Articles

Information Exchanges among Firms and Their Welfare Implications (Part I)

The Dual Relations between the Cournot and Bertrand Models

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I A. A. Cournot as the Great Founder of Oligopoly Theory —An Introduction

There is a historical episode by Joseph A. Schumpeter (1883-1950), one of the greatest economists in the 20th century. When he was a distinguished professor at Harvard University in the 1930s, he surprised all the attending students, one of whom was the young Paul A. Samuelson (1915-2005), by saying the following remark:[1)]

Listen, everybody. In the light of the history of economic science, it seems to me that there exist four great economists. Believe or not, three out of those four are Frenchmen! Can you guess who they really are?"

According to Schumpeter's opinion, F. Quesnay (1694-1774), A. A. Cournot (1801-77) and M. E. L. Walras (1834-1910) were strong candidates for such an exclusive economics club. Surely, Quesnay was very famous of newly inventing an economic table as the flow chart of all the economic activities in a national economy. Cournot was a born mathematician, later applying a mathematical approach to oligopoly theory. Walrus was regarded as a pioneer of modern general equilibrium theory, thus being admired so much by Lionel W. McKenzie (1923-2008) when I myself was a graduate student at the University of Rochester in the 1960s.

There remains one question unanswered: who should be the last of the four great economists according to Schumpeter's preference? One thing is for certain. The last person

1) See Yasui (1979).

should not be a Frenchman. He could have been Adam Smith (1723-90), the author of the epoch-making book *The Wealth of Nations* (1776). Or possibly, Carl Menger (1840-1921), an important member of the influential Austrian School? Or J. G. K. Wicksell (1851-1926), the founder of the outstanding Swedish School? Or J. M. Keynes (1883-1946), the author of the revolutionary book *The General Theory* (1936)? Or perhaps Schumpeter himself? No one really knows. Fortunately or unfortunately, Schumpeter kept his mouth shut until his death, thus contributing to the creation of a great mystery in the history of economic thought.

There was another interesting episode which connected Schumpeter with Cournot, when Schumpeter was teaching economics at the University of Bonn long before the Second World War. The following question was asked by Schumpeter to the young Nakayama, who was then a visiting foreign student at Bonn and later became a leading professor of modern economics in the Japanese academics. "Mr. Nakayama, please let me know how you have managed to study economics before coming to Germany. Nakayama's answer was simple, yet gave Schumpeter a really nice surprise, "Yes, Sir. I have carefully read the works of Cournot, Gossen and Walras under the direction of my Japanese mentor."

Honestly speaking, in spite of his monumental works, Cournot has been mostly underestimated with a few exceptions. One outstanding exception was Schumpeter, who as mentioned before, very highly evaluated Cournot. In such long survey of papers, we would like to gladly share this Schumpeter spirits, thus critically evaluating and freely extending Cournot's work on oligopoly to the world of risk and uncertainty . In this connection, it is also worthy of attention to record the following sentence by J. R. Hicks (1904-94), one of the greatest economists in the 20th century:

The generally increased interest in mathematical economics during the last few years has naturally turned attention back to the work of Cournot, the great founder of the subject, and still one of best teachers. It was Cournot's creation of elementary monopoly theory which was the first great triumph of mathematical economics; yet Cournot had left much undone. It is not surprising that the endeavor to complete his work have been an attractive occupation for his successors. (Hicks, 1935, p. 1).

As J.R. Hicks noted in his survey paper (1935), Cournot was regarded as the great founder of the theory of monopoly and oligopoly, and still one of the best teachers in the 1930s. It was quite fortunate rather than unfortunate that Cournot had left much undone, thereby the endeavor to make his work complete has been continuously an attractive task for his successors until today. It is my sincere hope that I will be one of his good successors.

More exactly speaking, almost 180 years have passed since the publication of Cournot's epoch-making book *Recherches sur les principes mathématiques de la theorie des richesses* (1938). It is appropriate as well as important to see how and to what extent Cournot's pioneering work has contributed to the economics profession. One of the main goals of this paper is to show that Cournot is academically alive and indeed very much alive, and continue to be so.

Cournot used to be called an insolent founding father. Even before the Marginal Revolution in the 1870s, he invented marginal concepts such as marginal revenue, marginal cost, demand elasticity, and the like. More than 100 years before the appearance of Game Theory, he made full use of a very important concept of equilibrium in non-cooperative games — Nash equilibrium. It is also worth mentioning that it took as many as 45 years for Cournot's great book to be reviewed by Bertrand (1883).

The present paper aims to overview and evaluate the problem of information exchanges in oligopoly models, one of the most fashionable topics in contemporary economics. It is intended to discuss a synthesis of the two important fields, the economics of imperfect competition and the economics of imperfect information.[2)]

Needless to say, the issue of information transmissions and exchanges among producers is important not only from a theoretical point of view, but also from antitrust policy implications. In the real economy, there exist several types of institutions in which producers exchange their private information with each other. Trade associations are among those information-pooling mechanisms. In order to determine under what conditions the information exchanges among producers should be encouraged or discouraged in terms of the consumer welfare or the total social welfare, it is first necessary to fully understand the working and performance of oligopoly markets under the conditions of imperfect information.

Let us consider a homogenous or a differentiated product market. Then this paper address to the following set of questions. First of all, are firms with different demand and/or cost functions willing to reveal or share information about demand or cost? Next, how and to what extent such information transmission affect consumers or the whole society? Finally, are the welfare implications of information exchanges sensitive to the number of participating firms?

There exist a growing number of papers that discuss those questions. The line of research was initiated by Basar and Ho (1974) and Ponssard (1979), and continued by the explosion of works in the 1980s including Clark (1983), Vives (1984), Okada (1984), Sakai (1985, 86, 87, 89) , Gal-Or (1985, 86, 87), Shapiro (1986), and many others. Besides, there have appeared a number of remarkable papers in the 1990s and even in the 2000s.[3)]

At the first glance, there appear no definite answers in the existing literature, so that the antitrust implications of information sharing in oligopoly might be far from clear. In some papers, firms are assumed to behave as Cournot competitors whereas in others, they are regarded as Bertrand competitors. There may exist a common risk or private (firm-specific) risks. Risk may be about the demand or cost side. Products may be homogenous or differentiated. Even if differentiated, they may be substitutes, independent or complements. When there exist more than two sources of risks, they may be positively or negatively correlated. Besides, the number of participation firms may be just two, three, ..., or any finite number.

It is generally expected that different models leads to different consequences. The problem of information exchanges in oligopoly models has no exception for such a universal rule.

2) The imperfect competition revolution took place in the 1930s, with J. Robinson (1933), Chamberlin (1933) and Stackelberg (1934) being its front runners. We would add to say that another equally important revolution — the imperfect information revolution — happened in the 1970s in which Arrow (1970), Akerlof (1970), Stiglitz (1975a, 75b), and Spence (1974) were primary promoters. The nature and significance of this new revolution was intensively discussed by Sakai (1982). For the economic thought of risk and uncertainty, see Sakai (2012).

3) See Sakai (1990), Vives (1990, 1992, 1999, 2001) and Kühn & Vives (1994).

Once a specific set of assumptions is made to describe an oligopoly model to work with, however, a definite set of answers will be obtained. The following set of items must be checked; (1) the type of competitors (Cournot-type or Bertrand-type), (2) the type of risks (demand or cost risks), (3) the nature of risks (a common risk or private risks), (4) the degree of physical and stochastic correlation among firms (positively or negatively correlated) , and (5) the number of firms (two, three or any finite number). In this paper, a wide variety of oligopoly models will successively be introduced, and the problem how a change in one of those assumptions may result in a corresponding change in some of welfare results will be the focus of investigation. For the sake of presentation and also subject to the space constraint, however, little or no attention will be paid to some other related issues such as those of risk aversion, measurement errors, partial sharing, garbling, first-mover versus second-mover advantages and the like.

While there may exist many possible models regarding information exchanges in oligopoly models as mentioned above, it is remarkable to see that there is only one mathematical approach to such problems, namely the approach based on game theory. As is well-known, game theory has played a key role in integrating the two branches of economics into one, the Economics of Imperfect Competition and the Economics of Imperfect Information. In fact, a recent body of work in oligopoly has been associated with the application of many concepts borrowed from game theory, with the concept of Nash equilibrium continuing to be a dominant one.[4)]

Even if each of various models aforementioned is set up on the stage, it is no easy task to systematically analyze all the welfare effects of information exchanges among firms, and to provide clear-cut and intuitive interpretations for the results. When the problem at issue is too complicated to seize the essence of the matter, it is a well-established wisdom to break it into several pats, and to examine the welfare results piecewise before knitting them together. As will be seen, the consequences of information exchanges among firms can be classified under the four headings; namely, own and cross variation effects, and own and cross efficiency effects.

Interestingly enough, the first effect or the own variation effect represents how information flows affect the variability of each firm's strategic variable (each output for Cournot models, or each price for Bertrand models), whereas the second effect or the cross variation effect shows how it influences the degree of strategic interdependence among firms. The third and fourth effects demonstrate in which direction information exchanges contribute to the efficiency of resources on an industry-wide basis. In particular, whereas the own efficiency effect is related to a better or worse correspondence between each stochastic parameter (the demand intercept or the unit cost) and its associate strategic variable, the cross efficiency effect is connected with a changed correspondence between the stochastic parameter of one firm and the strategic variable of the other firm. It will be seen that those four effects provide quite useful tools by which to trace the welfare implications of information exchanges among firms.

4) The theory of games was first invented as the joint product of a born mathematician, von Neumann, and a great economist, Morgenstern (1944), and later developed by the man with "beautiful mind," Nash (1951). For its applications to oligopoly problems, see Shubik (1980), J.M. Friedman (1977, 86), and more recent works by many others.

As is seen in the contents mentioned above, the long series of papers consist of three parts: namely, Part I, Part II, and Part III. The present paper corresponds to Part I, and aims to set up a basic framework of differentiated duopoly in the absence of any risks. The dual relations between the Cournot and Bertrand models will carefully be investigated. More specifically, it is noted that the Cournot model with substitutable (or complementary) goods is the dual of the Bertrand model with complements (or substitutes). Part I also prepares for later discussions on extensive comparisons of more general oligopoly models on the basis of the type of competitors, the type of risks, and the number and nature of risks.

Part II will be discussed in the next paper. It begins to deal with the world of risk and uncertainty, with a discussion of the most fundamental model: the Cournot duopoly model with a common demand risk. It will be followed by the same Cournot model with a common cost, and by the corresponding Bertrand models with a common demand or cost risk. Part III will be the target of the last paper of the series. In this last part, many possible extensions of the duopoly results to very general oligopoly situations will be carried out.

II Alternative Duopoly Models with and without Risk Factors

2-1. The Dual Relations between the Cournot and Bertrand Duopoly Models

— The World of Perfect Information

As was noted above, there exist two types of competition (Cournot or Bertrand), two more types of risk (demand or cost), and still two more types of information structures (a common value or private values). Therefore, when all the possible combinations are considered, eight different types of oligopoly models will have to be discussed. This could probably constitute a very repetitious and even tiresome task. Fortunately, there would be a great help from the duality argument! Indeed, as will be seen below, there exist the nice dual relations between the Cournot and Bertrand models in the world of perfect information.

Let us pick up any two models. Then if they share the same formal structure and differ only in the interpretation placed on variables and parameters, we say that they are dual. One natural consequence of such duality argument is that a proposition derived for one model can become a proposition for the other if variables and parameters are duly interchanged between the two models. In the light of the history of economic thought, it is Cournot himself who was close enough yet fell short of adopting what we may now call a dual approach to oligopoly theory.

Cournot has established the important proposition that the output supplied under duopoly is greater than the output supplied under pure monopoly. Since the market demand curve is usually downward sloping, this implies that the price charged under duopoly is lower than the price charged under pure monopoly.

While there is the wide range of physical interdependence between two outputs, Cournot discussed only the two extreme cases, namely, the case of perfect substitutes and the one of perfect complements. This clearly indicates the limitations of the original analysis of Cournot, thus showing the necessity to extend it to a wider range of intermediate cases between those of perfect substitutes and perfect complements.[5)]

The model we are going to analyze here is the following non-stochastic duopoly model with differentiated products and/or cost differences. On the production side, we have a duopolistic sector with firms 1 and 2, each one producing a differentiated product, and a competitive numéraire sector. Let x_0 be the output of the numéraire good, x_i be the output of the i th firm, and p_i be its unit price ($i = 1, 2$). The unit price of x_0 is of course unity, namely $p_0 = 1$.

On the consumption side, we have a continuum of consumers of the same type with utility functions which are linear and separable in the numéraire good. For tractability, it is assumed that the utility function U of the representative consumer is quadratic:

$$U = x_0 + \alpha_1 x_1 + \alpha_2 x_2 - (1/2)(\beta x_1^2 + 2\beta\theta x_1 x_2 + \beta x_2^2), \qquad (1)$$

where α_i and β are all positive, and the value of θ lies between -1 and 1.

Let us assume that the consumer is supposed to maximize U subject to the budget constraint, $x_0 + p_1x_1 + p_2 x_2 = m$, where is m denotes his given income. Then it can easily be seen that inverse demand functions are given by the following set of linear equations:

$$p_1 = \alpha_1 - \beta x_1 - \beta\theta x_2, \qquad (2)$$
$$p_2 = \alpha_2 - \beta x_2 - \beta\theta x_1. \qquad (3)$$

If we use matrix notation, we can summarize (2) and (3) as follows:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} - \beta \begin{bmatrix} 1 & \theta \\ \theta & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Now, assuming that $\alpha_1 - \alpha_2\theta > 0$ and $\alpha_2 - \alpha_1\theta > 0$, let us put

$$a_1 = (\alpha_1 - \alpha_2\theta) / \beta(1 - \theta^2),$$
$$a_2 = (\alpha_2 - \alpha_1\theta) / \beta(1 - \theta^2),$$
$$b = 1 / \beta(1 - \theta^2).$$

It is then easy to see that these newly introduced parameters, a_1, a_2 and b, are all positive. In the light of (2) and (3), it is not hard to obtain the following set of *direct* demand equations:

$$x_1 = a_1 - bp_1 + b\theta p_2, \qquad (4)$$
$$x_2 = a_2 - bp_2 + b\theta p_1. \qquad (5)$$

Alternatively, in matrix notation, we have

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} - b \begin{bmatrix} 1 & -\theta \\ -\theta & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}.$$

It is noted that the value of θ stands for a good measure of the substitutability of the two products. In fact, x_1 and x_2 can be regarded as substitutes, independent, or complements according to whether θ is positive, zero, or negative.

We assume that the technology exhibits constant returns to scale, so that firm i has constant unit cost κ_i. Profits of firm i are provided by $\Pi_i = (p_i - \kappa_i)\, x_i$. It is noted that Π_i is not symmetric in p_i and x_i unless κ_i vanishes. In general, the Π_i functions treat $(p_i - \kappa_i)$ and x_i symmetrically. In order to make symmetric treatment clearer, it is instructive to reformulate (2)–(5) in the following way:

5) There is now a growing body of literature dealing with the working and performance of oligopoly markets under product differentiation, centering around the duality and efficiency comparison between Cournot and Bertrand equilibriums. For its earlier works, see Singh & Vives (1984), Vives (1984), Okuguchi (1987), and Sakai (1986).

$$p_1 - \kappa_1 = (\alpha_1 - \kappa_1) - \beta x_1 - \beta\theta x_2\,, \qquad (2^*)$$
$$p_2 - \kappa_2 = (\alpha_2 - \kappa_2) - \beta x_2 - \beta\theta x_1\,, \qquad (3^*)$$
$$x_1 = (a_1 - b\kappa_1 + b\theta\kappa_2) - b(p_1 - \kappa_1) + b\theta(p_2 - \kappa_2)\,, \qquad (4^*)$$
$$x_2 = (a_2 - b\kappa_2 + b\theta\kappa_1) - b\,(p_2 - \kappa_2) + b\theta\,(p_1 - \kappa_1). \qquad (5^*)$$

In matrix notation, these four equations may be rewritten as follows:

$$\begin{bmatrix} p_1 - \kappa_1 \\ p_2 - \kappa_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 - \kappa_1 \\ \alpha_2 - \kappa_2 \end{bmatrix} - \beta \begin{bmatrix} 1 & \theta \\ \theta & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 - b\kappa_1 + b\theta\kappa_2 \\ \alpha_2 - b\kappa_2 + b\theta\kappa_1 \end{bmatrix} - b \begin{bmatrix} 1 & -\theta \\ -\theta & 1 \end{bmatrix} \begin{bmatrix} p_1 - \kappa_1 \\ p_2 - \kappa_2 \end{bmatrix}.$$

As is well-known, the Cournot equilibrium is the Nash equilibrium in outputs, whereas the Bertrand equilibrium is the Nash equilibrium in prices. In view of equations (2*)–(5*) or their matrix notations aforementioned, the dual relations between the Cournot and Bertrand models are given in Table 1.

There is an outstanding duality between the Cournot and Bertrand equilibriums: Cournot equilibrium with substitute (or complementary) outputs is the dual of Bertrand equilibrium with complements (or substitutes). Once the Cournot equilibrium strategies are determined, the Bertrand equilibrium strategies are also given by the duality argument. All we have to do is to replace x_i with $(p_i - \kappa_i)$, $(p_i - \kappa_i)$ with x_i,

$(\alpha_i - \kappa_i)$ with $(a_i - b\kappa_i - b\theta\kappa_j)$, β with b, and θ with $(-\theta)$ $(i, j = 1,2;\ i \neq j)$.

More specifically, the equilibrium concept we are going to use in this paper is the application of Nash equilibrium to many oligopoly models of Cournot and Bertrand types. In the absence of any risks, we say that the pair (x_1^C, x_2^C) of output strategies is an equilibrium if the following conditions are met:

$$x_1^C = \text{Arg Max}_{x_1}\ \Pi_1\,(x_1\,, x_2^C)\,,$$

$$x_2^C = \text{Arg Max}_{x_2}\ \Pi_2\,(x_1^C, x_2).$$

When the Cournot equilibrium is reached, no firm has an incentive to deviate from it. It is noted here that each firm's profit is given by the following equations:

$$\Pi_1(x_1, x_2) = (\alpha_1 - \kappa_1 - \beta x_1 - \beta\theta x_2)x_1\,,$$

$$\Pi_2(x_1, x_2) = (\alpha_2 - \kappa_2 - \beta x_2 - \beta\theta x_1)x_2\,.$$

Then the reaction functions of firms 1 and 2 are provided by

Table 1 The Dual Relations between the Cournot and Bertrand Models

Variables and Parameters	Cournot Model	Bertrand Model
Strategic Variables	x_1	$p_1 - \varkappa_1$
	x_2	$p_2 - \varkappa_2$
Dependent Variables	$p_1 - \kappa_1$	x_1
	$p_2 - \kappa_2$	x_2
Parameters	$\alpha_1 - \kappa_1$	$a_1 - b\kappa_1 + b\vartheta\kappa_2$
	$\alpha_2 - \kappa_2$	$a_2 - b\kappa_2 + b\vartheta\kappa_1$
	β	b
	θ	$-\theta$

$R_1^C : x_1 = (1/2\beta)\,(\alpha_1 - \kappa_1 - \beta\theta x_2)\,,$

$R_2^C : x_2 = (1/2\beta)\,(\alpha_2 - \kappa_2 - \beta\theta x_1)\,.$

The Cournot duopoly equilibrium under no risks can easily be depicted in Fig. 1. There are the two charts (A) and (B) in the figure. The left chart (A) indicates the case of substitutes (namely, $\theta > 0$) in which the reaction curves are negatively sloping.

In contrast, the right chart (B) shows the case of complements (i.e., $\theta < 0$) where the reaction curves are positively sloping.

As mentioned above, the Bertrand equilibrium with price strategies constitutes the dual of the Cournot equilibrium with output strategies. Therefore, we say that the (p_1^B, p_2^B) of price strategies is an equilibrium if the following conditions are satisfied:

$p_1^B = \text{Arg Max}_{p_1}\ \Pi_1(p_1\,, p_2^B),$

$p_2^B = \text{Arg Max}_{p_2}\ \Pi_2(p_1^B\,, p_2).$

Since we have

$\Pi_1(p_1, p_2) = (p_1 - \kappa_1)\,(a_1 - bp_1 + b\theta p_2),$

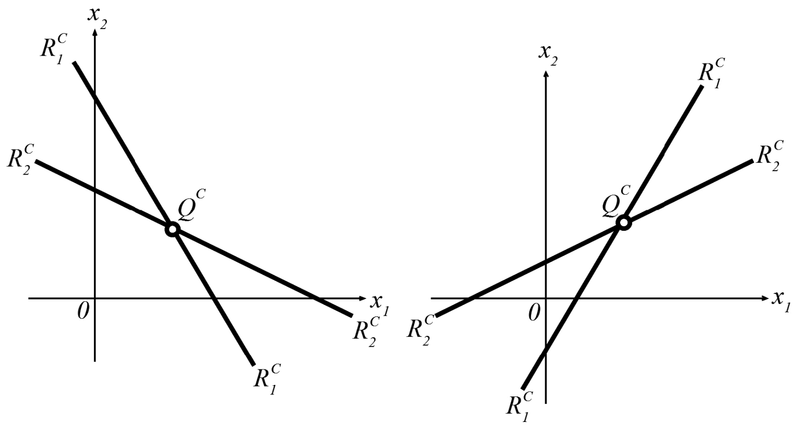
$\Pi_2(p_1, p_2) = (p_2 - \kappa_2)\,(a_2 - bp_2 + b\theta p_1)\,,$

the reaction functions of firms 1 and 2 are given by

$R_1^B : p_1 = (1/2b)\,(a_1 + b\kappa_1 + b\theta p_2)\,,$

$R_2^B : p_2 = (1/2b)\,(a_2 + b\kappa_2 + b\theta p_1)\,.$

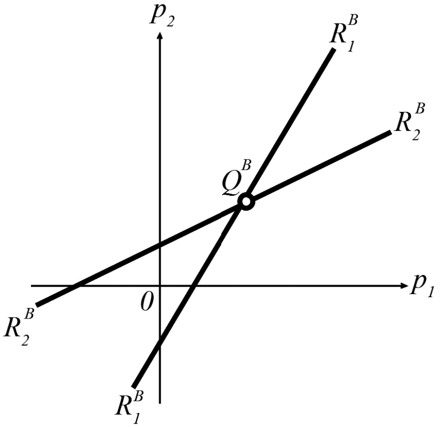
We can depict the Bertrand duopoly equilibrium under no risks in Fig. 2. The left chart (A) stands for the case of substitutes ($\theta > 0$), in which the reaction curves are positively sloping. The right chart (B) corresponds to the case of complements ($\theta < 0$), where the reaction curves are negatively sloping.



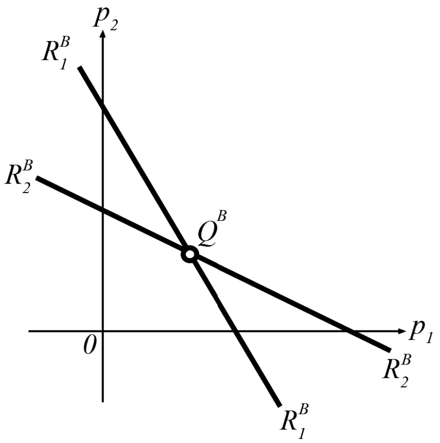
(A) $\theta > 0$:
the Case of Substitutes

(B) $\theta < 0$:
the Case of Complements

Figure 1 Cournot Duopoly Equilibriums under No Risks



(A) $\theta > 0$:
the Case of Substitutes



(B) $\theta < 0$:
the Case of Complements

Figure 2. Bertrand Duopoly Equilibriums under No Risks

Comparison of Fig. 1 and Fig. 2 shows the existence of dual relations between Cournot and Bertrand. It is quite interesting to see that Charts (A) and (B) of Fig. 1 respectively correspond to Charts (B) and (A) of Fig. 2 if price and output variables are interchanged.

The duality argument is very convenient and really powerful. However, it should not be almighty. It may sometimes break down. In fact, as will be seen below, when we discuss consumer surplus, it surely breaks down!

It is a rather common practice in economics that consumer surplus is measured by $CS = U - x^0 - \sum_i p_i x_i$. Therefore, if we make use of (1) – (3), we find the following CS formula:

$$\begin{aligned} CS &= (1/2) \sum_i (\alpha_i - p_i)\, x_i \\ &= (1/2) \sum_i \{\alpha_i - (p_i - \kappa_i)\}\, x_i \\ &\quad - (1/2) \sum_i \kappa_i\, x_i. \end{aligned} \qquad (6)$$

It is easy to see that the formula does not treat x_i and $(p_i - \kappa_i)$ symmetrically. Consequently, the duality argument applies only to profits and producer surplus, but not to consumer surplus and total surplus at all.

2-2. Introducing Risk Factors into Alternative Duopoly Models

We are now ready to introduce risk factors and investigate how the presence of demand or cost risk affect the working and performance of an oligopoly market. The problem here is that there are so many ways of introducing stochastic factors into our model, depending on the type of risk (demand or cost, a common value or private values) faced by firms.[6)]

First of all, let us assume that risk is about the demand side. For simplicity, suppose that α_1

6) In the light of the history of economic thought, there have been so many ways of introducing risk and uncertainty into economic models. For details, see Sakai (2010).

and α_2 are now random variables, and may be described in the following way:

$$\alpha_1 = \alpha + \varepsilon_1 , \ \alpha_2 = \alpha + \varepsilon_2 . \qquad (7)$$

Here α denotes a stochastic demand common to all the firms, where as ε_i shows a stochastic demand specific to the i th firm (i = 1, 2). It is noted that ε_1 and ε_2 may be positively or negatively correlated. For instance, suppose that x_1 and x_2 respectively represent "one week trip in New York and Boston" and one week trip in California" as two attractive goods in the tourist industry. Then α may mean the fluctuations of the yen-dollar exchange, whereas ε_1 and ε_2 respectively show the weather in the Eastern Coast and the one in the Western Coast.

It is recalled that the Cournot and the Bertrand models are dual. If α_1 and α_2 are stochastic parameters in the former model, so are a_1 and a_2 in the latter model. As was stated above, the relations between these two set of stochastic parameters must be indicated by the following formulas:

$$a_1 = (\alpha_1 - \alpha_2\theta)/\beta(1 - \theta^2),$$
$$a_2 = (\alpha_2 - \alpha_1\theta)/\beta(1 - \theta^2). \qquad (8)$$

Now, let us turn our attention to the case in which risk is about the cost side. Assume that κ_1 and κ_2 are stochastic parameters, being written as follows:

$$\kappa_1 = \kappa + \tau_1, \kappa_2 = \kappa + \tau_2. \qquad (9)$$

Here κ stands for a stochastic cost common to all the firms, whereas κ_i shows a stochastic cost specific to the i th firm (i = 1, 2). It is noticed that τ_1 and τ_2 may be positively or negatively correlated. For instance, let us consider the fluctuations of oil prices in the world. The common parameterκmay represent the dollar/yen exchange rate which fluctuates frequently but influences every firm's cost at the same ratio. Assume that τ_1 and τ_2 respectively stand for the imported price of Iraq oil and the one of Venezuela oil. The Iraq oil and the Venezuela oil may rise or decline in the same direction or in opposite directions, depending on the domestic conditions of each country.

The question which would naturally arise is whether or not the nice relationship between the Cournot and Bertrand models remain intact in the presence of risk. On the one hand, if risk is about the demand side, the parameters α_1 and α_2 are random in the Cournot model, the parameters a_1 and a_2 are random in the Bertrand model (see Table 1). As a result, the introduction of risk, whether it is common or firm-specific, does not change the dual relation between these two models.

On the other hand, if risk is about the cost side, a completely new situation will emerge since the simple duality argument is no longer applicable. As can be seen in Table 1, when κ_1 and κ_2 are random variables, they affect not only parameters but also *dependent* variables in the Cournot model, whereas they influence parameters as well as *strategic* variables in the Bertrand Model. Therefore, the way how cost risk changes the relations between strategic and dependent variables in the Cournot model must be different from the way how it changes these relations in the Bertrand model. So when cost risk is introduced into an oligopoly model, the Cournot equilibriums with substitute (or complementary) outputs are no longer the dual of the Bertrand equilibriums with complements (or substitutes).[7)]

In short, the duality argument is powerful, but not almighty. As common sense tells us, it may be helpful in some situations, it may not be so in other situations. This shows the necessity for differentiating the case of demand risk from the one of cost risk.

III Concluding Remarks

In the above, we have intensively discussed the information sharing in oligopoly and its welfare implications. It is hoped that such discussions will lead to a synthesis of the economics of imperfect competition and the economics of imperfect information.

At the memorial Third Congress of the European Economic Association at the University of Bologna in 1988, Gray-Bobo as an invited speaker impressed so many people by saying the following:[8)]

A 150 years old book, written 15 years after Ricardo's death by an almost entirely isolated man, can be so brilliantly argued that some of its parts are still discussed today. (Gray-Bobo, 1988, p. 2)

Almost 30 more years have passed since Gary-Bobo's interesting remark. It is true that Augustin A. Cournot spent his isolated life as a first-rate mathematician, later applied differential and integral calculus to the problem of oligopoly. His courageous attempt to synthesize powerful mathematics and practical economics , however, may now be regarded as a towering landmark in the history of economic thought. It would be fair to say that Cournot is so great because his doctrine is still alive after 180 yeas of its first publication.

In the absence of no risks, there exist the remarkable dual relations between Cournot and Bertrand oligopoly models. In fact, the Cournot equilibrium with substitutable (or complementary) outputs is the dual of the Bertrand equilibrium with complements (or substitutes). Once the Cournot equilibrium strategies are determined, so are the Bertrand equilibrium strategies by the duality argument. It is really one of main purposes of this paper to discuss whether and to what extent introduction of risk factors into the Cournot or Bertrand models would influence such duality analogy.

On the one hand, if risk is about the demand side, the introduction of risk, whether is of common type or of firm-specific type, does not change the dual relation between the two models. On the other side, if risk is about the cost side, a completely new situation has to emerge since the simple duality argument is no longer applicable.

In conclusion, we can say that the duality argument is powerful, but not almighty. It may be useful in some situations, but it may not be so in other situations. We must differentiate the case of demand risk from the one of cost risk.

In this paper, we have worked with admittedly simple oligopoly models with or without conditions of risk. We do believe, however, that the results obtained in this paper are fundamentally robust. Much work remains to be left for future research.

7) For this point, see Sakai and Yamato (1989, 1990). The usefulness and limitations of the duality argument must always be kept in mind. Everything has a sunny side as well as a shady side.

8) See Gray-Bobo (1988), page 20. I still remember how much I was excited when I read my paper on a new topic on oligopoly theory before the huge audience at the University of Bologna, Italy. I really left my heart in the presumably oldest university in the world. We can surely learn new lessons from old teachings!

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Information Exchanges among Firms and Their Welfare Implications (Part I)

The Dual Relations between the Cournot and Bertrand Models

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This long series of papers consist of three parts. Part I is concerned with the basic dual relations between the Cournot and Bertrand models. Part II begins to deal with the world of risk and uncertainty, with a discussion of the Cournot duopoly model with a common demand risk as a starting point. It then deals with other types of duopoly models with a common risk. Part III discusses more complicated problems such as private risks and oligopoly models. All these three parts taken together aim to carefully outline and critically evaluate the problem of information exchanges in oligopoly models, one of the most important topics in contemporary economics.

The present paper corresponds to Part I. We intend to give a synthesis of the economics of imperfect competition and the economics of imperfect information. The problem at issue is how and to what extent the information exchanges among firms influence the welfare of producers, consumers and the whole society. In the real world, trade associations may be regarded as typical information exchange mechanisms. It is hoped that the welfare implications obtained in the paper will shed a new light to the effectiveness and limitations of the industrial policies adopted by governments.

Key words: Information exchange, oligopoly models, welfare implications, dual relations, trade associations.