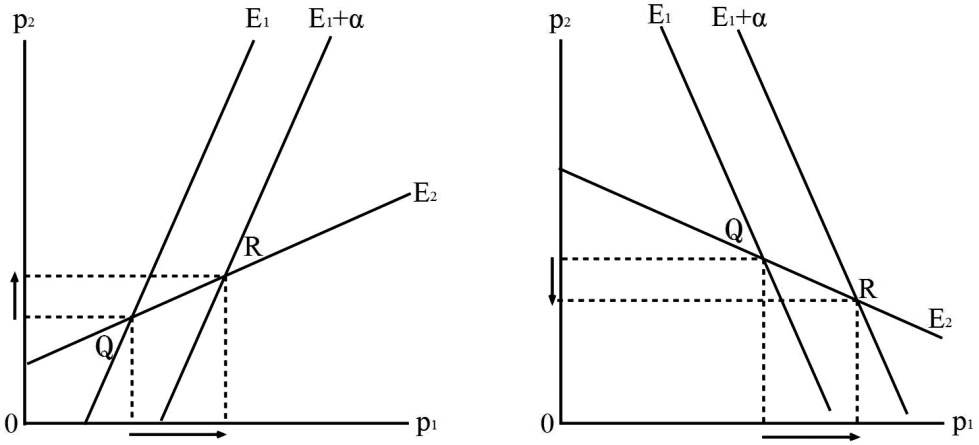
Let us turn our attention to the issue of comparative statics. This is a very important problem that was first intensively discussed by J.R. Hicks and later greatly developed by Michio Morishima.

First of all, let us assume that the two goods are substitutes like tea and coffee. Let us take a look at Fig. 7. In the left chart (A), the straight lines E_1 and E_2 respectively represent the excess demand lines of good 1 (tea) and good 2 (coffee). More specifically, let us consider the following set of equations:

$$E_1 : E_1 = -2p_1 + p_2 + 4 \quad (15)$$
$$E_2 : E_2 = p_1 - 3p_2 + 6 \quad (16)$$

If we put $E_1 = 0$ and $E_2 = 0$ and solve for p_1 and p_2, we find a pair of equilibrium prices: in fact, $(p_1^Q, p_2^Q) = (3.6, 3.2)$.

Now suppose that because of some reasons, the popularity of tea suddenly rises among the people. Then, the excess demand curve E_1 has



(A) The goods are substitutable (B) The goods are complimentary

Fig. 7 Comparative statics: changes in excess demand

to shift to the right. This implies that regarding the constant term of Eq, (15) above, it has to change its value from 4 to $(4 + \alpha)$:

$$\text{A change in } E_1 : E_1 + \alpha = -2p_1 + p_2 + 4 + \alpha \tag{17}$$

Let us take a look at Fig. 7. In Chart (A), the new equilibrium point R is found at the intersection of the two lines, $E_1 = 0$ and $E_2 = 0$. For convenience, let us specify $\alpha = 2$. Then, the coordinates of R are calculated as $(p_1^R, p_2^R) = (4.8, 3.6)$.

4-1 Hicksian Three Laws of Price Changes: The Case of Substitutable Goods

In his classic *Value and Capital*, Hicks (1939, 1946) succeeded in deriving the very famous three laws which stood for the core of his comparative static results. It was also remarkable to see that those Hicksian laws were later developed a great deal by his successor, Morishima (1984). It is the Hicksian laws that we will intensively investigate in this and following subsections.

We are most interested in comparing the two equilibrium points, the old point Q and the new point R. We are expected to obtain several results of great importance. First of all, we would like to pay attention to comparison of the following two relations:

$$p_1^R = 4.8 > 3.6 = p_1^Q \tag{18}$$
$$p_2^R = 3.6 > 3.2 = p_2^Q \tag{19}$$

To begin with, let us suppose that because of some reasons, tea becomes very popular among the people, whence its demand suddenly increases. To take an example, the opening of the Great Britain Exhibition in Kyoto may produce such a sudden surge for the popularity

of tea in Japan. Then, its immediate effect we can expect to have is that tea will be more expensive than before in response to the demand increase. Not only that, there would also be many other repercussions such as its indirect impact on other markets including the coffee market. Besides, those influences will invoke the third and forth rounds of repercussions as well. Those direct and indirect effects would make our repercussion calculations very hard and complicated. In order to make sure of this point, if we patiently compute the *proportional rates of increase* of the prices (in terms of per cent), then we will be able to derive the following equations:

$$(p_1^R - p_1^Q) / p_1^Q = (4.8 - 3.6) / 3.6 = 1/3 \quad (20)$$

$$(p_2^R - p_2^Q) / p_2^Q = (3.6 - 3.2) / 3.2 = 1/8 \quad (21)$$

Clearly, the ratio $(p_1^R - p_1^Q) / p_1^Q$ is greater than the ratio $(p_2^R - p_2^Q) / p_2^Q$, implying that the rising rate of tea exceeds the one of coffee.

We must bear in mind that whether or not the excess demand functions are linear, all the comparative static results derived above generally hold. J.R. Hick first obtained all those results which Michio Morishima rightfully called the "Hicksian Three Laws of Price Changes." They are really outstanding consequenses of the Hicks-Morishima approach, which can be summarized as the following theorem.

Theorem 1
(Hicks-Morishima on substitutable goods)

Assume that the two goods, x_1 and x_2, are substitutable and that the own effects overpower the cross effects. Then the point of general equilibrium, if it exists, must be stable. When the excess demand for good 1, E_1, increases, the following three properties must hold:

(i) The price of good 1, p_1, will rise.
(ii) The price of good 2, p_2, will rise too.
(iii) The price of good 1 will rise proportionately more than the price of good 2; namely, the price ratio (p_1 / p_2) will rise.

4.2 The Case of Complementary Goods: Hicksian Three Laws Continued

Now, let us turn to the case of complementary goods Let us assume that goods 1 and 2 respectively represent tea and sugar. Then, as Chart (B) of Fig. 7 above tells us, the two excess demand lines, E_1 and E_2, are both downward sloped. If we suppose that the own effects outweigh the cross effects, then the old intersection point Q becomes a stable point.

Now, suppose that a sudden tea boom happens among the people, resulting in a drastic increase in tea demand. This is the first direct effect of tea price rise. Moreover, such rise will lead to a sequence of secondary repercussions since it will dampen the demands for both tea and sugar. Recall here that complementary goods have to go in the same direction. This is the second indirect effect. If we employ a similar reasoning, then we would also take account of third and fourth round of repercussions that must have less influence than the second round.

In Chart (B), let us shift the excess demand function to the right, from E_1 to $E_1 + \alpha$. Then, since such shift changes the equilibrium point in a right and downward direction, from Q to R, it will have to cause a big rise in p_1 together with a small change in p_2. Consequently, in absolute value terms, the rising rate of p_1 will exceed the falling rate of p_2. We omit a more detailed calculation here.

The final results for the case of complementary goods will be summarized as the following theorem:

Theorem 2
(Hicks-Morishima on complementary goods)

Suppose that the two goods, x_1 and x_2, are complementary, and that the own effects overpower the cross effects. Then, the point of general equilibrium, if it exists, must be stable. When the excess demand for good 1, E_1, increases, the following three properties must hold:

(i) The price of good 1, p_1, will rise.
(ii) The price of good, p_2, will fall a bit.
(iii) The rising rate of good 1 will be greater than the falling rate of good 2; namely, price ratio (p_1 / p_2) will rise a great deal.

As I mentioned before, the exact derivation of Theorems 1 and was the main theme of the Young Morishima. For that purpose, he literally racked his brains. In this paper, I have intended to limit my analysis on a simple yet useful case of *linear* excess demand functions. Besides, following the Morishima spirits, I have often made use of many eye-catching graphical approaches. I can say, however, that the extension of my analysis to the more general nonlinear cases should not be so difficult, requiring for a bit more calculation and patience.

V Morishima Towards a New Economics

This paper may be regarded as an hommage to my two great mentors, Professors John R. Hicks and Michio Morishima. The Hicks-Morishima approach was adopted and developed by Morishima in his book (1984). For this book, Morishima gave an interesting subtitle "An Introduction to New Economics." Because it was far more than "an introduction" and contained "several new and original results," it seemed to me that the subtitle per se was "too modest, and even cheating." In fact, Morishima himself made several remarks :

> The approach of this book differs to a considerable degree from standard ecnomics. ... This book attempts to analyze the price mechanism in accordance with reality and at the same time to introduce students directly to the major problems of economics – an analysis of the way in which the real economy operates and the best way to bring about a change in direction in this operation.
> (Morishima, 1984, Preface)

Morishma's later book (1999) could be thought of as one of his "last swans," in which he greatly lamented the recent Japanese society as one approaching at a "dangerous stage." Here again, he made some intriguing remarks:

> This book is the one I have constantly wanted to write for a long time. In fact, it is a sort of a "Grand Integral Social Science" in the

sense that it contained economics, sociology, education, history and many other related fields. It could rightly be called an "Academic Symphony." .. Let us take a careful look at many non-economic fields such as the worlds of politics, religion, and ideology. Then you will see that many main actors such as political, religious , and military bosses may play very large roles in our society. Unfortunately, economics and other related fields are so powerless to investigate those important problems. So, I should say that the existing social science remains in an under-developed state, thus lacking its very essential core." (Morishima, 1999, Appendix)

Honestly speaking, the right way toward the "Grand Integral Social Science" a la Morishima seems to be too far for me to take. Even a right direction for the road has not been found yet. A few years ago, when I published my book (2010) *The Economic Thought of Risk and Uncertainty* written in Japanese, I sent one copy to Professor Tetsuya Nose, a close friend of Morishima. Nose immediately replied to me, "This looked like 'an academic symphony' a la Morishima! " This was really the highest praise on my academic work.

I sincerely hope that this paper would serve very well as one small step toward Morishima's dream of establishing the Grand Integral Social Science. I strongly believe in the following golden saying: "Life is an adventure! Life is really a challenge! "

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The Hicks-Morishima Approach Reconsidered

Another Look at the Interdependence of Several Markets

Yasuhiro Sakai

This paper aims to shed some new light on the Hicks-Morishima approach to the interdependence of several markets. In spite of its rather simple and ambitious framework for the interdependence of several markets, it is quite unfortunate that this approach has been rather neglected in the academic circle. I suppose that there are several reasons for this. First, the traditional general equilibrium approach developed by Lionel W. McKenzie, Gerald Debreu and Kenneth J. Arrow exclusively works with the good space rather than the price space. In contrast, the Hicks-Morishima approach based on Hicks' classical book *Value and Capital* exclusively operates on the price space, thus against the current main stream of economic theory. Next, the majority of economics readers are usually familiar with the straightforward notion of demand and supply curves, but not with the twisted concept of excess demand curves. It is one of my main purpose to mend such unfortunate tendency, presumably proceeding toward the establishment of a new grand system of social science. We can learn new lessons from old teachings.

Keywords: J.R. Hicks, M. Morishima, *Value and Capital*, excess demand curves, general equilibrium analysis, comparative statics

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